Direct Measurement of the Group Velocity of Light

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We propose and demonstrate a method to directly measure with high accuracy the group velocity of light. We make use of the natural fluctuations of light and determine in a correlation experiment the coincidence of fluctuations of light passing through a sample path and a reference path with known delay. It is shown that this coincidence can be detected by using amplitude interferometers such as a Michelson interferometer, or intensity interferometers. Accuracies of the order of 10^{-6} are practically possible. That is orders of magnitude better than previous methods allow. Using a Nd:glass laser as light source and an intensity interferometer for detecting coincidence, we demonstrate our method by measuring the group velocity of light in water within 2×10^{-4} using a sample length of only 15 cm. The limitations of this new method caused by second-order dispersion are discussed quantitatively; higher-order effects are discussed qualitatively.

I. INTRODUCTION

Phase and group velocity are well-known concepts in the theory of wave propagation. The former is related to the phenomenon of refraction, the latter to the propagation of signals. For dispersive media, these velocities may differ substantially.

Direct and very accurate measurements of the phase velocity (refractive index) by refraction or reflection methods are now standard procedures within large ranges of wavelength λ. A convenient way for determining the group index n_g is to calculate it indirectly from the refractive index n_p according to

$$n_g = n_p - \lambda \frac{\partial n_p}{\partial \lambda} \quad . \tag{1}$$

In some cases, however, a direct measurement of the group index might be advantageous. In particular, if the bulk value of n_p is not accessible, or if n_p does not give sufficient information as in waveguides, a direct measurement is the only alternative.

The classical approach for a direct measurement of the group velocity is to modulate the light and to detect the time delay of that modulation after the light has traveled a given distance z through the sample. The accuracy of such a measurement is limited by practical distances and modulator-detector responses.

The first experiments to measure the light velocity in transparent samples were carried out more than a century ago.[1-3] Since at that time light could only be modulated by mechanical means, the accuracy was only a few percent. Towards the end of the nineteenth century the velocity of light was still being measured mechanically, but as a function of wavelength.[4] The first electronic modulation with a Kerr cell did not result in greater accuracy,[5] but when acoustic modulation was introduced, measurements with a relative error of 10^{-3} became possible.[6] We propose and test in this paper a method in which accuracies of 10^{-6} can, in principle, be obtained.

This is possible by making use of the natural fluctuations of ordinary light, and carrying out a correlation measurement. In every light beam with frequency $\nu = c/\lambda$ and frequency width $\Delta\nu$, the light intensity fluctuates with a characteristic time $1/\Delta\nu$. If we use a relative bandwidth of 1% in the visible, for instance, the characteristic fluctuation time is of the order of 10^{-13} sec which is far below electronic resolution times. Whether fluctuations of such short durations are coincident in time or not can be determined with optical correlators. Such a device splits the original beam into two beams A and B. One of the beams, say B, contains a variable delay

$$\tau = \Delta l/c \, . \tag{2}$$

By changing the distance Δl, the relative delay between beams A and B can be varied. That enables us to measure the mutual correlation function of the two light beams. The correlation function has a maximum when the fluctuations coincide. If the correlator is linear, i.e., an amplitude correlator, one measures the mutual coherence function; if it has a quadratic characteristic, one measures the intensity correlation function.[7] Nonlinear correlators have recently become popular for measuring the duration of ultrashort light pulses and investigating the phase relations between spectral components of light sources. The accuracy of such correlation measurements is due to the magnitude of the light velocity which transforms fluctuations on a picosecond scale to distances on a tenth of millimeter scale.

In this paper we want to show that amplitude or

intensity correlators can also be used advantageously to measure directly the group velocity in transparent samples. If such a sample is inserted into beam A, the fluctuations are retarded and coincidence is found for a different distance Δl in beam B. We show that this difference corresponds always to the group delay suffered in the sample path A. This allows us to measure the group index n_g with high accuracy, since it will become evident that the accuracy is only limited by higher-order terms of the dispersion which alter the fluctuation behavior in the sample path.

Our analysis is carried out for both types of correlator units. In Sec. II we describe the use of an amplitude interferometer. In Sec. III we describe the use of an intensity interferometer employing second-harmonic generation for measuring the correlation function. The accuracy to be expected from these devices is discussed in Sec. IV. As a test of the method, we report in Sec. V the direct measurement of the group velocity in water using a Nd : glass laser as light source and an intensity interferometer as correlator.

II. AMPLITUDE INTERFEROMETERS

Without loss of generality we can assume a light source with its analytic signal represented by the spectrum

$$V = E_0 \sum_n e^{-2(n\delta\nu/\Delta\nu)^2} e^{2\pi i(\nu+n\delta\nu)t+\Psi_n} . \tag{3}$$

If the phases Ψ_n of the individual modes are statistically independent, (3) is within one repetition period $1/\delta\nu$ a Gaussian fluctuating light beam with center frequency ν and full width $\Delta\nu$ between half-peak intensity points. The mode index n is an integer $-\infty < n < +\infty$, and $\delta\nu$ is the mode spacing. We will assume that $\delta\nu \ll \Delta\nu$, so that we can replace the sum by an integral in the following calculations. If the relative phases Ψ_n are constant, (3) represents an ideal bandwidth-limited short pulse in time t.

The light beam is split within the correlator into two beams with the analytic signals V_A and V_B. The spectral components of V_B experience the time delay (2). The spectral components of V_A experience in the sample of length z a phase change

$$\Phi_n = \varphi + n\varphi' + n^2\varphi'' , \tag{4}$$

where according to Ref. 8

$$\varphi = -2\pi n_p z/\lambda \tag{5}$$

is the phase angle at the center frequency,

$$\varphi' = -2\pi\left(n_p - \lambda\frac{\partial n_p}{\partial\lambda}\right)\left(\frac{\delta\nu}{\nu}\right)\frac{\partial}{\lambda} = -2\pi n_g\frac{\delta\nu}{c}z \tag{6}$$

is the contribution from first-order dispersion, and

$$\varphi'' = -2\pi\frac{\lambda^2}{2}\frac{\partial^2 n_p}{\partial\lambda^2}\left(\frac{\delta\nu}{\nu}\right)^2\frac{\partial}{\lambda} \tag{7}$$

is the contribution from second-order dispersion.

For amplitude interferometers, the superposition of beams A and B yields an intensity

$$I = |V(t-\tau,\Psi) + V(t,\Psi+\Phi)|^2 . \tag{8}$$

The time average of I measured by any detector with finite response time is simply obtained by calculating the intensity profile of each spectral component separately and then integrating over all intensity profiles. After some algebra,[9] one obtains

$$\bar{I} = (\sqrt{\pi})\frac{\Delta\nu}{\delta\nu}E_0^2\left\{1 + [1+(\gamma z)^2]^{-1/4} \times \exp\left[-\left(\frac{\pi\Delta\nu}{2}\right)^2\frac{(\tau - n_g z/c)^2}{1+(\gamma z)^2}\right]\cos\chi\right\} , \tag{9}$$

where

$$\gamma = \frac{\pi}{4}\lambda\frac{\partial^2 n_p}{\partial\lambda^2}\left(\frac{\Delta\nu}{\nu}\right)^2 \tag{10}$$

and

$$\chi = 2\pi\left(\nu\tau - n_p\frac{z}{\lambda}\right) + \tfrac{1}{2}\arctan(\gamma z) - \gamma z\left(\frac{\pi\Delta\nu}{2}\right)^2\frac{(\tau - n_g z/c)^2}{1+(\gamma z)^2} . \tag{11}$$

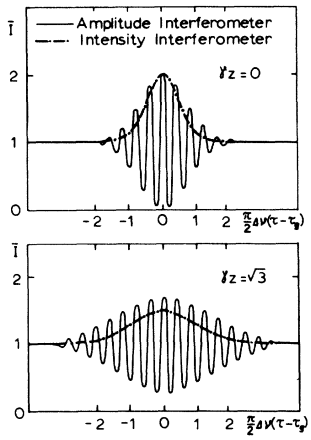

FIG. 1. Theoretical correlation functions for group-velocity measurements using amplitude and intensity interferometers. If the second-order dispersion is negligible ($\gamma z = 0$, upper diagram), the width of the correlation function is determined entirely by the spectral width of the light source. The broadening of the correlation peak due to second-order dispersion (lower diagram) was calculated for a Gaussian fluctuating source in the case of an intensity interferometer. The time τ is a variable delay, τ_g is the group delay produced by insertion of the sample into one arm of the interferometer, and $\Delta\nu$ is the full width of the light source between $1/e$ intensity points.

In (9), $\cos\chi$ describes the fine structure (or fringes) as τ is varied, and the exponential describes the envelope of the fringes in a Michelson interferometer loaded by dielectric. This is shown schematically in Fig. 1. The envelope has its maximum for a delay

$$\tau_g = n_g z/c\,. \tag{12}$$

The fringe contrast decreases to $1/e$ of its maximum within a delay $\delta\tau = \pm\,\delta l/2c$. The width of that fringe contrast is from (9)

$$\delta l = 2c[1+(\gamma z)^2]^{1/2}/\pi\Delta\nu\,. \tag{13}$$

Note that for $z=0$ or $\partial^2 n_p/\partial\lambda^2 = 0$, (13) is the coherence length of the light.

We notice here that a Michelson interferometer measures always the group velocity of light, since the maximum fringe visibility occurs at the group delay time τ_g. We were unable to find this statement in modern textbooks. Understandably there was confusion in Michelson's time about this point.[10] We further notice that this result is completely independent of any phase relations among the Ψ_n. Whether the light source is thermal, a mode-locked laser, or an FM beam does not matter. Only the bandwidth $\Delta\nu$ is important.

III. INTENSITY INTERFEROMETERS

If second-harmonic generation is used for detecting the correlation function,[11] the superposition of beams A and B yields an intensity

$$I_{SH} = \sigma I(t-\tau, \Psi) I(t, \Psi+\Phi)\,, \tag{14}$$

where the constant σ includes conversion efficiency and other factors. As a first case we consider statistically independent phases Ψ_n, i.e., a Gaussian fluctuating light beam. Inserting (3) and (4) into (14) one obtains

$$\begin{aligned} I_{SH} = \sigma E_0^4 \sum_{nmkl} &\exp\left[-2\left(\frac{\delta\nu}{\Delta\nu}\right)^2 (n^2+m^2+k^2+l^2)\right] \\ &\times \exp[2\pi i\delta\nu t(n-m+k-l)] \\ &\times \exp[i(\Phi_n - \Phi_m + \Phi_k - \Phi_l)] \\ &\times \exp\{i[2\pi\delta\nu\tau(m-n) + \varphi'(k-l) + \varphi''(k^2-l^2)]\}\,. \end{aligned} \tag{15}$$

Only those terms will contribute to a time average $\bar{I}_{SH}$ which have $n-m+k-l=0$. Since we have assumed that the phases are statistically independent, the contributions to the sum vanish in the ensemble average, except for the two cases where $n=m$ and $k=l$ simultaneously, or $n=l$ and $m=k$. Replacing the sums by integrals, we obtain

$$\bar{I}_{SH}(n=m;k=l) = \sigma\frac{\pi}{4}\left(\frac{\Delta\nu}{\delta\nu}\right)^2 E_0^4 \tag{16}$$

and

$$\bar{I}_{SH}(n=l;m=k) = \sigma\frac{\frac{1}{4}\pi(\Delta\nu/\delta\nu)^2 E_0^4}{[1+(\gamma z)^2]^{1/2}} \times \exp\left[-\frac{(\pi\Delta\nu)^2}{2}\frac{(\tau - n_g z/c)^2}{1+(\gamma z)^2}\right]. \tag{17}$$

Both terms contain also all terms with $n=m=k=l$. However, the number of those terms is small compared to the number of terms contained in (16) and (17), and the final result is

$$\bar{I}_{SG} = \sigma\frac{\pi}{4}\left(\frac{\Delta\nu}{\delta\nu}\right)^2 E_0^4\left[1+[1+(\gamma z)^2]^{-1/2} \times \exp\left(-\frac{(\pi\Delta\nu)^2}{2}\frac{(\tau - n_g z/c)^2}{1+(\gamma z)^2}\right)\right]. \tag{18}$$

In contrast to the case of the amplitude interferometer (9), we see that the high-frequency factor $\cos\chi$ is absent. The maximum of $\bar{I}_{SH}$, however, occurs again at the group delay time (12). The full width of the correlation peak at $1/e$ points is

$$\delta l_{SH} = \sqrt{2}c[1+(\gamma z)^2]^{1/2}/\pi\Delta\nu\,, \tag{19}$$

and the contrast ratio is

$$R \equiv \bar{I}_{SH}(\tau_g)/\bar{I}_{SH}(\tau) = 1+[1+(\gamma z)^2]^{-1/2}\,, \tag{20}$$

where

$$|\tau - \tau_g| \gg \delta l/c\,.$$

Schematically, the correlation peak is shown in Fig. 1.

As a second case we consider that of constant relative phases Ψ_n, the emission of a perfectly mode-locked laser with bandwidth-limited short pulses. For $\Psi_n = 0$, we obtain from (3) by integration of the analytic signals $V(t-\tau, \Phi)$ closed forms for evaluating the intensities $I(t-\tau, \Phi)$ of Eq. 14. For the vacuum path we obtain

$$I(t-\tau, 0) = \frac{\pi}{2}\left(\frac{\Delta\nu}{\delta\nu}\right)^2 E_0^2 e^{-[\pi\Delta\nu(t-\tau)]^2}\,. \tag{21}$$

This corresponds to the intensity of just one displayed pulse from the laser's pulse train. Likewise, we obtain from (3) and (4) for the sample path

$$I(t, \Phi) = \frac{\frac{1}{2}\pi(\Delta\nu/\delta\nu)^2 E_0^2}{[1+(2\gamma z)^2]^{1/2}}\exp\left(-\frac{[\pi\Delta\nu(t-n_g z/c)]^2}{1+(2\gamma z)^2}\right). \tag{22}$$

Inserting (21) and (22) into (14) which gives the intensity of the second-harmonic signal, and integrating over time $-\infty < t < +\infty$ yields the time-averaged second-harmonic pulse intensity

$$\bar{I}_p = \sigma\frac{\frac{1}{4}\pi(\frac{1}{2}\pi)^{1/2}(\Delta\nu/\delta\nu)^4 E_0^4}{[1+2(\gamma z)^2]^{1/2}}\exp\left(-\frac{[\pi\Delta\nu(\tau - n_g z/c)]^2}{2[1+2(\gamma z)^2]}\right). \tag{23}$$

The maximum occurs again for the group delay (12). The contrast ratio R is infinite, and the width of the correlation function is

$$\delta l_p = \sqrt{2}\, c[1+2(\gamma z)^2]^{1/2}/\pi\Delta\nu. \tag{24}$$

It is seen that the width for pulses is larger than that for Gaussian fluctuating beams if $\gamma z \neq 0$.

IV. IMPLICATIONS FOR MEASUREMENTS OF GROUP VELOCITY

We have shown that a first-order dispersion $\partial n_p/\partial\lambda$ shifts the correlation peak to a position corresponding to the group delay (12) of the sample [see Eqs. (9), (17), and (23)]. We have further shown for the cases considered here that a second-order dispersion broadens the correlation peak without affecting its position.

Logically, a third-order term will distort any waveform transmitted through the sample. This distortion must be small or we must redefine the group velocity.

If we evaluate the dispersions of transparent materials (quartz, sapphire, and LiF) around 1 μ wavelength,[12] we obtain the following typical values:

$$0.01 < \lambda\, \frac{\partial n_p}{\partial\lambda}\,, \tag{25}$$

$$0.01 < \lambda^2\, \frac{\partial^2 n_p}{\partial\lambda^2}\,, \tag{26}$$

$$0.05 < -\lambda^3\, \frac{\partial^3 n_p}{\partial\lambda^3}\,. \tag{27}$$

Since the third-order term (27) will always be multiplied with the factor $(\Delta\nu/\nu)^3$ in real problems, third- and higher-order terms can usually be neglected for group-velocity measurements if $\Delta\nu/\nu < 10^{-2}$.

The second-order term (26), however, is normally not negligible. It is just this term which limits the accuracy of a group-velocity measurement. In the case of a Gaussian fluctuating beam, for instance, we find that the width (19) of the correlation peak has a minimum

$$\delta l_{\mathrm{SH\,min}} = 2c/\pi\Delta\nu \quad \text{if } \gamma z = 1\,. \tag{28}$$

Associated with this minimum is the relative bandwidth

$$\Delta\nu/\nu = \left(\tfrac{1}{4}\pi z\lambda\, \frac{\partial^2 n_p}{\partial\lambda^2}\right)^{-1/2} \tag{29}$$

which is plotted in Fig. 2 as a function of z/λ and $s = \lambda^2|\,\partial^2 n_p/\partial\lambda^2\,|$. Although the contrast ratio is decreased to $R = 1.707$ for this bandwidth, we still can expect to be able to determine the position of the correlation peak with reasonable accuracy. Beyond the bandwidth (29) the contrast ratio decreases further, whereas the width of the correlation peak increases again. Therefore, we may denote (29) as the maximum bandwidth for accurate group-velocity measurements.

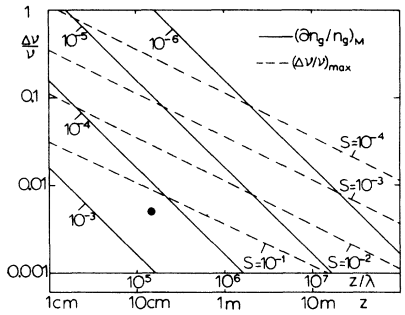

FIG. 2. Diagram to evaluate the accuracy [Eq. (31)] of a direct group-velocity measurement as a function of bandwidth $\Delta\nu$ and sample length z. Valid are readings below that dotted line [Eq. (29)] which corresponds to the actual second-order derivative $S = \lambda^2|\,\partial^2 n_p/\partial\lambda^2\,|$. A second scale for actual sample lengths has been provided which is valid for 1-μ wavelength. The dot refers to the example (Sec. V).

Now we can calculate the expected accuracy of direct group-index measurements. Since the correlation peaks measured with and without the sample are a distance $\Delta l = (n_g - 1)z$ apart, we obtain for the error

$$\frac{\delta n_g}{n_g} = \frac{\delta z}{z} + \frac{\delta l}{n_g z(N-1)}\,. \tag{30}$$

We have assumed that both the distance Δl and the sample length z can be measured with an accuracy δz. However, this error is not specifically connected with our method. For the method-dependent error we assumed that the accuracy of determining the actual position of the correlation peak increases with the number of measurements N which fall within δl. With (28) we obtain for this error

$$\eta = \left(\frac{\delta n_g}{n_g}\right)_M = \frac{2}{\pi n_g(N-1)}\left(\frac{z}{\lambda}\right)^{-1}\left(\frac{\Delta\nu}{\nu}\right)^{-1}. \tag{31}$$

For $N = 5$ and $n_g = 1.5$, (31) is plotted for some situations in Fig. 2. It shows which combinations of relative bandwidth and sample length are required to give a desired accuracy with only five correlation points. Valid are all readings below the maximum bandwidth whose limit is set by the relevant second-order dispersion $\lambda^2\partial^2 n_p/\partial\lambda^2$, according to Eq. (29). For instance, if one expects $\lambda^2\partial^2 n_p/\partial\lambda^2 = 10^{-3}$, one obtains an accuracy of 10^{-5} with a sample length of 13 cm and a relative bandwidth of 10%, or an accuracy of 3×10^{-6} with a sample length of 1.5 m and a relative bandwidth of 3%. Thus, Fig. 2 shows that accuracies up to 10^{-6} are practically possible. This is comparable to the accuracy of

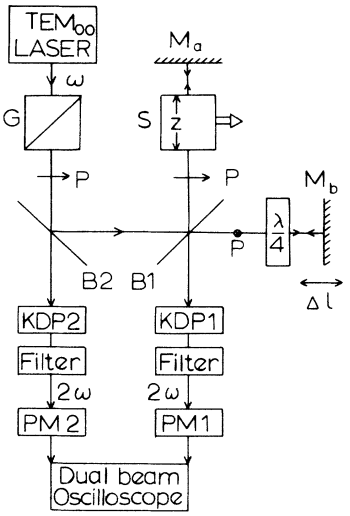

FIG. 3. Arrangement used for measuring the group velocity in the sample S. The Glan-Thompson prism G, the beam splitters B, and the $\frac{1}{4}\lambda$ plate provide the polarizations P. The light source is flash-lamp-pulsed Nd:glass laser with a longitudinal-mode spacing of approximately 150 MHz. The KDP crystals generate the second-harmonic signals which are separated from the fundamental by filters F, and integrated over the laser flashes with photomultipliers. The correlation function is determined from the ratio of the second-harmonic signals by changing the position of mirror M_b.

precision wave-index measurements in the visible spectrum.[13,14]

Finally, one can find the absolute value of the second-order dispersion directly by measuring the width of the correlation peak. With a relative bandwidth of 1%, for instance, one obtains an accuracy of 10% with a sample length of 5 m if $|\lambda^2\partial^2 n_p/\partial\lambda^2| \sim 10^{-2}$.

V. EXAMPLE

In order to demonstrate one of our two methods to measure the group velocity in a sample directly, we have used an intensity interferometer[11] with a flash-lamp–pulsed Nd: glass laser, and measured the group velocity in water. The arrangement is shown in Fig. 3. The laser light is polarized by a Glan-Thomson prism G and divided into two parts by beam splitter $B2$. One of them generates a second-harmonic signal in KDP2 (potassium dihydrogen phosphate) which is used as reference signal for the laser power. The other part enters the correlation unit, a modified Michelson interferometer. The beam is split again into two parts. One of them passes twice through the sample S, a cuvette filled with water where $z = 15.2814 \pm 0.0002$ cm. The other part passes twice through a $\frac{1}{4}\lambda$ plate and experiences a rotation of its polarization by 90°. Both parts exit the correlator orthogonally polarized to each other and have a variable delay between them. The orientation of KDP1 is chosen such that the second-harmonic signal is only generated by the product of the two beam intensities emerging from the correlator.

The interferometer signal from photomultiplier PM1 and the monitor signal from PM2 are integrated over the duration of each flash. The ratio of these signals yields the correlation function if the position of mirror M_b is varied. Since, in contrast to the case of the amplitude interferometer, the measured ratios follow a smooth curve and do not show oscillations (see Fig. 1), only a few shots are needed to determine the position of the correlation peak. This was done for the empty and the full cuvette. It is advantageous to determine in advance the approximate position of the correlation peaks by taking into account the group velocity of the cuvette walls, the beam splitter, the quarter wave plate, and a possible tilt of the cuvette. It is further advantageous to use plane parallel beam splitters with one surface antireflection coated rather than a wedge which we actually used. A plane parallel beam splitter makes the alignment of the correlator unit independent of the wavelength. Antireflection coating decreases the interference between the reflections from both surfaces of the beam splitter. Interferences from the cuvette and the $\frac{1}{4}\lambda$ plate surfaces should also be avoided, e.g., by tilting these devices, because the absorption of a 30-cm path of water is so large (15 dB) at 1-μ wavelength that the residual transmission is comparable to a reflection from an uncoated glass surface. Finally, the laser should not oscillate in too many transverse modes since that would decrease the contrast ratio because of spatial incoherence.[15] Fundamental transverse-mode operation was attempted by using flat mirrors spaced 1 m apart and an iris with 0.25-cm diam.

We measured the group velocity for two different temperatures. At 19.7 °C we found $n_g = 1.3406$, and at 26.6 °C $n_g = 1.3372$. This yields by interpolation at 20 °C $n_g = 1.3404$. From (31) we obtain for $z = 15$ cm, $N = 5$, and a measured linewidth of 1.5 THz (or 50 Å) for our laser, an error of $\delta n_g = 2\times10^{-4}$. The error of measurement $\delta z/z$ was much smaller. Our experimental result is, therefore, $n_g = 1.3404 \pm 0.0002$. From the wave index[16] which is $n_p = 1.32405$ at 20 °C for $\lambda = 1.0623$ μ, we calculate with (1) $n_g = 1.3404 \pm 0.0001$. The two results agree perfectly.

VI. CONCLUSIONS

We have shown that amplitude and intensity interferometers can be used to measure the group velocity of light directly and with high accuracy. Our technique makes use of the natural fluctuations of

light, i.e., additional modulation or measurements at different frequencies are not required. In the case of amplitude interferometers, incoherent lamps with suitable filters and slits can be used as light sources. The correlation function appears in the form of interferometer fringes which are sensitive to adjustments within fractions of a wavelength. In the case of intensity interferometers, at least medium-powered pulsed lasers must be used to provide sufficient bandwidth and second-harmonic power. The correlation function is not sensitive to adjustments within fractions of a wavelength, and is relatively insensitive to beam alignment if the laser oscillates in the fundamental transverse mode.

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Kinetic Energies of Atomic Motions in Liquid ^{4}He†

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It is shown that average atomic kinetic energies in liquid helium can be deduced, as a function of temperature, in an elementary fashion, using existing experimental data and an empirical interatomic potential. The resulting kinetic energies are compared to the results of fitting recent high-energy neutron scattering data with the simple model of Puff and Tenn.

In past years, many macroscopic thermodynamic properties of ^{4}He have been measured in the laboratory as a function of temperature, and information on the structure and dynamics at the atomic level has been obtained from x-ray and neutron scattering experiments. The purpose of the present paper is to point out that it is possible to deduce average kinetic energies of atoms in liquid ^{4}He as a function of temperature by using these existing thermodynamic and scattering data. While only elementary theoretical principles are involved, these computations seem not to have been reported previously. The resulting kinetic energies are not only of interest in their own right but also provide results which can be used to test future theoretical computations for $T > 0$. As an independent check of our results we also compare our kinetic energy values with those deduced by fitting the model of Puff and Tenn[1] to the high-energy neutron scattering data of Harling.[2,3]

A formal expression for the average kinetic energy per atom can be deduced by considering liquid ^{4}He in thermodynamic equilibrium under its own vapor. In the vapor, the enthalpy per atom, h_v, is well represented by $\frac{5}{2}kT$, while in the liquid the enthalpy per atom, h_l, can be written as $\mathrm{KE} + V + p/\rho$, where KE is the kinetic energy per atom, V the potential energy per atom, p the pres-