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Low-Velocity Moderation of Σ^- Hyperons in Hydrogen†

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A measurement of the low-velocity moderation of Σ^- hyperons in hydrogen has been performed in a bubble-chamber experiment. The technique employed determines very accurately the Σ^- velocity at decay, $\Delta\beta = \pm 0.004$, by using $\Sigma^- \rightarrow \pi^- + n$ decays accompanied by neutron scatters with visible proton recoils. The results of the study, based on 23 200 Σ^- leading to 151 events with recoils in the region $\beta_{\Sigma^-} < 0.10$, indicate that the Σ^- moderation time for $\beta > 0.02$ agrees with the predictions of energy-loss theory using the first-order Born approximation and with experimental energy-loss data for protons. However, for the region $0.01 < \beta < 0.02$, the moderation-time data for Σ^- (and π^-) in hydrogen no longer agree with calculations based on the Born approximation or with experimental data for protons. The results for hydrogen are discussed and interpreted as being related to an "excess-range" effect previously observed for negative particles in emulsion.

I. INTRODUCTION AND BACKGROUND

This paper is a description of a study of the low-velocity moderation of charged particles in a region where there have been few experimental data. The theoretical treatment of the energy-loss phenomena of charged particles in matter originated with the work of Bethe,[1] which resulted in a Born-approximation expression for the energy loss per unit path length

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mv^2} NZ \ln\left(\frac{2mv^2}{I}\right). \quad (1)$$

In this expression, ez is the charge of the incident particle, v is its velocity, m is the electron mass, Z and N are the atomic number and density of the moderator, respectively, and I is the effective ionization potential of the moderator. The factor $Z\ln(2mv^2/I)$ is called the stopping number of the moderator and given the symbol B, and $-(dE/dx)/N$ is called the stopping cross section of the moderator. Equation (1) is valid when the velocity of the incident particle is much larger than the velocity of the bound electrons in the moderator. In a hydrogen atom, the orbital electron velocity is $v_0 = \alpha c \approx \frac{1}{137} C$, and for incident velocities greater than v_0, virtually all energy loss occurs via the Coulomb scattering of the incident particle with the electrons The argument of the logarithmic factor in Eq. (1) should actually contain the additional factor $\gamma^2 = [1-(v/c)^2]^{-1}$ and a density-effect correction developed by Sternheimer.[2] However, these relativistic effects do not contribute in the low-velocity region which is the subject of this study.

In general, the mean ionization potential I is taken to be the main parameter of energy-loss (or stopping-power) theory. There have been considerable experimental and theoretical efforts directed toward the measurement and calculation of I for a wide range of elements and materials. An article by Whaling[3] contains an extensive compilation of stopping-power measurements and there are other experimental and theoretical reference studies available.[4,5]

The work of Livingston and Bethe,[6] Brown,[7] Walske,[8] Walske and Bethe,[9] and Fano[10] has resulted in correction terms to Eq. (1) in the low-velocity region. The original calculations of Walske[8] are valid for K-shell electrons and yield an expression (using first-order Born approximation) to order η^{-2} for the stopping number of atomic hydrogen:

$$B = \ln\eta + 1.2893 - \eta^{-1} - \tfrac{25}{6}\eta^{-2}\ , \qquad (2)$$

where $\eta = mv^2/2I$. Walske has extended Eq. (2) for materials with $Z > 1$, and has performed a similar calculation for L-shell electrons.

Several direct measurements have been made of the low-velocity moderation of protons and heavy ions in gases which yield values of dE/dx accurate to a few percent for various incident energies.[11–15] These direct measurements were made by injecting a momentum-analyzed beam into a column of moderating gas and measuring the momentum or energy of the beam upon exit from the moderator. Equivalent proton energies are in the region 10 keV–6 MeV (corresponding to $0.004 < \beta < 0.1$) in hydrogen, and are suitable for comparison with the present study.

A second technique,[16] although less direct, has been used to test the theory in solids and liquids as well as in gases. This technique involves measuring the stopping ranges for beams of different incident energies. Equations (1) or (2) can be integrated to obtain the continuous-slowing-down approximation (CSDA) for range

$$R(E_0) = R_1 + \int_{E_1}^{E_0} -\left(\frac{dE}{dx}\right)^{-1} dE\ , \qquad (3)$$

where R_1 and E_1 are generally empirically determined for a given moderator. Because Eq. (2) diverges at low velocity, Eq. (3) cannot be numerically integrated from $E = 0$, but there is very little contribution to the range from small values of E. By measuring stopping ranges for different velocities in the range $\beta \gg \alpha$, the constant R_1 can be determined, and then Eq. (3) yields good information about dE/dx at higher velocities. This technique has been used in metal foils and solids, and is particularly well suited for nuclear emulsions where very accurate range measurements can be made. Once R_1 and E_1 have been determined for a given emulsion stack, range measurements alone determine the initial energy of a stopping particle.

In the course of measuring the Σ hyperon masses, Barkas, Dyer, and Heckman[17] determined the stopping ranges of Σ^+ and Σ^- hyperons in emulsions and found that the Σ^- has an anomalous range relative to Σ^+. Earlier emulsion determinations of the pion masses contained similar results,[18,19] and a recent pion range experiment by Heckman and Lindstrom[20] has confirmed this "excess-range" effect for negative pions. Barkas, Dyer, and Heckman suggested that the excess range of the negative particles may be due to a departure from the Born approximation [used in Eqs. (1) and (2)] for incident-particle velocities comparable to the bound-electron velocities in the emulsion, and that the neglected higher-order Born-approximation terms depend on the sign of the charge of the incident particle.

Another method which can test energy-loss theory in a sensitive way uses the moderation time of unstable particles. The basic technique involves using a particle of known lifetime, and measuring the decay rate as a function of velocity. In this way the number of decays can be converted into the time spent in a particular velocity interval. The general technique was first used by Fields *et al.*[21] to measure the cascade time of π^- mesons in a liquid-hydrogen bubble chamber. This experiment was repeated by Doede *et al.*[22] and by Bierman *et al.*[23] and has also been done for π^- in liquid deuterium[24] and helium.[25] Similar experiments have been done in hydrogen[26] and helium[25] with K^- mesons. The primary object of these experiments was to measure the atomic and nuclear cascade times of the mesons and test the cascade-time theories of Wightman,[27] Day, Snow, and Sucher,[28] Russel and Shaw,[29] and Leon and Bethe.[30] But some secondary information about differential moderation times in the low-velocity region was obtained.

The study we report involves a measurement of the moderation time of the Σ^- hyperon. The technique used is similar to that of Burnstein, Snow, and and Whiteside[31] who measured the cascade time of the Σ^- hyperon and at the same time found that the number of low-momentum Σ^- decays was anomalous. This evidence, along with reported anomalous ranges of Σ^- in emulsion and liquid hydrogen[32] prompted the present investigation of the Σ^- moderation time in the velocity region $\beta < 0.10$. Section II of this paper is devoted to a description of the experimental technique, and to the method of analysis of the data. Section III presents the results along with a comparison with theory and with other experimental data on the energy loss of charged particles.

II. EXPERIMENTAL PROCEDURE AND ANALYSIS

A test of the validity of energy-loss theory in the low-velocity region can be obtained by accurately measuring the (proper) time Δt necessary for a particle to slow down from a velocity v_1 to a velocity v_2. In the case of an unstable particle like the π, K, or Σ, this increment of time is conveniently measured by counting the number of decays ΔN in the time Δt, corresponding to the velocity interval (v_1, v_2). By the definition of the mean lifetime τ, we have

$$\Delta N = N(1 - e^{-\Delta t/\tau}) , \tag{4}$$

where N is the number of decaying particles which start with velocity v_1.

In this experiment, the Σ^- studied were produced and decayed via the following sequence of reactions in hydrogen:

$$K^- + p \to \Sigma^- + \pi^+ . \tag{5a}$$

$$\searrow \pi^- + n , \tag{5b}$$

$$n + p \to n + p . \tag{5c}$$

Reaction (5b) is the normal decay mode[33] of the Σ^-, and $\approx 88\%$ of the Σ^- produced by K^- at rest via reaction (5a) will decay via reaction (5b). The remaining 12% will come to rest in the liquid and are absorbed. The usual procedure for determining the Σ velocity at decay involves using the momentum of the decay pion in reaction (5b), measured from curvature, to do a zero-constraint calculation of the momentum of the Σ^- at decay. By adding momentum information from the measured length of the Σ^-, via the range-energy relations, a one-constraint fit may be done to the decay reaction (5b). The measurement error in the Σ length, along with the highly nonlinear nature of the range-energy relations at low velocities, results in a one-constraint decay fit which poorly determines the Σ velocity at decay.

Our experimental technique uses decay neutrons which elastically scatter in the liquid ($\approx 10\%$ probability) via reaction (5c) and produce a visible proton recoil. By measuring the range of the proton and the direction of the neutron in the scatter reaction, one can calculate the magnitude of the momentum of the decay neutron. Then using this information together with the value of the π^- momentum measured from curvature (but not using any information from the Σ^- range) a three-constraint fit to decay reaction (5b) can be performed. This procedure results in a precise determination of the velocity of the Σ^- at decay which is independent of the low-velocity range-energy relations. By repeated measurements, we have found the error in the Σ^- decay momentum to be < 5 MeV/c, corresponding to a velocity resolution of $\Delta\beta = \pm 0.004$.

The Σ^- studied in this experiment were produced by the interaction of K^- at rest on protons in the Saclay 81-cm liquid-hydrogen bubble chamber[34] with a hydrogen density of 0.0624 g/cm^2 at the CERN proton synchrotron.[35] An average of about two examples of Σ^- production and decay appear in each photograph. We have scanned about 10 000 pictures in two views with the third view available for resolving questionable topologies or partially obscured tracks. The scanning was done on Recordak 35-mm microfilm projectors[36] using a projected image approximately twice chamber size. Events were measured on a standard film-plane digitizer, and geometric reconstruction and kinematic fitting were done in the IBM 7094 program PACKAG.[37] In a previous scan of the film all examples of Σ^- production and decay were recorded with an efficiency of finding K^- at-rest reactions of $(90 \pm 1)\%$. We restricted ourselves in this experiment to stopping-K^- events, since less than 10% of the K^- interacted in flight.

The scanners were instructed to check every Σ^-

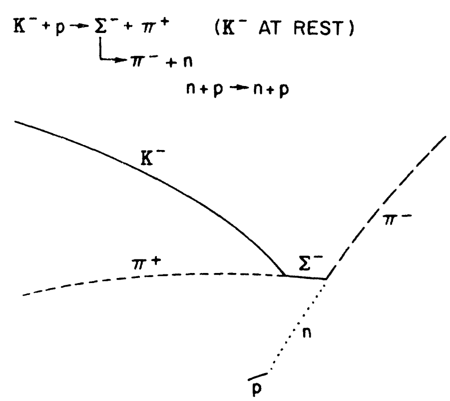

FIG. 1. Line drawing of an example of Σ^- production and decay with an associated (n, p) elastic scatter.

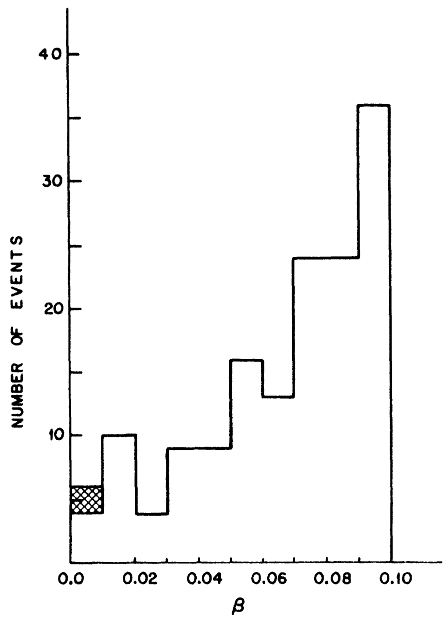

FIG. 2. Decay velocity distribution of Σ^- hyperons in liquid hydrogen. Cross-hatched events are consistent with at-rest decays.

decay vertex recorded for any possibly related recoil protons. Figure 1 is a line drawing of an event which passes the scanning and measuring criteria described in the Appendix. About one-third of the film was independently rescanned and the scanning efficiency for good neutron recoil events passing all acceptance criteria was found to be $(85 \pm 5)\%$. To ensure a reasonable efficiency for finding genuinely associated recoils, and to eliminate some background proton recoils, the scan was done using a template applied to the projected image in each of the two views scanned. The template restricted the scanner to a narrow pie-shaped sector originating at the decay point of each Σ^-. This area contained practically all the neutron recoils from Σ^- with $\beta < 0.10$ at decay.

The geometric criteria resulted in the rejection, both before and after measurement, of some *bona fide* events. For this reason, it was necessary to calculate a weight factor to be applied to each accepted event to obtain the "true" number of events. For each event accepted in the sample, a Monte Carlo simulation of 500 Σ^- decays was generated at the point in the chamber where the event was located, and with the actual value of the decay momentum of the Σ^-. The simulated events were propagated in the chamber and a probability for observing a recoil, reaction (5c), subject to the scanning criteria imposed, was calculated. Further details concerning the geometric criteria are presented in the Appendix.

III. EXPERIMENTAL RESULTS AND CONCLUSIONS

This study is based on 23 200 Σ^- which resulted in a total of 151 examples of the reaction sequence (5a)–(5c), a Σ^- decay with an associated neutron recoil, which pass all our acceptance criteria in the velocity region $\beta < 0.10$. The Σ^- decay velocity distribution of these events is presented in Fig. 2.[38] Weight factors were calculated and applied to each event to correct for scanning losses due to our scanning criteria, and to account for the probability of observing a proton recoil. In this way a corrected number of decays in each velocity interval was determined. Inverting Eq. (4) gives

$$\Delta t_i = \tau \ln[N_i/(N_i - \Delta N_i)] , \qquad (6)$$

where τ is the Σ^- lifetime and N_i is the number of Σ^- which enter the ith velocity interval. N_i was determined for each interval by propagating the initial number of Σ^- produced through each successive velocity interval. The time used in this calculation was obtained by numerically integrating the energy-loss relations in Eqs. (1) and (2):

$$\Delta t = -\frac{M}{c}\int_{\beta_1}^{\beta_2} \left(\gamma^2 \, d\beta \Big/ \frac{dE}{dx}\right) , \qquad (7)$$

where Δt is a proper time interval, but dE/dx is in the laboratory frame. M is the mass of the moderated particle.

The Σ^- differential moderation time, presented in Fig. 3, was obtained by correcting for over-all geometric criteria imposed on measured events

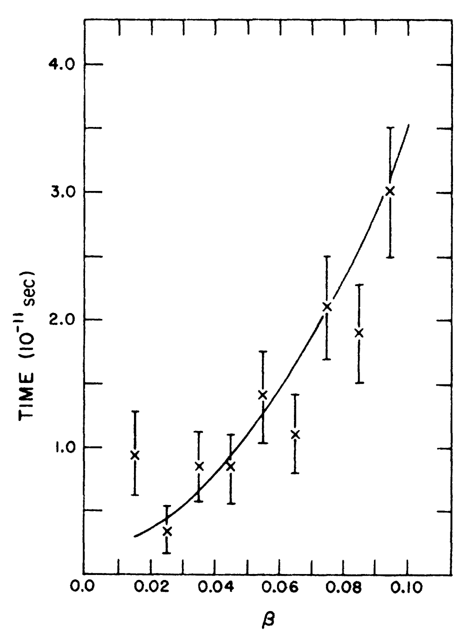

FIG. 3. Differential moderation time for Σ^- hyperons in liquid hydrogen.

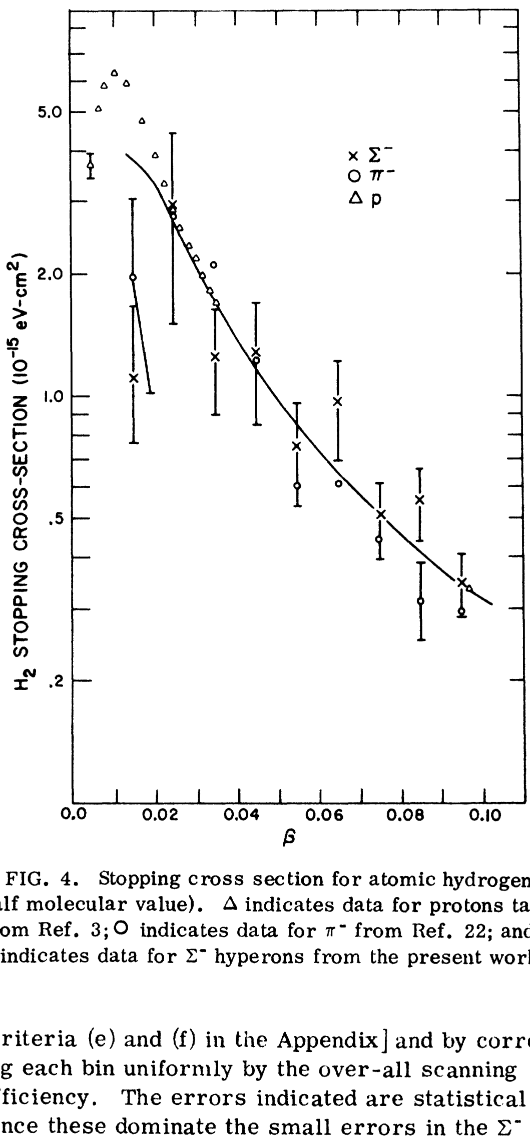

FIG. 4. Stopping cross section for atomic hydrogen (one-half molecular value). Δ indicates data for protons taken from Ref. 3; O indicates data for π^- from Ref. 22; and × indicates data for Σ^- hyperons from the present work.

[criteria (e) and (f) in the Appendix] and by correcting each bin uniformly by the over-all scanning efficiency. The errors indicated are statistical since these dominate the small errors in the Σ^- mass and lifetime, as well as those of the bubble-chamber density and the number of incident Σ^-. The curve plotted in Fig. 3 is Eq. (7) for a velocity interval of width $\Delta\beta = 0.01$. A value of 18.3 eV[15] has been used for the mean ionization potential of hydrogen.

A comparison of the results of this experiment may be made with other experiments which have measured the energy loss of protons and pions in hydrogen. From Eq. (7), the mean value of the stopping cross section of hydrogen $[-(1/N)(dE/dx)]$ in a given velocity interval may be determined, thus allowing a direct comparison with other experiments as shown in Fig. 4. The curve in Fig. 4 was computed from Eq. (2), the first-order Born approximation, and is not expected to agree exactly with experiment for $\beta \approx \alpha$ since Eq. (2) diverges in this region. The significant feature of Fig. 4 is that in the velocity interval $0.01 < \beta < 0.02$, the measured values of the stopping cross section for both π^- and Σ^- do not agree with the first-order Born approximation and with the proton measurements by 1.5–2 standard deviations. The errors indicated for the present experimental data are statistical. The bin width of $\Delta\beta = 0.01$ was chosen to be greater than the experimental resolution of $\Delta\beta = \pm 0.004$. Systematic errors, if any, would not be expected to affect the results since they would vary slowly across the entire velocity range studied.

Figure 4 indicates that Σ^- hyperons and π^- mesons exhibit a lower rate of energy loss in hydrogen than protons at values of $\beta \lesssim 0.02$.[39] This conclusion may be compared with other experiments which have directly or indirectly measured the energy-loss differences of positively and negatively charged particles. We do not include data for $\beta < 0.01$ in Figs. 3 and 4 since it becomes difficult to measure the differential moderation time for $\beta < 0.01$ because of the velocity resolution of $\Delta\beta = 0.004$, which results in an ambiguity between at-rest and in-flight Σ^- decays.

A useful quantity which may be calculated for comparing various experiments is the excess range, or the difference between the ranges of negative and positive particles of the same mass and initial momentum. We have calculated the excess range implied by our moderation-time measurement neglecting the contribution from the region $\beta < 0.01$ because the maximum contribution from this region would be small—much less than the statistical error in our result. In the velocity region $0.01 < \beta < 0.02$, we obtain a Σ^- differential moderation time $\Delta t = (0.95 \pm 0.30) \times 10^{-12}$ sec. At an average velocity $\langle \beta \rangle = 0.015$, this implies a travel distance $d = \beta c \Delta t = 43 \pm 14\ \mu$. Equation (7) indicates that $t \propto 1/(dE/dx)$ and by performing the same calculation using the proton data of Fig. 4, one finds that a positively charged particle of the Σ^- mass would have a travel distance $d = 10\ \mu$ implying an excess range for the negative particle $\Delta R_{\Sigma^-} = 33 \pm 14\ \mu$. There have been no other experiments on Σ^- moderation time in hydrogen which allow a direct comparison with our results. However, a recent experiment by Schmidt[32] measuring particle masses in a hydrogen bubble chamber has been cited by Schmidt and by Heckman and Lindstrom[20] as offering evidence for an excess-range effect. This experiment reports a Σ^- mass of 1196.53 ± 0.24 MeV using range measurements on stopping Σ^- produced in reaction (5a). The value of the Σ^- mass obtained using other data with a seven-parameter fit, including the measured ranges of stopping protons but not the range of the Σ^-, was 1197.43 ± 0.11 MeV. This mass difference of 0.90 ± 0.26 MeV implies an increase in range, or excess range, for the Σ^- of $230 \pm 65\ \mu$, which is not in agreement with our value determined from moderation-time measurements. A careful analysis of Schmidt's experiment reveals that his excess-range result could be due to systematic errors known to be possible for simple-range measurements.[40] In addition, experiments in hydrogen bubble chambers based on Σ^--range measurements[41] to date have not had the statistical resolution necessary to detect an effect as small as is indicated by our moderation-time measurement, which implies a Σ^- mass from range discrepancy of 0.1 ± 0.04 MeV.

A comparison can also be made with emulsion results. Since Eq. (2) is an expansion in η, one would expect the first-order Born approximation to break down at about the same value of η in all materials. In our experiment we find a significant departure from the first-order Born approximation and from proton measurements at $\beta_H < 0.02$. Since $\eta \propto (\beta^2/I)^{1/2}$, one would expect to see a corresponding departure from the first-order Born approximation, and a consequent difference in measurements on positive and negative particles, in emulsion at $\beta_E \lesssim (I_E/I_H)^{1/2}\beta_H$, or $\beta_E \lesssim 4\beta_H \approx 0.08$. In fact, Heckman and Lindstrom[20] find that for π^- mesons compared to π^+ in emulsion, a statistically significant lower value of dE/dx occurs for $\beta < 0.08$. Heckman and Lindstrom measured a maximum difference in dE/dx in emulsion for π^- compared to π^+ of $(14 \pm 4)\%$ in their lowest velocity interval where $\langle \beta \rangle = 0.51$ and $\langle \eta \rangle = 2.1$. In hydrogen, we have measured a difference in dE/dx of $(80^{+10}_{-15})\%$ for Σ^- compared to protons, and the measurement of Doede *et al.*[22] implies a difference for π^- compared to protons of $(60 \pm 15)\%$ in the region where $\langle \beta \rangle = 0.015$ and $\langle \eta \rangle = 3.1$.

While the magnitude of the departure of dE/dx for negative particles from that of positive particles is greater in hydrogen than in emulsion, the percentage increase in range is very much greater in emulsion. For example, a 172-MeV/c Σ^- produced in reaction (5a) has its range in emulsion increased by 25 ± 5 μ [17] out of an expected 625 μ, whereas in hydrogen the increase is 33 ± 14 μ out of an expected 10 500 μ. A similar comparison can be made for π mesons. Using the measured Δt of Doede *et al.*[22] from their moderation-time experiment yields an excess range $\Delta R_{\pi^-} = 4 \pm 0.5$ μ. Measurements and calculations for emulsion[20] give values of the excess range of π^- compared to π^+ of $3-6 \pm 1.5$ μ. We interpret the similarity of the excess range of negative particles in hydrogen and emulsion as a coincidence caused by the fact that the ratio $\langle \beta \rangle / \Delta(dE/dx)$, which is proportional to the range, is the same for the two materials. [$\Delta(dE/dx)$ is the change in dE/dx and is inversely related by Eq. (7) to the "extra" time spent by the negative particle in the affected velocity interval.] One would not expect this ratio, and therefore the excess range, to be the same in all other materials. However, there are no theoretical predictions of the magnitude of the excess-range effect and further work is clearly needed to derive the form of the higher-order Born-approximation terms which might describe the nature and expected magnitude of the observed effect for various materials. On the other hand, it is known that the effect occurs in the region where the Born approximation breaks down (at the same value of η) for both emulsion and hydrogen.

A further experimental study of the effect could be made by using the technique described here to simultaneously study the low-velocity energy loss of Σ^+ and Σ^- hyperons. This would yield a direct comparison of two particles with comparable masses while eliminating any systematic errors, since Σ^+ and Σ^- have essentially identical production and decay topologies. However, this suggested experiment would be formidable since Σ^+ events in the very low-velocity region occur at a much lower rate than do Σ^- events,[42] and roughly 100 times the present data would be necessary to obtain a Σ^+ measurement with statistics comparable to our present study.

Finally, it should be noted that the apparent difference in the rate of energy loss between positive and negative particles does not alter the conclusions of most bubble-chamber experiments where momenta are generally determined by curvature rather than by range, and where the range-energy relations are generally applied at higher velocities than those of this study. However, in those precision low-velocity experiments where the momenta are determined from the ranges of stopping particles, the range effects discussed in this work could make very small systematic alterations in the results.

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APPENDIX: SCANNING AND MEASURING ACCEPTANCE CRITERIA

The scanning template imposed the following criteria on each projected image: (a) The projected length of the Σ^- on the scanning table was > 8 mm; (b) the projected length of the recoil on the scanning table was > 4.5 mm; (c) the projected angle on the scanning table between the directions of the decay neutron and the decay pion in reaction (5b) was

> 140°; (d) the (n,p) scatter [reaction (5c)] was within a projected distance on the scanning table of 14 cm from the decay vertex.

In addition, two criteria were imposed on the data after measurement and geometric reconstruction: (e) The absolute value of the dip of the Σ^- was < 60°; (f) the absolute value of the cosine of the space angle between the Σ^- and K^- [reaction (5a)] was < 0.867 (|space angle| > 30°).

Criterion (a), together with (e) and (f), assured a good measurement of the production reaction (5a) and consequently eliminated any reactions where the K^- was not at rest. In addition, (a) eliminated most short Σ^-, which have momenta > 120 MeV/c at decay ($\beta > 0.1$), which are not the subject of this study.

Criterion (b) assured a measurable proton recoil. Criterion (c) was imposed to eliminate most of the background resulting from stray proton recoil tracks in the chamber. For a Σ^- decaying at 120 MeV/c, the maximum space angle between the neutron and pion is 144° (60 MeV/c corresponds to 162°), and very few events with space angles greater than this minimum will have a projected angle less than 140°. Criteria (c) and (d) together were applied by the scanner by aligning a template directly on the projected image. These criteria restricted the scanning to a small sector about each Σ^- where the scanner could carefully search for a recoil. If a potential recoil was found, its projected length and that of the Σ^- were checked, and if all criteria (a)–(d) were met, the event was recorded for measurement.

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[39]The effect does not appear in the meager data for π^- in deuterium (see Ref. 24). However, the measurement in the velocity interval $0.01 < \beta < 0.02$ is based on only one π^- decay remaining after a background subtraction of one

decay.

[40]A possible systematic error which could account for Schmidt's large excess-range effect is a small increase in the measured length of "stopping" or "connecting" tracks. A measured Σ^- length that is too long leads to a lower-than-"true" value of the Σ^- mass. Schmidt's experimental stopping-Σ^--range distribution, read from Fig. 10 of Ref. 32, is 1.08 ± 0.04 cm. If his Σ^- range were systematically long by about one-half the radius of a bubble ($\approx 200\,\mu$), his result would be consistent with ours. On the other hand, the same type of systematic measuring error for stopping protons from Σ^+ and Λ decays would lead to increases in the masses above their "true" values. These mass changes would, however, be a second-order effect because the proton ranges are part of a seven-parameter fit with other quantities, including a fixed proton mass, involved.

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[42]The number of Σ^+ which survive to $\beta = 0.02$ is $\approx 4\%$ of those produced and only $\approx 1\%$ survive to $\beta = 0.01$; the corresponding numbers for Σ^- are 26 and 12% based on Σ^+ and Σ^- lifetimes. The Σ^+ event rate is further reduced from the Σ^- rate by a factor of about 4 because only one-half as many Σ^+ are produced in K^-, p interactions at rest, and half of these Σ^+ decay via the uncharged pion mode, $\Sigma^+ \to p + \pi^0$.

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Lifetimes of Some Doubly Excited Levels in Neutral Helium†

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We have searched for radiative transitions from doubly excited levels in He I, using the beam-foil method. The spectra showed the $1s2p\ ^3P$-$2p^2\ ^3P$ transition at 320.4 Å and some weaker lines, which are also interpreted as transitions from the doubly excited system. Radiative lifetimes were measured for three lines. Our value for the $2p^2\ ^3P$ level, 0.080 ± 0.007 nsec, is in agreement with the recent theoretical value of 0.0803 nsec. The possibilities of observing transitions from doubly excited levels in the He I isoelectronic sequence are briefly discussed.

INTRODUCTION

Evidence of doubly excited states in neutral helium was first obtained from studies of the arc spectrum of helium[1,2] and from energy-loss measurements in electron-helium collisions.[3] Compton and Boyce[1] and Kruger[2] reported unidentified spectral lines at 309.04 and 320.38 Å. The former remained unclassified, whereas Kruger tentatively assigned the latter to the $1s2p\ ^3P$-$2p^2\ ^3P$ transition in He I. This identification was later supported by Wu's calculations[4] of auto-ionization probabilities for various doubly excited terms in helium, of which those of the type $2pnp\ ^1P$, ^3P, and $2pnd\ ^1D$, ^3D are not expected to auto-ionize via Coulomb interaction.

In recent years, the auto-ionizing doubly excited He I levels have been the subject of many experimental and theoretical investigations. The review article by Fano[5] gives a detailed list of references. Using synchrotron radiation to excite neutral helium, Madden and Codling[6,7] observed four Rydberg series in the doubly excited ^1P system. Two of the series converged to the $n = 2$ limit of He$^+$ and were described by Cooper *et al.*[8] as $sp2n \pm (n \geq 3)$, being symmetrized mixtures of the $2snp$ and $2pns$ series members. Several of these and other doubly excited levels have also been observed as resonances in electron-helium[9–13] and ion-helium[14] collisions. Most of the doubly excited levels observed so far can autoionize to the continua above the $1s\ ^2S$ ground state of He$^+$. From their Fano-type line profiles,[15] Madden and Codling[7] were able to deduce the auto-ionization probabilities for the $2s2p\ ^1P$ and $sp23 +\ ^1P$ levels, obtaining good agreement with theory.[16] The radiative deexcitation probabilities for several auto-ionizing ^1P and ^3P states have been calculated by Knox and Rudge[17] and Dickinson and Rudge.[18]

Experimental and theoretical results for the non-auto-ionizing doubly excited He I states are not as numerous. The energy of the $2p^2\ ^3P$ level has been calculated by, among others, Holøien,[19] Midtdal,[20] and Drake and Dalgarno.[21] Their eigenvalues give further support to the assignment of the 320-Å line. Drake and Dalgarno have also calculated the energy of the $2p3p\ ^1P$ level and the lifetimes of both these levels. They identify the 309-Å line, observed by Compton and Boyce,[1] as the $1s3p\ ^1P$-$2p3p\ ^1P$ transition. The need for further experimental studies of such exactly quantized doubly excited levels has been emphasized by Holøien.[22] This article de-