

Isotope effects in inelastic 1.5-keV  $\text{He}^+$ -( $\text{H}_2, \text{D}_2$ ) collisions

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Linear- and circular-polarization measurements have been made for the  $3^3P \rightarrow 2^3S$  transitions in helium resulting from the collision processes,  $\text{He}^+ + \text{H}_2/\text{D}_2 \rightarrow \text{He}(3^3P) + \text{H}_2^+/\text{D}_2^+$ . This was carried out at an incident-ion lab energy of 1.5 keV and for laboratory scattering angles ranging from 0.50 to 2.33 deg. The behavior of linear polarization for  $\text{H}_2$  and  $\text{D}_2$  targets as a function of scattering angle was found to be different, contrary to what one would expect from simple models that depend only upon the electronic structure of the system. Some correlation of the circular polarization between the two isotopes was found.

## I. INTRODUCTION

The last decade has shown a marked increase in the utilization of polarized photon-scattered particle coincidence measurements to elucidate mechanisms in inelastic ion-atom collision processes.<sup>1</sup> From measurements of  $P$ -state excitation, one can often determine the magnetic substate cross sections  $\sigma(M_L)$ , and the phase differences  $\Delta\phi$  of the associated scattering amplitudes.<sup>2</sup> Such results have shed new light on excitation processes since the observed radiation from the final state can be associated with the relative populations of the intermediate states of the temporary molecule formed during the collision.<sup>3</sup> The analysis of such photon-correlation data, within the context of the independent electron model has been particularly fruitful.

The use of polarized photon-scattered particle correlation measurements to study inelastic processes in ion-atom-molecule collisions has been more recent.<sup>4-6</sup> Owing to the added degrees of internal freedom of the molecule, the physical interpretation of the measured properties of the emitted radiation, in terms of collision mechanisms, is considerably more difficult than in the ion-atom case. Despite this difficulty, new insights into the collision process have begun to emerge.<sup>7,8</sup>

We report here on photon-correlation measurements at 1.5-keV incident ion energy of the process  $\text{He}^+ + \text{H}_2/\text{D}_2 \rightarrow \text{He}(3^3P) + \text{H}_2^+/\text{D}_2^+$  that make direct comparisons between the two isotopes. Electronically, the two target systems are the same, and any differences in the cross sections for the two targets should result from differences in mass, the rotational and/or vibrational structure. In the region of the reported incident ion energy, the collision time is small compared to the typical vibrational and rotational frequencies.

## II. EXPERIMENT

The experimental apparatus used for the present measurements has been discussed in earlier publications.<sup>4,9</sup> A 1.5-keV  $\text{He}^+$  ion beam is incident on a target gas jet of  $\text{H}_2$  (or  $\text{D}_2$ ). The 3889-Å radiation from the  $3^3P \rightarrow 2^3S$  transition of He is detected in a direction perpendicular to the

collision plane; the collision plane being defined by the initial  $\text{He}^+$  momentum and the final  $\text{He}(2^3S)$  momentum. Delayed coincidence measurements between the 3889-Å radiation of a specific polarization and the scattered  $\text{He}(2^3S)$  are made for laboratory scattering angles, varying between 0.50 and 2.33 deg. Circular-polarization measurements are made by inserting a quarter-wave plate before analyzing the polarization. The number of coincidences is normalized to the number of scattered neutral particles, thus minimizing errors introduced by fluctuations in ion beam intensity and scattering gas density. It should be noted that no analysis is done on the initial- or final-state distribution of the targets  $\text{H}_2/\text{D}_2$ .

Examples of typical sets of linear polarization data are shown in the polar plots of the radiation intensity for four specific linear polarization angles, at selected  $\text{He}(2^3S)$  scattering angles (see Fig. 1).

## III. RADIATION MEASUREMENTS

Our task in this section is to relate the measured radiation characteristics to atomic source parameters. There are several ways that this can be done. However, one would like to choose a representation that could possibly

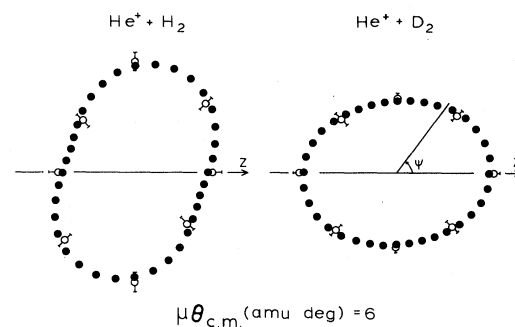


FIG. 1. Polar plots of the linear intensity  $I(\psi)$  for  $\text{H}_2$  and  $\text{D}_2$  at two different scattering angles. The white circles with error bars are data, and the solid circles without error bars are least-squares fits of the data to Eq. (7).

lead to insights into the excitation mechanisms of the collision process. In the case of ion-atom collisions, this is often easy in that a one-to-one correspondence exists between the temporal molecular state quantum numbers to those of the final radiating atomic states. In the case of ion-atom-molecule collisions, the molecular states of the temporal three-particle complex are generally not known.

In an attempt to extend and relate our knowledge and understanding of binary systems to the three heavy-particle collision systems, we have chosen to represent measured polarization intensities in terms of the relative population of the magnetic sublevels of the  $3^3P$  state.

The ensemble of atoms in a single "pure" state formed by a collision, can be represented by a single wave function. When more than one state of the total system (target plus projectile) is accessible, such as in the present  $\text{He}^+ + \text{H}_2/\text{D}_2$  systems, then the ensemble is characterized by a weighted incoherent superposition of pure states.<sup>10</sup> The radiation from such an ensemble of excited atoms can be expressed in terms of alignment and orientation parameters, where the components represent average values over the ensemble.<sup>11,12</sup>

The intensity of this radiation measured in the detector frame is<sup>11,12</sup>

$$I = \frac{1}{3}CS[1 - h^{[2]}G^{[2]}(A_0^{\text{det}} - 3A_{2+}^{\text{det}} \cos 2\beta) + \frac{3}{2}h^{[1]}G^{[1]}O_0^{\text{det}} \sin 2\beta], \quad (1)$$

where  $C$  and  $S$  are constants;  $\beta$  describes the polarization, with  $\beta=0, \pm\pi/4$  for linear, left, and right circularly polarized light, respectively.  $A_0^{\text{det}}$ ,  $A_{2+}^{\text{det}}$ , and  $O_0^{\text{det}}$  are the components of the alignment and orientation parameters as measured in the detector frame.  $G^{[k]}$  is a time-averaged expression accounting for the depolarization of light due to the fine-structure interaction. For the  $3^3P \rightarrow 2^3S$  transition in helium,  $h^{[1]}=2$ ,  $h^{[2]}=-2$ ,  $G^{[1]}=\frac{1}{2}$ , and  $G^{[2]}=\frac{5}{18}$ .

The alignment and orientation parameters in Eq. (1) are defined in terms of the expectation values of the state multipoles.<sup>11</sup> They are related to the intrinsic properties of the radiation source and are a function of the angular coordinates of the photon detector. With the detector along the  $+Y$  axis ( $\theta=\phi=\pi/2$ ), the intensity of the linear radiation ( $\beta=0$ ) can be expressed:

$$I = \frac{1}{3}CS[1 + h^{[2]}G^{[2]}A_0^{\text{col}} - \frac{3}{2}h^{[2]}G^{[2]}(A_0^{\text{col}} - A_{2+}^{\text{col}})\sin^2\psi + \frac{3}{2}h^{[2]}G^{[2]}A_{1+}^{\text{col}}\sin 2\psi], \quad (2)$$

where

$$A_{11} = \langle L_x L_y + L_z L_y \rangle / L(L+1),$$

$$A_0 = \langle 3L_z^2 - L^2 \rangle / L(L+1),$$

$$A_{2+} = \langle L_x^2 - L_y^2 \rangle / L(L+1),$$

of the source.

The resulting polarization intensities can also be described in terms of the set of Stokes parameters,  $P_1, P_2, P_3$ , defined by<sup>13</sup>

$$\begin{aligned} P_1 &= \frac{I(0,0) - I(90,0)}{I(0,0) + I(90,0)}, \\ P_2 &= \frac{I(45,0) - I(135,0)}{I(45,0) + I(135,0)}, \\ P_3 &= \frac{I(45,90) - I(135,90)}{I(45,90) + I(135,90)}, \end{aligned} \quad (3)$$

where  $I(\psi, \alpha)$  is the intensity of the radiation polarized along a direction  $\psi$ , measured, in this case, with respect to the beam axis, and  $\alpha$  is an added phase shift before the intensity is measured.

The parameter  $P_3$  is often called the degree of circular polarization. Positive  $P_3$  corresponds to right-hand circular polarization and a negative value to a left-hand circular polarization.

In terms of the alignment and orientation parameters,

$$\begin{aligned} P_1 &= \frac{3h^{[2]}G^{[2]}(A_0^{\text{col}} - A_{2+}^{\text{col}})}{4 + h^{[2]}G^{[2]}(A_0^{\text{col}} + 3A_{2+}^{\text{col}})}, \\ P_2 &= \frac{6h^{[2]}G^{[2]}A_{1+}^{\text{col}}}{4 + h^{[2]}G^{[2]}(A_0^{\text{col}} + 3A_{2+}^{\text{col}})}, \\ P_3 &= \frac{6h^{[1]}G^{[1]}O_{1-}^{\text{col}}}{4 + h^{[2]}G^{[2]}(A_0^{\text{col}} + 3A_{2+}^{\text{col}})}. \end{aligned} \quad (4)$$

Because more than one final state of the collisional system (target plus projectile) is accessible, such as we would expect for the  $\text{He}^+ - \text{H}_2/\text{D}_2$  systems, the resulting Stokes parameters  $P_n$  are the sum of the parameters characterizing the radiation from each pure state  $i$ ,<sup>14</sup>

$$P_n = \sum_i P_n^i, \quad (5)$$

where each final pure state will be represented by a collection of quantum numbers given by  $i$ . Included in this collection are the various possible initial- and final-state rotational and vibrational quantum numbers of the  $\text{H}_2/\text{D}_2$  and  $\text{H}_2^+/\text{D}_2^+$  molecule.

Experimental results for collision systems are often given in terms of the alignment and orientation parameters.<sup>6</sup> However, we prefer to present our results using Stokes parameters and interpret the results in terms of magnetic substate populations and their relative phases. Since the spin-orbit interaction in He is weak, the  $3^3P$  state can be constructed from linear combinations of the orbital angular momentum eigenstates in the uncoupled representation. Assuming reflection symmetry, the correct quantum-mechanical expression for the linear polarization of the radiation emitted perpendicular to the scattering plane for a single final state of the collisional system in  $\text{He}(3^3P \rightarrow 2^3S)$  transitions can be written<sup>9</sup>

$$\begin{aligned} I^i(\psi) &= C_i[(28\sigma_0^i + 26\sigma_1^i) + (30\sigma_1^i - 15\sigma_0^i)\sin^2\psi \\ &\quad + 15(2\sigma_0^i\sigma_1^i)^{1/2}\cos(\Delta\phi^i)\sin 2\psi]. \end{aligned} \quad (6)$$

$I^i(\psi)$  represents the intensity of radiation from the  $i$ th pure state.  $\sigma_0^i$  and  $\sigma_1^i$  are the differential cross sections for exciting the  $m_L=0$  and  $\pm 1$  magnetic sublevels of the  $i$ th pure state, respectively, and  $\Delta\phi^i$  is the phase angle between the scattering amplitudes for  $m_L=0$  and  $\pm 1$  excitation of

the  $i$ th pure state. The constant  $C_i$  incorporates the relative weighing factor for each state.

The resulting linear intensity from an incoherent superposition of pure states is then

$$I(\psi) = \sum_i I_n^i(\psi) = A + B \sin^2 \psi + C \sin 2\psi, \quad (7)$$

where  $n$  represents the collection of relevant quantum variables and the final momentum of the scattered particle for each pure state and  $A, B, C$  are functions of  $\sigma_0, \sigma_1$ .

The Stokes parameters can now be expressed in terms of the magnetic substate scattering cross sections:

$$P_1 = \frac{k \sum_i (\sigma_0^i - 2\sigma_1^i)}{\sum_i (\sigma_0^i + 2\sigma_1^i)} \equiv k \frac{\sigma(0) - 2\sigma(1)}{\sigma(0) + 2\sigma(1)},$$

$$P_2 = \frac{k \sum_i (2\sigma_0^i \sigma_1^i)^{1/2} \cos(\Delta\phi^i)}{\sum_i (\sigma_0^i + 2\sigma_1^i)}, \quad (8)$$

$$P_3 = \frac{k' \sum_i (2\sigma_0^i \sigma_1^i)^{1/2} \sin(\Delta\phi^i)}{\sum_i (\sigma_0^i + 2\sigma_1^i)}.$$

$k$  and  $k'$  depend on the particular transition. For the He( $3^3P \rightarrow 2^3S$ ) transition,  $k = 15/41$  and  $k' = 27/41$ .

The degree of polarization,<sup>13</sup> expressed in terms of these normalized Stokes parameters, is given by

$$P^2 = 1 - 4 \frac{\sum_{i < j} \sum_{i < j} \frac{2\sigma_0^i \sigma_0^j + 2\sigma_1^i \sigma_1^j - 2(2\sigma_0^i \sigma_0^j)(2\sigma_1^i \sigma_1^j) \cos(\Delta\phi^i - \Delta\phi^j)}{\left[ \sum_i (\sigma_0^i + 2\sigma_1^i) \right]^2}}{\quad} \quad (10)$$

In general  $P^2 < 1$ , since the sum is a positive quantity. However, the condition of complete coherence ( $P^2 = 1$ ) can be met if  $\Delta\phi^i = \Delta\phi^j$  and  $\sigma_0^i/\sigma_1^i = \sigma_0^j/\sigma_1^j$ . That is, the coherency condition is satisfied when the phase differences in each possible final state are the same and when the overall intensities from each pure state of the statistical distributions differ from each other by a multiplicative constant. A statement that  $|P| = 1$ , when many final states are accessible, implies that the excitation to He( $3^3P$ ) is independent of the state variables of the collisional partner or that only one state takes part in the reaction. This seemingly trivial result will later allow us to make statements about the collision process.

#### IV. EXPERIMENTAL RESULTS

In comparing the experimental differences between the H<sub>2</sub> and D<sub>2</sub> targets, one has a difficulty in deciding what physical variables should be chosen. Using the past results of ion-atom collisions as a guide, the relative velocity of the projectile target should be an important variable; therefore, we have performed all our measurements at the same laboratory energy.

Comparisons of differential (in angle) measurements are considerably more difficult because the exact form of the scattering potential is not known. For the same c.m. scattering angle, different regions of the interaction poten-

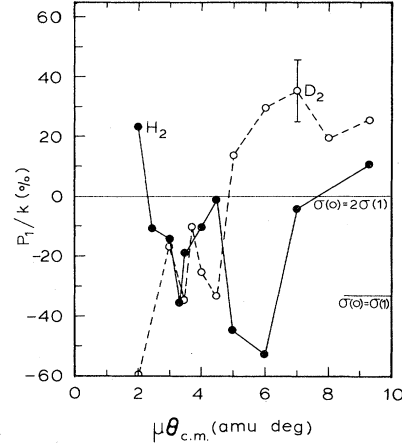


FIG. 2. Stokes parameter  $P_1/k$  for He( $3^3P$ ) resulting from He<sup>+</sup>-(H<sub>2</sub>,D<sub>2</sub>) collisions for selected values of  $\mu\theta_{c.m.}$ .

$$P^2 = (P_1/k)^2 + (P_2/k)^2 + (P_3/k')^2 = (I_{pol}/I_{tot})^2. \quad (9)$$

When a single final state is observed with a single phase  $\Delta\phi^i$ , the above expression reduces identically to the coherency condition:  $P^2 = 1$ . The coherency matrix  $P$ , then, is the ratio of the intensity of polarized light to the total intensity. For completely coherent radiation,  $|P| = 1$ .

Using Eq. (8), the coherency matrix can be written

tial are probed depending upon the state of the target. Again, using the binary systems as a guide, we can attempt some comparisons. The classical c.m. scattering angle  $\theta_{c.m.}$  for a given relative velocity  $v$  and reduced mass  $\mu$  can be written in terms of the classical impact parameter  $\rho$  and the interaction potential  $V(r)$  (assuming a central potential and small scattering angles):

$$\theta_{c.m.} = \frac{\rho}{\frac{1}{2}\mu v^2} \int_{\rho}^{\infty} \frac{dV(r)}{dr} \frac{dr}{(r^2 - \rho^2)^{1/2}}. \quad (11)$$

If we wish to compare scattering that probes the same region of the potential, we compare data for the same impact parameter  $\rho$ ; thus we plot our comparative H<sub>2</sub>/D<sub>2</sub> data relative to  $\mu\theta_{c.m.}$ .

For large impact parameters and some target orientations in which a central potential assumption is valid, this result should hold. Results of our measurements of the linear polarization in 1.5-keV He<sup>+</sup> + H<sub>2</sub> and He<sup>+</sup> + D<sub>2</sub> collisions for selected laboratory scattering angles are shown in Fig. 1. At each scattering angle, the coincidence rate is measured at four polarizer settings, and the intensity distribution is obtained by a best least-squares fit of the data to Eq. (7).

The measured values of  $P_1, P_2$ , and  $P_3$  as a function of  $\mu\theta_{c.m.}$  are shown in Figs. 2, 3, and 4, respectively. The re-

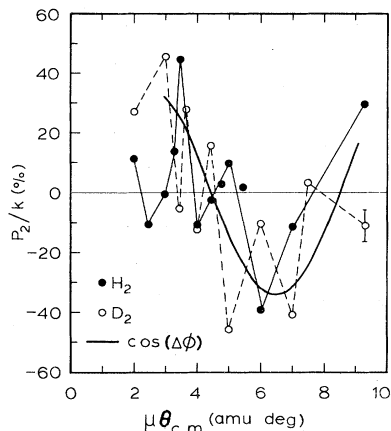


FIG. 3. Stokes parameter  $P_2/k$  for  $\text{He}(3^3P)$  resulting from  $\text{He}^+(\text{H}_2, \text{D}_2)$  collisions for selected values of  $\mu\theta_{c.m.}$ .

sulting values of the polarization  $|P|$  are shown in Fig. 5.  $P_1$  and  $P_2$  are seen to vary rapidly with  $\mu\theta_{c.m.}$ , while the change in  $P_3$  is less abrupt. It is clear that the behavior of  $P_1$  is distinctly different for the two targets.  $P_3$ , the degree of circular polarization, can be seen to change from left-handed to right-handed with increasing  $\mu\theta_{c.m.}$  angle for both  $\text{H}_2$  and  $\text{D}_2$ . This occurs at the same "impact parameter" for both isotopes.

To accentuate further the difference between the  $\text{H}_2$  and  $\text{D}_2$  targets, we compare the electron charge distributions of the  $\text{He}(3^3P)$  formed in the collision. From measured values of  $P_1$ , as shown in Fig. 2, it is clear that the shape of the electron cloud around the excited He is different for the two targets. For  $\mu\theta_{c.m.}=6$  scattering,  $\sigma(0)=0.66\sigma(1)$  for  $\text{H}_2$  while for the same value of  $\mu\theta_{c.m.}$ ,  $\sigma(0)=4.15\sigma(1)$  for  $\text{D}_2$ .

The probability for total  $\text{He}(3^3P)$  excitation as a function of  $\mu\theta_{c.m.}$  is shown in Fig. 6. To make certain that the structure in the total probability of  $\text{D}_2$  is real, we show two sets of total intensity measurements. The linear measurements are  $I(0,0)+I(90,0)$ , the denominator of  $P_1$ , and  $I(45,90)+I(135,90)$ , the denominator of  $P_3$ . By comparing the linear and circular polarization measurements, we find that the quarter-wave plate transmission to be 87.2%. The probability can be seen to increase with

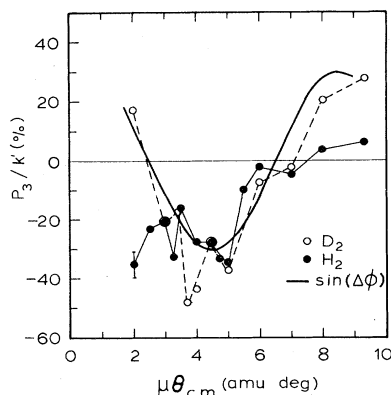


FIG. 4. Stokes parameter  $P_3/k$  for  $\text{He}(3^3P)$  resulting from  $\text{He}^+(\text{H}_2, \text{D}_2)$  collisions for selected values of  $\mu\theta_{c.m.}$ .

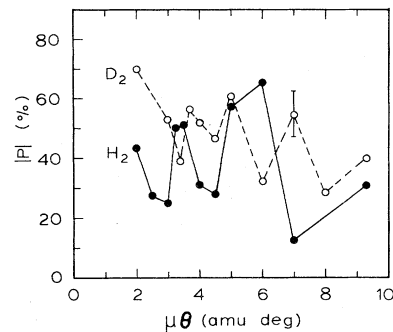


FIG. 5. Degree of polarization  $|P|$  for  $\text{He}(3^3P)$  resulting from  $\text{He}^+(\text{H}_2, \text{D}_2)$  collisions at selected values of  $\mu\theta_{c.m.}$ .

scattering angle, but is larger for  $\text{D}_2$  at all values but one of  $\mu\theta_{c.m.}$  studied. The probability for scattering from  $\text{D}_2$  also shows some oscillatory structure not present in  $\text{H}_2$ .

## V. DISCUSSION

One striking feature of the experimental data is the agreement of  $P_3$  between the two isotopes, as shown in Fig. 4, while  $P_1$ , as indicated in Fig. 2, shows little, if any, such correlation. This is because  $P_1$  and  $P_3$  reflect different properties of the  $\text{He}(3^3P)$  and in turn are sensitive to different aspects of the collision process.

If we consider Eq. (11), we note that the values of  $P_3$  for the two isotopes are approximately the same for equal values of  $\mu\theta_{c.m.}=m_1\theta_{lab}$  over the range of 3–8 amu deg, where  $m_1$  is the projectile mass. If the inelastic processes, measured as a function of  $\theta_{lab}$ , behaved as small-angle potential scattering,  $P_1$ ,  $P_2$ , and  $P_3$  should be independent of target mass. Experiments by Rille *et al.*<sup>15</sup> verify this for  $\text{H}^+/\text{D}^+-\text{H}/\text{D}$  elastic collisions.

Considering Eq. (8), we note that  $P_3$  is dominated by the  $\sin(\Delta\phi^i)$  terms and is clearly the reason why  $P_3$  is negative over a range of  $\mu\theta_{c.m.}$ . The denominator,  $\sum_i(\sigma_0^i+2\sigma_1^i)$ , is a more or less uniformly increasing function of  $\mu\theta_{c.m.}$  as indicated by the total probability behavior, and thus will not dominate the behavior of  $P_3$ . Because of the rather high degree of coherence, as indicat-

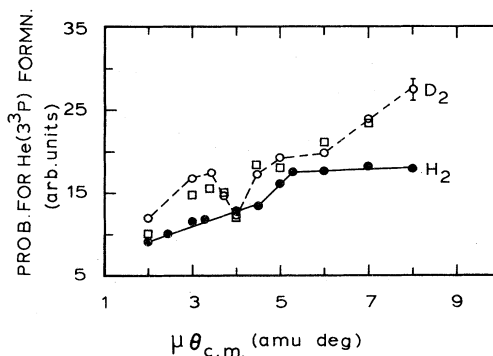


FIG. 6. Probability for  $\text{He}(3^3P)$  formation in  $\text{He}^+(\text{H}_2, \text{D}_2)$  collisions at selected values of  $\mu\theta_{c.m.}$ . Open circles and open squares for  $\text{D}_2$  represent the probability computed from linear and circular polarization measurements, respectively.

ed by the values of  $|P|$  around 40–50%, the sum  $\sum_i (2\sigma_0^i \sigma_1^i)^{1/2} \sin(\Delta\phi^i)$  is dominated by one phase difference of the scattering amplitudes. The experimental results of  $P_3$  suggest that this phase is the same for  $\text{H}_2$  and  $\text{D}_2$  at the same impact parameter.

The results of  $P_2$ , as shown in Fig. 3, also indicate the dominance of the  $\cos(\Delta\phi^i)$  term. To more clearly show the effect of the  $\sin(\Delta\phi)$  and  $\cos(\Delta\phi)$  terms we have superimposed over  $P_3$  and  $P_2$  these functions. Note that when the  $\sin(\Delta\phi)$  term is a minimum at about  $\mu\theta_{\text{c.m.}} = 4.50$  the  $\cos(\Delta\phi)$  term is going through zero as it should. Variations in  $P_3$  and  $P_2$  about these two functions are primarily due to the variations in the coefficients,  $(2\sigma_0^i \sigma_1^i)^{1/2}$ . To further elucidate this effect we have plotted  $\sigma(0)$  and  $\sigma(1)$  separately in Fig. 7 as obtained from the measured values of  $P_1$ . We note that the principal difference in  $P_3$  at 6–9 amu deg is the result of the increased value of  $\sigma(0)$  for  $\text{D}_2$  over that for  $\text{H}_2$ . From Fig. 7, it is clear there is no apparent correlation in  $\sigma(0)$  and  $\sigma(1)$  when comparing the two isotopes, except some overall general increase in  $\sigma(0)$  with increasing  $\mu\theta_{\text{c.m.}}$ .

Taulbjerg *et al.*<sup>16</sup> have derived a scaling law for rotational coupling, a possible mechanism for exciting  $\sigma_1$  in the present systems. Although it is not clear whether the rotational coupling scaling law applies to the present system, we attempted to fit  $\sigma_1$  for  $\text{H}_2$  and  $\text{D}_2$  to it. Their scaling law suggests that our present results should scale as  $\theta_{\text{c.m.}}$  which was not found to be the case.

Our data suggest that the assumption of the fixed nucleus approximation that  $\text{H}_2$  and  $\text{D}_2$  are equivalent, for the inelastic process measured, is not a good one. One possible internal parameter of  $\text{H}_2$  and  $\text{D}_2$  that might affect the process is the initial population distribution of the rotational quantum numbers at room temperature. Summing contributions to the process over all fixed nucleus orientations is equivalent to summing over all magnetic substates  $M_N$  of any one possible rotational state  $N$ . If we allow for the possibility of interference from different orientations within any  $M_N$  state, one could obtain differences between the isotopes, since at room temperature, the

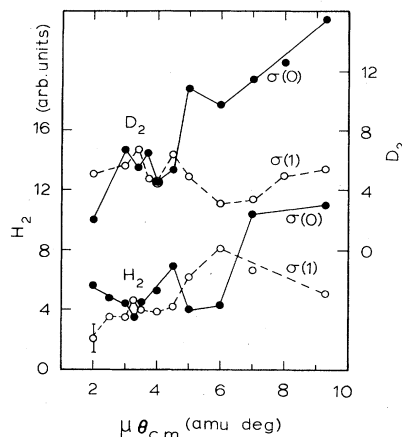


FIG. 7. Values of  $\sigma(0)$  and  $\sigma(1)$  formed in  $\text{He}^+(\text{H}_2, \text{D}_2)$  collisions at selected values of  $\mu\theta_{\text{c.m.}}$ . Scales on the right and left correspond to  $\text{D}_2$  and  $\text{H}_2$ , respectively.

rotational populations are distributed differently. This system merits further investigation so that the role of the internal  $\text{H}_2/\text{D}_2$  variables can be explored, perhaps by carrying out the measurements with cooled targets or with cooled ortho- and parahydrogen separately.

## VI. CONCLUSION

We can conclude that the fixed nucleus approximation is insufficient to explain the differences in  $\text{H}_2$  and  $\text{D}_2$ , namely that the magnetic substate cross sections,  $\sigma(0)$  and  $\sigma(1)$ , behave differently as functions of  $\mu\theta_{\text{c.m.}}$ , while the phase differences  $\Delta\phi$  are rather similar. This suggests that the excitation mechanisms are the same for the two isotopic targets; however, the relative degree to which these mechanisms operate is different.

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