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Use of cavities in squeezed-state generation

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Cavity degenerate-parametric-amplifier and cavity four-wave-mixing configurations capable of producing a large amount of squeezing near the threshold of oscillation are discussed. Cavities allow one to reduce the pump power delivered to the gain medium, or to select a gain medium which has less gain per unit length, but may be better suited for squeezed-state production in other respects.

I. INTRODUCTION

The use of degenerate parametric amplifiers[1-5] in the production of squeezed states has been discussed by a number of authors. Recently, Milburn and Walls[6] published an analysis of a parametric amplifier in which cavity losses were included. With their configuration the squeezing in the signal mode was at most a factor of 2. It is shown here that, by using a cavity configuration in which the quality factor is determined by losses through only one port, the port used for both the input and the output, a large amount of squeezing may be obtained when the degenerate-parametric amplifier is operated near the threshold of oscillation. The input signal and the squeezed output may be separated outside the cavity using a circulator.

Four-wave mixing has been proposed by Yuen and Shapiro[7] as a means of producing squeezed states. In their scheme squeezed light is produced by combining the output beams of the four-wave-mixing medium via a 50/50 beam splitter. In this paper alternative four-wave-mixing schemes are presented in which the four-wave-mixing medium is placed inside a cavity. This leads to simpler optical configurations which may make the experimental demonstration of squeezed-state production more feasible, particularly since high-Q cavities can be exploited to reduce the pump power required to achieve squeezing. Alternatively, high-Q cavities allow one to use four-wave-mixing or parametric-gain media with less gain per unit length, but that may be more suitable for squeezed-state generation.

II. SQUEEZED STATES

In this section some simple mathematical relationships[8] are presented which will make it easy to recognize when the output of a given device is squeezed. These results will greatly simplify the discussion of the specific devices treated in the following sections. Let a and b denote, respectively, the photon annihilation operators for the electromagnetic field mode entering the input port and the mode leaving the output port of some optical device. Suppose the device transforms the a mode into the b mode via the canonical transformation

$$b=\mu a+\nu a^\dagger \ , \tag{2.1}$$

where μ and ν are complex:

$$\mu=|\mu|e^{i\phi_\mu} \ , \quad \nu=|\nu|e^{i\phi_\nu} \ . \tag{2.2}$$

Since the commutation relations $[a,a^\dagger]=1$ and $[b,b^\dagger]=1$ must be maintained, one requires that

$$|\mu|^2-|\nu|^2=1 \ . \tag{2.3}$$

Physically, one requires $|\mu|\geqslant 1$. Let $|0_a\rangle$ denote the vacuum state for the a mode:

$$a|0_a\rangle=0 \ , \tag{2.4}$$

then for some phase the output of the device is squeezed, provided $\mu>1$, as is now demonstrated.

Let

$$\chi_1(\theta_x)=e^{i\theta_x}b+e^{-i\theta_x}b^\dagger \ , \quad \chi_2(\theta_x)=ie^{i\theta_x}b-ie^{-i\theta_x}b^\dagger \ , \tag{2.5}$$

denote the operators for the two quadrature field amplitudes of the b mode. Then one can readily show

$$\langle 0_a|\chi_1^2(\theta_x)|0_a\rangle=|\mu|^2+|\nu|^2+2|\mu||\nu|\cos(2\theta_x+\phi_\mu+\phi_\nu) \ , \tag{2.6a}$$

$$\langle 0_a|\chi_2^2(\theta_x)|0_a\rangle=|\mu|^2+|\nu|^2-2|\mu||\nu|\cos(2\theta_x+\phi_\mu+\phi_\nu) \ , \tag{2.6b}$$

The fluctuations in $\chi_1(\theta)$ and $\chi_2(\theta)$ for the state $|0_a\rangle$ are, respectively, maximized and minimized when

$$2\theta_x+\phi_\mu+\phi_\nu=0 \ . \tag{2.7}$$

As shown by Yuen and Shapiro[9] a homodyne detector measures $\chi_1(\theta_x)$ or $\chi_2(\theta_x)$. The phase angle θ_x can be chosen to satisfy (2.7) by adjusting the local oscillator phase.

Using Eqs. (2.3) and (2.7), Eqs. (2.6a) and (2.6b) reduce to

$$\langle 0_a|\chi_1^2|0_a\rangle=2|\mu|^2-1+2|\mu|(|\mu|^2-1)^{1/2} \ , \tag{2.8a}$$

$$\langle 0_a|\chi_2^2|0_a\rangle=2|\mu|^2-1-2|\mu|(|\mu|^2-1)^{1/2} \ . \tag{2.8b}$$

Let $|0_b\rangle$ denote the vacuum state for the b mode, $b|0_b\rangle=0$, then

$$\langle 0_b|\chi_1^2|0_b\rangle=\langle 0_b|\chi_2^2|0_b\rangle=1 \ . \tag{2.9}$$

For $|\mu|>1$ it is easily demonstrated that the fluctuations in χ_2 are reduced below the vacuum fluctuations of Eq. (2.9). For large $|\mu|$, (2.8b) becomes, to lowest order in $|\mu|^{-2}$,

$$\langle 0_a|\chi_2^2|0_a\rangle\simeq\frac{1}{4|\mu|^2}+O\left(\frac{1}{|\mu|^4}\right) \ . \tag{2.10}$$

Hence, for large $|\mu|$, the quantum fluctuations in χ_2 are reduced by a factor of $4|\mu|^2$ over the vacuum fluctuations, that is, the signal becomes highly squeezed. To summarize, when the input-output relation for an optical device is of the form (2.1) with $|\mu| > 1$, then squeezing occurs in one quadrature component of the signal, and the amount of squeezing becomes large as $|\mu|$ becomes large.

III. CAVITY DEGENERATE-PARAMETRIC AMPLIFIERS

The degenerate-parametric-amplifier configuration to be considered is shown in Fig. 1. The cavity consists of one partly reflecting mirror and one totally reflecting mirror. The input optical mode is a_R and the output optical mode is a_L. The subscripts L and R indicate, respectively, whether the mode is propagating to the left or to the right. A delay line is included as a convenient way, from a computational point of view, of adjusting the cavity length. The nonlinear medium providing the parametric gain is pumped at twice the signal frequency. It is now demonstrated that the device of Fig. 1 is capable of producing a large amount of squeezing near the threshold of oscillation.

The scattering matrices for the various components of this optical system are given below. For the partly reflecting mirror

$$b_R = K a_R - (1-K^2)^{1/2} b_L \ , \qquad a_L = (1-K^2)^{1/2} a_R + K b_L \ , \tag{3.1}$$

where, for convenience, K is taken to be real and positive. For the delay line

$$c_R = e^{i\theta} b_R \ , \qquad b_L = e^{i\theta} c_L \ , \tag{3.2}$$

where θ is the delay-line phase shift. For the parametric medium[3,4]

$$d_R = G_s c_R + e^{i\phi}(G_s^2-1)^{1/2} c_R^\dagger \ , \qquad c_L = G_s d_L + e^{i\phi}(G_s^2-1)^{1/2} d_L^\dagger \ , \tag{3.3}$$

where the gain G_s is taken to be real, and $G_s > 1$. For the totally reflecting mirror,

$$d_L = d_R \ . \tag{3.4}$$

Equations (3.3) and (3.4) can be solved to express c_L totally in terms of c_R. The result is

$$c_L = G c_R + e^{i\phi}(G^2-1)^{1/2} c_R^\dagger \ , \tag{3.5}$$

where the round trip gain is related to the single pass gain via

$$G = 2G_s^2 - 1 \ .$$

Equations (3.1), (3.2), and (3.5) can be solved to express a_L in terms of a_R. The result is

$$a_L = G' a_R + e^{i\phi}(|G'|^2-1)^{1/2} a_R^\dagger \ , \tag{3.6}$$

where G' is a complex number given by

$$G' = \frac{2(1-K^2)^{1/2} + G e^{i2\theta} + (1-K^2) G e^{-i2\theta}}{2 - K^2 + 2(1-K^2)^{1/2} G \cos 2\theta} \ . \tag{3.7}$$

The gain G' can be made very large by making the denominator of (3.7) go to zero provided the numerator remains finite. The denominator goes to zero when

$$\cos 2\theta = \frac{K^2-2}{2(1-K^2)^{1/2} G} \ . \tag{3.8}$$

The numerator of (3.7) remains finite, taking on the value

$$-\frac{K^2}{2(1-K^2)^{1/2}} + iK^2 G \sin 2\theta \ ,$$

when (3.8) is satisfied. Equation (3.6) has the form (2.1), and it has been shown that G' can be made large provided K and G are chosen such that an angle θ can be found satisfying (3.8), that is, one requires

$$\left| \frac{K^2-2}{2(1-K^2)^{1/2} G} \right| < 1 \ . \tag{3.9}$$

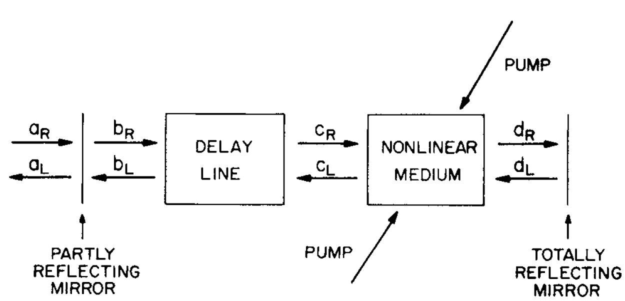

FIG. 1. Cavity degenerate-parametric-amplifier or four-wave-mixing configuration capable of producing a highly squeezed output. The partly reflecting mirror serves both as the input and output port. a_R, b_R, c_R, and d_R denote the annihilation operators for right propagating optical modes. Similarly, a_L, b_L, c_L, and d_L denote the annihilation operators for left propagating modes.

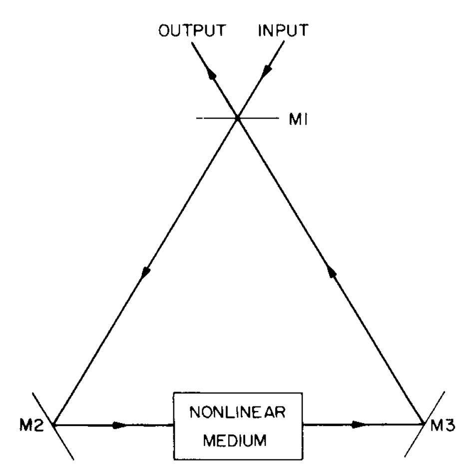

FIG. 2. Ring configuration for a degenerate-parametric amplifier capable of producing a highly squeezed output. This configuration has the advantage that the input and output beams are already separated. The input and output beams in the device of Fig. 1 can, of course, be separated with a circulator. Mirrors M2 and M3 are totally reflecting.

Hence, from the discussion of Sec. II, one sees that a large amount of squeezing is produced in the a_L mode when the gain and cavity parameters are adjusted to bring the system close to the threshold of oscillation. The incoming a_R and outgoing a_L modes can be separated using a circulator. Alternatively, as was pointed out to the author by Milburn,[10] a ring configuration as shown in Fig. 2 can be used to obtain separated input and output beams. The analysis for this parametric-amplifier configuration follows essentially the same lines as above.

IV. CAVITY FOUR-WAVE MIXERS

In this section the case is considered when the third-order nonlinear polarization of the nonlinear medium of Fig. 1 is used to do four-wave mixing of the two pump beams with the two signal beams. In this case the nonlinear medium is pumped at the signal frequency. It is shown in this section that this device is also capable of producing a large amount of squeezing near the threshold of oscillation. Instead of (3.3) the scattering matrix for the nonlinear medium now has the form[7,11]

$$d_R = e^{i\phi}(G_s^2-1)^{1/2}d_L^\dagger + G_s c_R \ ,$$
$$c_L = G_s d_L + e^{i\phi}(G_s^2-1)^{1/2}c_R^\dagger \ . \tag{4.1}$$

With the use of (3.4) Eqs. (4.1) can be solved to express c_L only in terms of c_R. The result is

$$c_L = Gc_R + e^{i\phi}(G^2-1)^{1/2}c_R^\dagger \ , \tag{4.2}$$

where G is now given by

$$G = \frac{G_s^2}{2-G_s^2} \ . \tag{4.3}$$

Note that G becomes very large as $G_s^2 \rightarrow 2$. In this limit the reflectivity $(G_s^2-1)^{1/2}$ of the gain medium becomes unity. Hence, the device depicted in Fig. 3, consisting of a totally reflecting mirror, and the four-wave-mixing medium which acts as a phase conjugating mirror, produces a large amount of squeezing near the threshold of oscillation. This configuration requires a reflectivity near unity to produce substantial squeezing. In contrast, the device of Fig. 1 can achieve a large amount of squeezing even when the reflectivity of the four-wave-mixing medium is small. This can easily be seen by noting that (4.2) is identical to (3.5). Hence, the discussion surrounding Eqs. (3.6) through (3.9) applies equally well to the cavity four-wave mixer.

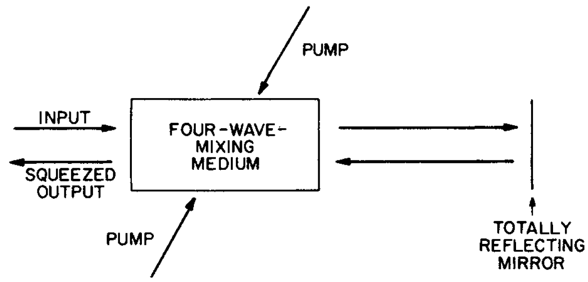

FIG. 3. Four-wave-mixing configuration whose output becomes highly squeezed as the reflectivity of the nonlinear medium, acting as a phase-conjugating mirror, approaches unity.

In summary, it has been shown that a large amount of squeezing can be obtained from a weak parametric-gain medium, or a four-wave-mixing medium with low reflectivity provided the medium is enclosed in a cavity of sufficiently high Q that the system can be brought near the threshold of oscillation. The ideal case when the Q is determined solely by one cavity port which is used as both the input and output port of the device has been treated here. Other losses will, in general, degrade the performance of the device. From a practical standpoint high-Q cavities allow one to select, from a much broader range of materials, nonlinear media suitable for squeezed-state generation. Alternatively, they allow one to reduce the pump power which, in turn, will reduce the problem of self-focusing or defocusing in the nonlinear medium and reduce the stray light entering the detectors from the pump beam.

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