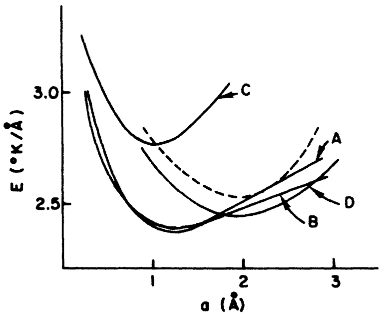

FIG. 1. Energy of a vortex line versus variational core parameter in a cylinder with radius 6 Å. Curves A, B, and C are results of Ref. 1. Dashed line is the representation of Ref. 2 by Ref. 1. Curve D is the actual result of Ref. 2.

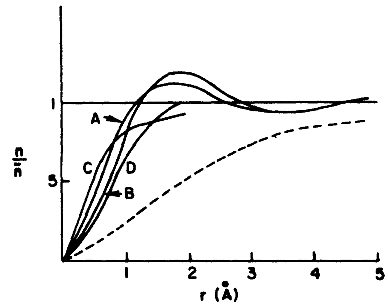

FIG. 2. Density versus distance from the axis of the vortex line. Curves A and B are results of Ref. 1. Dashed line is the representation of Ref. 2 by Ref. 1. Curve C is Fetter's density pattern (Ref. 3). Curve D is the result of Ref. 2.

of our previous results).[2] It is of the order of 2–3%, which is insignificant. The difference in the position of the minimum is merely due to the difference in the nature of the parameter a in Eq. (3.6) of Ref. 2. This is manifest in Fig. 2. In this figure we have superposed curves C and D on Fig. 2 of Ref. 1. Curves A and B represent the density profile of the vortex line obtained by Chester *et al.* Curve C is Fetter's ansatz for the profile,[3] Eq. (3.5) of Ref. 2. Curve D represents our results, Eq. (3.7) of Ref. 2. Figure 2 shows that the only difference between the Hartree calculation and that of Chester *et al.* is in the nonmonotonic behavior of the density beyond 1 Å. There is no significant difference in the size of the core. Notice that the dashed line represents our results in Ref. 1. The nonmonotonic behavior of the density is a result of the inclusion of the pair correlations. This was not attempted in the calculation of Ref. 2. One can account for it in the Hartree approximation by including higher moments of $V(\vec{x}-\vec{y})^2$ than the zeroth one. This will be reported elsewhere.

It is important to emphasize that owing to the extreme simplicity of the Hartree theory, it is very desirable to find and extend the limits of its applicability. In contrast to the obvious objection quoted above, one often finds that hydrodynamic descriptions apply far beyond the regions of variables in which they can be *a priori* justified.

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Evaporation from Helium II

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The report by Johnston and King[1] that He^4 atoms evaporated from the liquid may have a mean temperature exceeding that of the source raised the hope that such experiments might probe the quasiparticle spectrum of He II. It is now known[2] that their findings were due to He-He scattering in the vapor, and that their experimental arrangement is probably unsuitable for observing the effect sought, because of a concentration of normal fluid near the helium surface. However, it seems not without interest to calculate what might be expected under ideal circumstances.

Following Anderson's[3] suggestion that one should observe single-particle elastic emission from the roton excitations on the basis of a phase-space argument, Griffin[4] derived an expression for the evaporation rate in analogy with standard solid-state tunneling theory. The critical quantity in this ex-

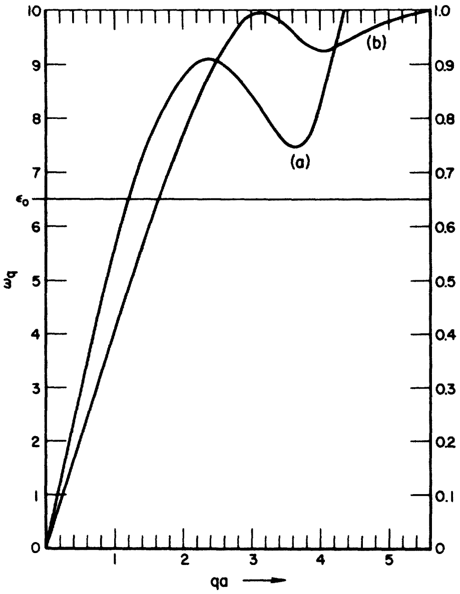

FIG. 1. (a) Quasiparticle spectrum ω_q. (b) Bogolubov coefficient $1-g_q$.

pression is the spectral density function for the liquid,

$$A(\vec{q}, \omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} \langle [A_{\vec{q}}(t),\ A_{\vec{q}}^\dagger(0)]\rangle , \qquad (1)$$

where $A_{\vec{q}}$ is the anihilation operator in the superfluid. In this paper we take advantage of a recent approximate theory of a strongly interacting Bose gas[5] to calculate $A(\vec{q}, \omega)$ explicitly and see what features are most relevant to an ideal evaporation experiment. We find that the crucial quantity is the momentum-dependent coefficient which enters into the Bogolyubov transformation to a quasiparticle representation, and that it is the roton branch which contributes most strongly, as suggested by Anderson.

By assuming that only those He atoms which are emitted essentially perpendicular to the surface are detected and by treating the tunneling coefficients as constant, we find that Griffin's[4] equation (4) can be simplified to

$$\dot{N}(\epsilon p) \propto \int_0^{\infty} e^{-\epsilon p/kT} A(q, \epsilon_0 + \epsilon_p)\, dq , \qquad (2)$$

where ϵ_0 is the latent heat of evaporation at $T = 0$. Following Novaco,[5] and making a Bogolyubov transformation to quasiparticle operators, we find

$$A(q, \omega) = 2\pi(1 - g_q^2)^{-1} [\, \delta(\omega - \omega_q) + g_q^2 \delta(\omega + \omega_q)]. \qquad (3)$$

Next we use Novaco's theory in a phenomenological sense. That is, we adopt his expressions relating g_p and ω_p, the quasiparticle spectrum, but we take ω_p as given by experiment.[6] Thus,

$$g_p = -(A_p - \omega_p)/B_p, \quad \omega_p = (A_p^2 - B_p^2)^{1/2} \qquad (4)$$

and $B_p = 12.14p^{-1} \sin p$. (We adopt reduced units $2ma^2/\hbar^2 = 1$, where a is the boson hard-sphere radius.) This gives the results shown in Fig. 1. Only the first term on the right-hand side of (3) contributes to the integration in (2), which can simply be carried out by inspection of Fig. 1. This leads to the behavior shown in Fig. 2(a), where we have plotted relative intensity against the kinetic energy of the evaporated atom. Since energy $\propto$ (time of flight)$^{-2}$, this leads to an evaporation profile like that in Fig. 2(b). It is interesting to note that this differs from that observed by Johnston and King in the absence of the large exponential tail.

A calculation similar to ours has already been carried out by Hyman, Scully, and Widom.[7] Although also based on a tunneling approach, their calculation differs from ours in that they work in terms of excitations having finite lifetimes rather than in terms of stable quasiparticles. In spite of the predictions of these theories are remarkably alike, as evidenced by the similarity of the roton peak in Fig. 2 of their paper to our Fig. 2(a). (We have not included the phonon contribution to the evaporation rate, as it is quite small in the energy region of interest, but our theory also predicts behavior similar to that shown by Hyman *et al.*) Hyman *et al.* estimate the Lorentzian linewidth γ of the roton excitation from a calculation of roton-

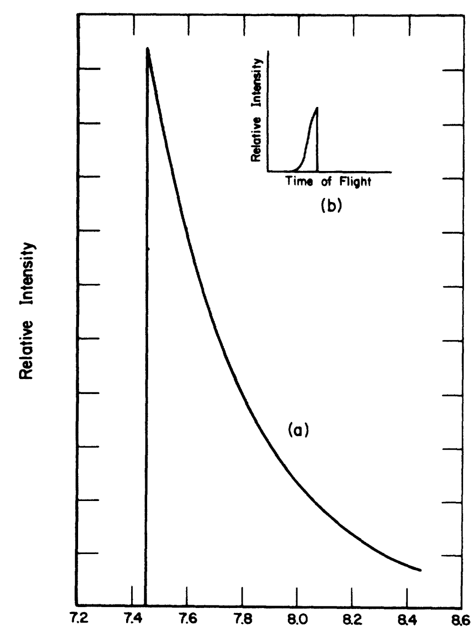

FIG. 2. (a) Equation (2). $\dot{N}$ is in arbitrary units, ϵ_p in dimensionless units. (b) Schematic behavior of predicted evaporation profile for a time-of-flight experiment.

phonon collisions, following Landau and Khalatnikov. They find $\gamma/k_B = 0.001\,^\circ$K. Our curve differs from theirs essentially in width, and for comparison, our result corresponds roughly to $\gamma/k_B \cong 0.005\,^\circ$K. This is quite reasonable, for in adapting a quasiparticle approach, we effectively include additional scattering mechansims leading to a decrease in the lifetime of the excitations.

ACKNOWLEDGMENT

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He^3 Quasiparticle Energy in Superfluid He^4

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A simple classical model is used to estimate the deviations of the He^3 quasiparticle energy in He^4 at $T=0$ from the parabolic form $\epsilon(p) = -E_0 + p^2/2m$. The result is in reasonable agreement with a recent experimental determination of this effect.

A recent analysis[1] of the second-sound velocity in dilute solutions of He^3 in He^4 demonstrates that the He^3 quasiparticle energy deviates from purely parabolic behavior as a function of the momentum p. As pointed out in Ref. 1, this effect can be described by continuing the expansion of the energy in powers of p^2 beyond the first-order term which appears in the theory of Landau and Pomeranchuk[2]:

$$\epsilon(p) = -E_3 + \frac{p^2}{2m}\left[1 - \chi(p^2/p_c^2)\right] \quad , \tag{1}$$

where m is the inertial mass of the quasiparticle in the limit $p \to 0$ and E_3 is the binding energy of the He^3 atom in He^4 at $T=0$; χ is a dimensionless constant and p_c is the characteristic momentum $p_c = m_4 s$, s being the first-sound velocity in He^4 and m_4, the He^4 atomic mass. Brubaker *et al.* find that their data are consistent with a value of $\chi = 0.14 \pm 0.05$.

In this paper a calculation of χ based on a simple classical hydrodynamic model is given. The result is that $\chi \approx 0.18$, in reasonable agreement with the measured value. Specifically, consider the energy associated with the motion of a compressible sphere of radius a_0 (the He^3 atom) through a compressible ideal fluid (the superfluid He^4) at velocity $w_0\hat{e}_3$. The radius a_0 is chosen so that the volume of the sphere is that volume of liquid displaced by a He^3 atom in He^4,

$$\tfrac{4}{3}\pi a_0^3 = (1+\alpha)\, v_4^0/N_0 \quad ,$$

where v_4^0 is the molar volume of He^4 and N_0 is Avogadro's number; the experimental value[3] of α is 0.28. Clearly, this model can be equally well applied to the motion of other impurities in He^4 by changing α accordingly. To determine χ we must keep terms in this energy which are of order w_0^4; i.e., it is necessary to keep the first-order nonlinear corrections in the hydrodynamic equations. The compressibility of He^4 is $K = 1/\rho_0 s^2$, where ρ_0 is the density of pure He^4, while the compressibility of the He^3 impurity K_b may be found from measurements[4] of the molar volume of dilute solutions under pressure. Defining a velocity by $1/s_b^2 \equiv \rho_0 K_b$, this procedure leads to $s^2/s_b^2 \simeq 1.1$.

It is clear that this model is too simple to give a satisfactory quantitative description of the effect found in Ref. 1; nevertheless, it is believed that the calculation is qualitatively valid and of interest as such. The reader should be reminded that the same model predicts a value of $m = 1.83 m_3$, in comparison with the experimental result $m = 2.28 m_3$ found in Ref. 1.

The hydrodynamic equations are the usual ones[5] for He^4; at $T=0$, He^4 is all superfluid and the equations simplify to those for an ideal classical fluid[6]:

$$\nabla \times \vec{v} = 0 \quad , \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \nabla(\rho\vec{v}) = 0 \quad , \tag{3}$$