Electron-hydrogen scattering in an intense laser field

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The dependence of the differential cross section for laser-assisted elastic electron-hydrogen scattering on the direction of propagation of the laser beam is studied by discarding the usual dipole approximation for the incident electron wave function. Representative numerical results are presented for a Nd-glass laser, which display a significant dependence at intensities of the order of 10^{11}–10^{12} W/cm^2.

Scattering of electrons from hydrogen atoms in the presence of a laser field has been studied in the dipole approximation by many authors.[1,2] We have recently carried out detailed calculations[3] employing the Volkov wave function for the incident electron and a second-order perturbation solution for the atomic wave function.[1] All these calculations fail to bring out any possible dependence of the scattering cross sections on the direction of the laser beam because of the underlying dipole approximation. The present work focuses on the effect of spatial variations of the electromagnetic field on the elastic scattering process, insofar as it modifies the incident electron wave function, since, at optical laser frequencies, the dipole approximation should be adequate as far as the bound system is concerned. Further, confining to field strengths of the order of 10^7 V/cm, which is much less than typical atomic field strengths, one may use the same wave function referred to earlier,[1] given explicitly by

$$\psi_n(\vec{x},t)=\phi_n(\vec{x})\exp\left[-i(E_n+\Delta E_n)t+i\xi_n\sin 2\omega t + i\frac{\vec{x}\cdot\vec{A}_0}{c}\cos\omega t\right] , \quad (1)$$

where ϕ_n denotes the unperturbed atomic wave function corresponding to energy E_n. (We use atomic units throughout.) The electromagnetic field is described, in the Coulomb gauge, by a vector potential

$$\vec{A}=\vec{A}_0\cos\omega\nu, \quad \nu=t-\hat{n}\cdot\vec{x}/c$$

propagating in the direction $\hat{n}$ [which does not figure in Eq. (1) owing to the dipole approximation]. The constants ΔE_n and ξ_n are of no consequence here, since they drop out of the matrix elements for elastic scattering. For the incident electron, we may take the solution of Ehlotsky,[4] which is accurate enough for values of A_0 and ω we are concerned with in this work, and has the form

$$\chi_{\vec{k}}(\vec{x},t)=\exp[-i(Et-\vec{k}\cdot\vec{x})-\alpha\sin\omega\nu+\beta\sin 2\omega\nu+\gamma\nu] , \quad (2)$$

where

$$\alpha=\mu\vec{k}\cdot\vec{A}_0/A_0\omega\delta, \quad \mu=A_0/c ,$$

$$\delta=1-\vec{k}\cdot\hat{n}/c, \quad \beta=\mu^2/8\omega\delta, \quad \gamma=2\omega\beta ,$$

and $\vec{k}$ denotes the incident momentum.

The S matrix for direct electron-hydrogen elastic scattering in the Born approximation is then given by $(i\neq f)$

$$S_{fi}=-i\int dt\, d\vec{r}_1\, d\vec{r}_2\, \chi^*_{\vec{k}_f}(\vec{r}_1)\psi_0(\vec{r}_2) \times\left[\frac{1}{|\vec{r}_1-\vec{r}_2|}-\frac{1}{r_1}\right]\chi_{\vec{k}_i}(\vec{r}_1)\psi_0(\vec{r}_2) . \quad (3)$$

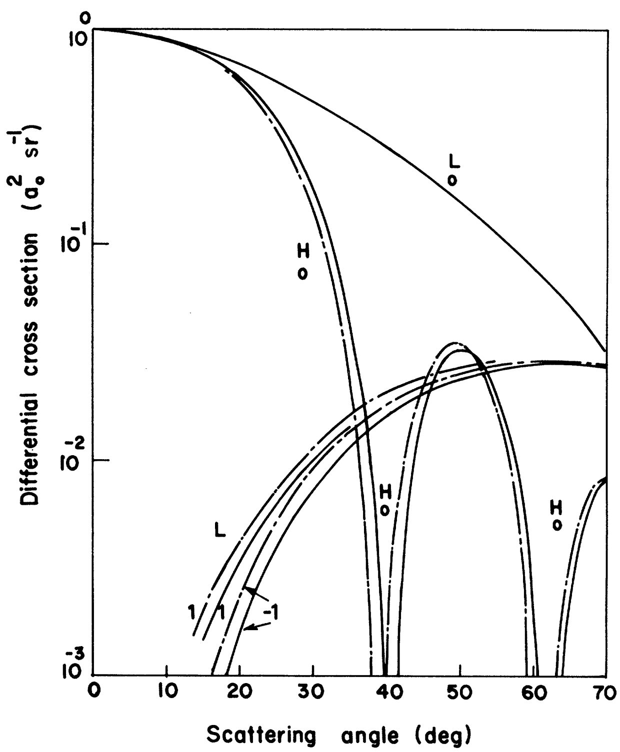

FIG. 1. Differential cross sections for the elastic scattering of 54.4-eV electrons from hydrogen atoms in a Nd-glass laser field, plane polarized along the incident electron direction. Solid lines are for $\vec{k}_f\cdot\hat{n}>0$ and dash-dot lines correspond to $\vec{k}_f\cdot\hat{n}<0$. The numerals give the values of n, the number of photons emitted (positive) or absorbed (negative). L refers to a field strength of 10^7 V/cm and H to 5×10^7 V/cm.

As usual, separating out the time dependence by means of the Bessel function expansion and using the correct incident flux[4] leads to the differential scattering cross section for the n-photon process (n positive for emission and negative for absorption).

$$\frac{d\sigma_n}{d\Omega}=4\frac{k_f(n)}{k_i}\left[\frac{Q_n^2+8}{(Q_n^2+4)^2}\right]^2\left\{\left[\sum_{r=-\infty}^{\infty}J_{n-2r}(\alpha_i-\alpha_f)J_r(\beta_i-\beta_f)\right]^2\Big/\left[1+\mu^2\left[\frac{\vec{k}_f(n)}{k_f^2(n)}+\frac{\vec{k}_i}{k_i^2}\right]\cdot\frac{\hat{n}}{4c}\right]\right\}, \tag{4}$$

where

$$\vec{Q}_n=\vec{k}_f(n)-\vec{k}_i+\hat{n}(\gamma_f-\gamma_i+n\omega)/c$$

and $k_f(n)$ is given by

$$k_f^2(n)=k_i^2+2(\gamma_i-\gamma_f-n\omega) .$$

In Eq. (4) the denominator in square brackets is correct only to terms of order μ^2, which is clearly sufficient for the present calculations.

The corresponding cross section in the dipole approximation is readily obtained from Eq. (4) by dropping all $\hat{n}$-dependent terms.

Some representative numerical results for electron scattering at $k_i=2$ ($E_i=54.4$ eV) in the presence of a Nd-glass laser field, plane polarized along $\vec{k}_i$, are presented in Fig. 1. The scattering plane is the one containing $\vec{k}_i$ and $\hat{n}$. The numerals indexing the various curves indicate the values of n. At a field strength of 10^7 V/cm, the zero photon process (which completely dominates small-angle scattering[3]) is practically insensitive to the beam orientation, and is well represented by the single curve labeled L in the figure. Moreover, the curve also represents the results of the dipole approximation. However, the dependence on the beam direction is manifest in the pair of curves marked H, which refers to the same event, but with a field 5 times as strong. For $n\neq 0$, this dependence is present even for a field of 10^7 V/cm, as illustrated in Fig. 1 for $n=\pm 1$. (The dipole results in all these cases lie in between the corresponding pair of curves.) An order of magnitude estimation of the various factors in Eq. (4) reveals that for field strengths considered here, the $\hat{n}$ dependence of all terms except $J_n(\alpha_i-\alpha_f)$ is of little consequence. The observed modifications are due, in the main, to the fact that a small change in $(\alpha_i-\alpha_f)$ produces a significant change in $J_n(\alpha_i-\alpha_f)$, where the latter is falling or rising rapidly. In conclusion, the results presented in this paper would suggest that a marked dependence of the cross section on the relative direction of the laser beam may be expected, with laser intensities of the order 10^{11}–10^{12} W/cm^2.

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