Unusual transition sequence in Taylor wavy vortex flow

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When the Reynolds number for a Taylor-Couette flow with a radius ratio $\eta = 0.782$ is ramped quasistatically, a wavy vortex flow with an azimuthal m number $m = 2$ is excited at a transition Reynolds number of $R = 137$. This mode disappears at $R = 161$, and the flow returns to the Taylor vortex state and remains in this state until a new wavy state with $m = 3$ is excited at $R = 322$. This unusual sequence of flow evolution seems to be consistent with the stability boundaries calculated by Jones.

The wavy vortex state of Taylor-Couette flow has received considerable attention from experimentalists in recent years.[1-5] The driving force behind much of these efforts has been the desire to understand the transition to turbulence in this model system. Most of these experiments were done in "narrow-gap" apparatuses characterized by $\eta > 0.85$, where η is the ratio of the inner radius to the outer radius. The Taylor-Couette flow in a narrow-gap regime evolves from a simpler state to a state of greater complexity through a series of transitions. The wavy state, having evolved from the Taylor vortex state with a small azimuthal m number, undergoes a series of transitions, in succession, where the azimuthal m number increases at each transition. States with differing m numbers are connected by narrow regions of m-number indeterminacy characterized by increased spectral noise.[6] Eventually, the flow obtains another frequency incommensurate with the first. At a still higher Reynolds number, the system becomes weakly turbulent.[7]

We report here an observation of a different sequence of flow evolution of in an apparatus with a gap which apparently lies on the boundary that separates the "wide-gap" and the "narrow-gap" regimes. Our Couette apparatus has glass inner and outer cylinders of radii $R_1 = 2.413$ cm and $R_2 = 3.085$ cm with gap $d = 0.672$ cm and radius ratio $\eta = 0.782$. The length of the annular region is fixed by a pair of nonrotating movable plugs. The fluid consists of a glycerol-water mixture with a kalliroscope solution added (3% by volume) as a tracer. The kinematic viscosity of the solution is 6.905 cS (centistokes) at 20 °C with a slope of -0.33 cS/°C in the temperature range of ~ 19–21 °C. The entire apparatus is immersed in a water bath, and its temperature is continually recorded and used later to calculate the Reynolds number. The typical temperature excursion encountered during a 12-h period is 0.2 °C. The rate of change in the Reynolds number due to this temperature drift is negligibly small in our measurements.

A focused beam of light from a helium-neon laser is directed to the center of a vortex located at the middle of the apparatus. The light scattered by the Kalliroscope flakes is gathered by $f/3.5$ optics and detected by an RCA 1P21 photomultiplier. The output signal is then amplified and fed to a digitizer. Timing signals for the entire experiment, including the motor drive pulses for the inner cylinder, are derived from a single real-time clock with 1-μsec timing accuracy. Except for the initialization phase described below, the entire management of the experiment, including data acquisition and on-line data analysis, is done by a microcomputer.

The initialization process for each run consists of the following three steps. First, the flow is ramped upward from rest to $R/R_c \simeq 1.1$ with an acceleration of one Reynolds number per second. Second, the length of the apparatus and the desired number of vortex pairs are then set by adjusting the top plug manually. Third, the Taylor vortex flow is allowed to equilibrate for 30 min, and then cathetometer readings are taken of locations of pair boundaries. Upon completion of this initialization step, the measurement phase is started. The Reynolds number is ramped upward to a specified final value with the rate dR/dt as a parameter of each run. During ramping, at specified times (and therefore at specified Reynolds numbers), the reflectance signal is sampled and digitized 40 times during one revolution of the inner cylinder. The rms value of the ac component of this signal (which corresponds to the wave amplitude) is evaluated and stored.

One such set of data is presented in Fig. 1. Here, the length $L = 32.42$ cm, the number of Taylor vortex pairs $N = 25$, and the acceleration $dR/dt = 0.0097$ Reynolds numbers per second (this unit shall be shortened simply as $\sec^{-1}$ in the rest of this paper). Three locations marked by arrows are of particular interest. A wavy vortex flow sets in at $R = 137$, only to vanish at $R = 161$, and then to reappear at $R = 322$. These transitions are extremely sharp as shown in Fig. 2. The six panels show Fourier spectra of the reflec-

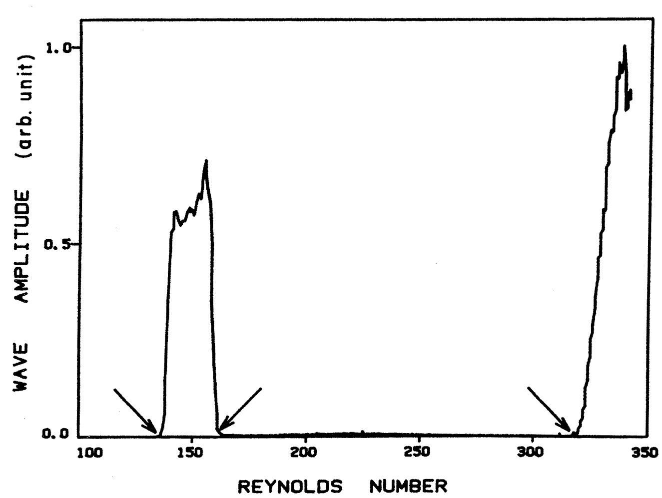

FIG. 1. Wave amplitude (in arbitrary units) as a function of Reynolds number for an acceleration $dR/dt = 0.0097$ $\sec^{-1}$, $\Gamma = 48.26$, and $N = 25$.

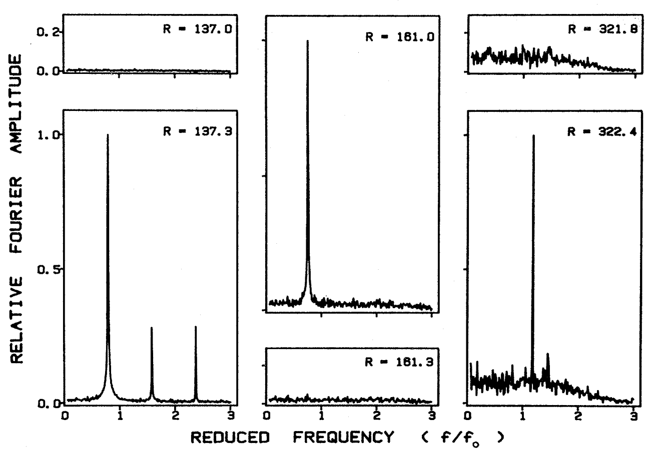

FIG. 2. Fourier spectra near the three arrows in Fig. 1. Each vertical pair have common ordinate scaling and therefore the Fourier amplitudes may be compared directly. The symbol f_0 stands for the frequency of rotation of the inner cylinder.

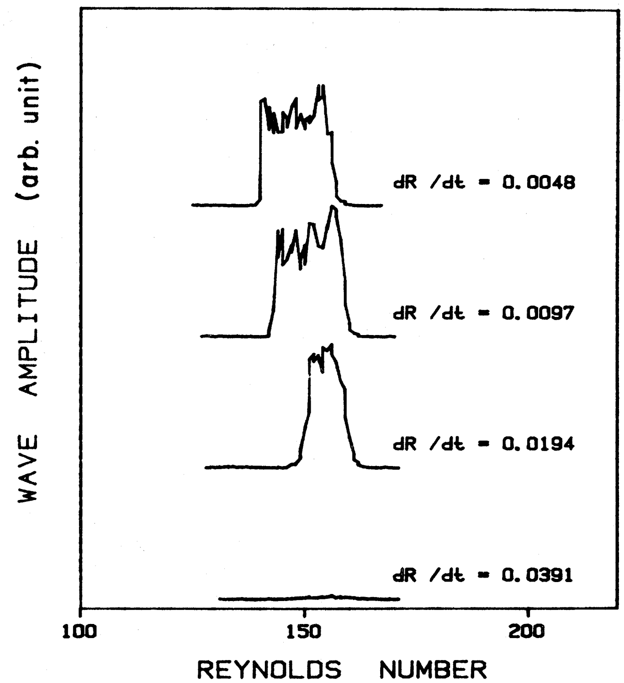

FIG. 3. Wave amplitude vs Reynolds number for four different ramping rates. In these runs $\Gamma = 48.73$ and $N = 25$.

tance signal near these transition points. These spectra are taken of stationary flows; i.e., no ramping is in progress during the data acquisition. We note that a change in Reynolds number of something less than 0.3% is sufficient to cause these transitions.

The existence of the wavy mode in the range $137 < R < 161$ was not a complete surprise. About five years ago, we were alerted to a calculation by Jones,[8] which suggested stability boundaries that could lead to this type of behavior within a certain range of radius ratios. Our search for this mode failed. The reexamination of experimental procedure used then reveals that the failure was due to too rapid a ramping rate. Figure 3 illustrates this point. Four runs, differing only by the ramping parameter, are shown. From analyses of data from other sets of runs where slower accelerations were used, we conclude that an acceleration value of 0.0048 $\sec^{-1}$ is as small as it needs to be. The lowest trace in Fig. 3, taken with $dR/dt = 0.0391$ $\sec^{-1}$ shows that this mode can be missed entirely when the system is ramped too rapidly. It is to be noted that the value of $dR/dt = 0.0391$ $\sec^{-1}$ corresponds to a change of Reynolds number by 1 in 25 sec. The ramping rate we used five years ago, which was then considered a very slow rate, is about 20 times faster than this value.

The first wavy mode is an $m = 2$ mode, while the second mode at $R = 322$ is an $m = 3$ mode. Table I compares the present results with the theoretical calculation of Jones.[8] The disagreement regarding the $m = 1$ mode has to do with the stationary ends in our apparatus. These ends should act as effective stabilizers against the $m = 1$ wavy mode, and the discrepancy is therefore of no consequence. The onset of a wavy mode with $m = 3$ is outside the range of calculations presented by Jones, and agreement may yet be demonstrated in a further calculation. What is remarkable is the qualitatively good agreement that exists between the experiment and theory regarding the $m = 2$ mode. Theoretical calculations using an infinite-length approximation invariably give results that disagree with experiments which, of necessity, are done in finite-length apparatuses. The success of Jones's prediction of this interesting stability boundary for the $m = 2$ mode encourages us to examine other parts of his calculations. Several related measurements are in progress, and will be reported later.

We investigated the dependence of this mode on the aspect ratio $\Gamma = L/d$ using dR/dt of 0.0097 $\sec^{-1}$. We are unable to excite this mode in geometries where $\Gamma < 41.3$. The wavy mode at $R = 322$, however, is still there at these shorter lengths. In the range of Γ from 43.3 to 49.5 (the latter being the largest value attainable in our apparatus),

TABLE I. Comparison of experiment and theory (entries are Reynolds numbers where appropriate).

	Experiment	Theory
Aspect ratio, L/d	48.26	∞
Pair size in units of d	1.916	1.993
Onset of $m = 1$	Not seen	110
Onset of $m = 2$	137.3	120
$m = 2$ gone	161.3	163
$m = 1$ gone	Not seen	169
Onset of $m = 3$	322	?

TABLE II. Dependence of transition Reynolds numbers on aspect ratio and pair size.

L/d	N	Pair size	Onset at	Gone at
41.32	21	1.95	Not seen	Not seen
43.32	22	1.95	144	151
45.39	23	1.96	140	153
47.42	24	1.96	141	157
49.46	25	1.95	146	160
	25	1.94	142	162
	25	1.93	143	160
	25	1.91	137	161
	25	1.89	142	161
	25	1.84	145	160
	25	1.80	147	162

the transition points vary little as shown in Table II. We also investigated the dependence on the vortex size. While holding $N=25$, L is varied, thus obtaining vortex pair sizes in the range of $1.95\times d$ to $1.80\times d$. Again, we observe a weak dependence as shown in Table II. The Reynolds number entries in this table are obtained from the ac amplitude measurements described above, and therefore are reliable only to about 5%.

We have reported here a measurement of unusual stability boundaries for a wavy vortex mode with $m=2$. There is a limited but an important agreement between our results and Jones's nonlinear calculations.[8] This mode is observable only when the Reynolds number is varied quasistatically, in an apparatus of sufficient length to accommodate 22 or more pairs of vortices.

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