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Transient noise-induced optical bistability

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We solve numerically the simplest Fokker-Planck equation which describes the effects of amplitude noise in absorbitive optical bistability. When the semiclassical solution exhibits critical slowing down, the probability distribution becomes two-peaked in a sizable time interval during the approach to the one-peaked steady-state distribution. The switching time undergoes considerable fluctuations, and on the average it is shorter than predicted by semiclassical theory. These noise effects seem accessible to experimental observation.

Starting from Ref. 1 a large number of works were devoted to the analysis of fluctuations in optical bistability (OB). One of the most interesting results is the two-peaked character of the steady-state intensity probability distribution in the bistability region. However, there is only one experimental work[2] which reports a double-peaked distribution (DPD). This is obtained by using a hybrid device in which fluctuations are artificially and skillfully introduced and controlled. On the other hand, an observation of the DPD at steady state with ordinary fluctuations seems difficult because the lifetime of the two metastable states is tremendously long.

In this paper we propose a procedure that produces a novel kind of OB, such that the observation of the DPD is experimentally accessible. This situation occurs *in the transient,* and in correspondence to values of the incident intensity for which the system has only one semiclassical stationary state. Precisely, let us consider the typical experiment on critical slowing down in which the incident field is abruptly switched on to a value slightly larger than the up-switching threshold y_M of the system (Fig. 1). As is well known from the semiclassical theory (Ref. 3 and Fig. 2), the time evolution exhibits a long lethargic stage, followed by a rapid switching.

As we show in this paper, the statistical treatment predicts that there is an observable time interval during which the intensity probability distribution becomes two peaked, which corresponds to an OB of statistical type. Another result is that the switching time undergoes remarkable fluctuations, such that the average switching time can be, according to the noise level in the system, sensibly smaller than the one predicted by the semiclassical theory (e.g., by a factor 2). This is relevant for the overall switching behavior, because it shows that the lengthening of the evolution due to the critical slowing down can be in part counteracted by noise.

We stress that this transient bimodality, which arises exclusively from noise, is a phenomenon of general type. Nicolis and collaborators[4] first predicted it in the case of combustion and suggested that the same phenomenon arises

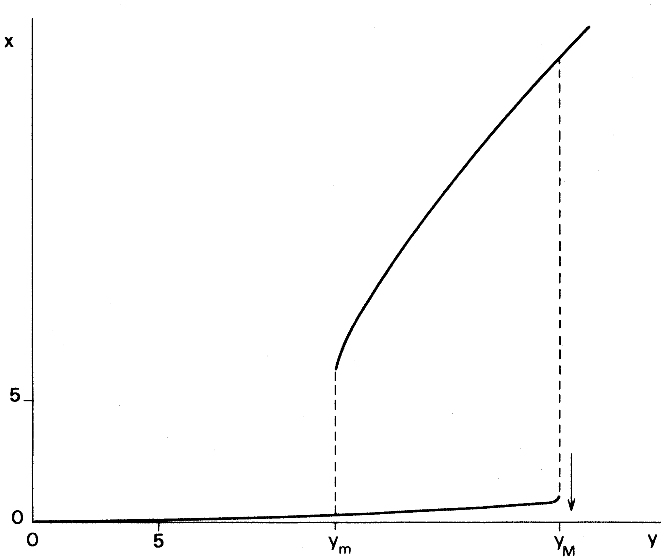

FIG. 1. Hysteresis cycle of normalized transmitted field x as a function of normalized incident field y for $C=20$, according to the mean-field state equation of absorbitive OB $y=x+2Cx/(1+x^2)$. The arrow indicates a value of the incident field slightly larger than the up-switching threshold $y=21.0264$.

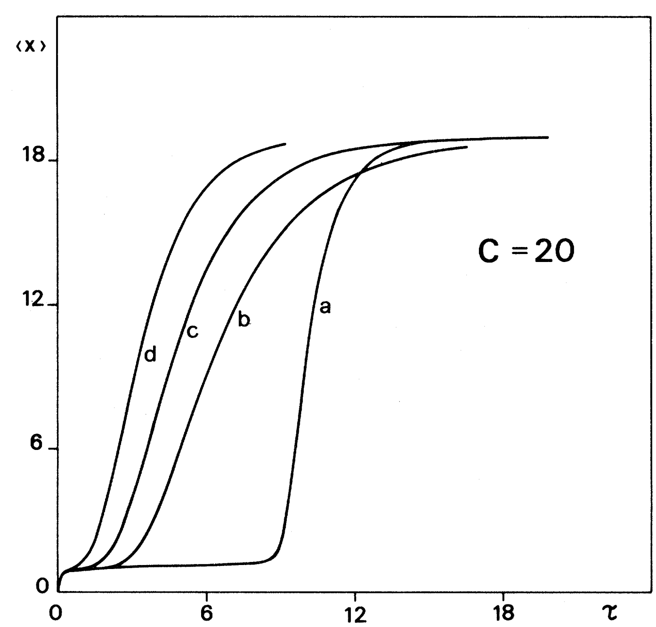

FIG. 2. Time evolution of the mean value $\langle x\rangle$ of the transmitted field when the incident field is changed stepwise from zero to the value $y=21.04$. Time is expressed in units of the cavity buildup time. (a) Semiclassical theory, $q=0$, (b) $q=0.005$, (c) $q=0.02$, (d) $q=0.1$.

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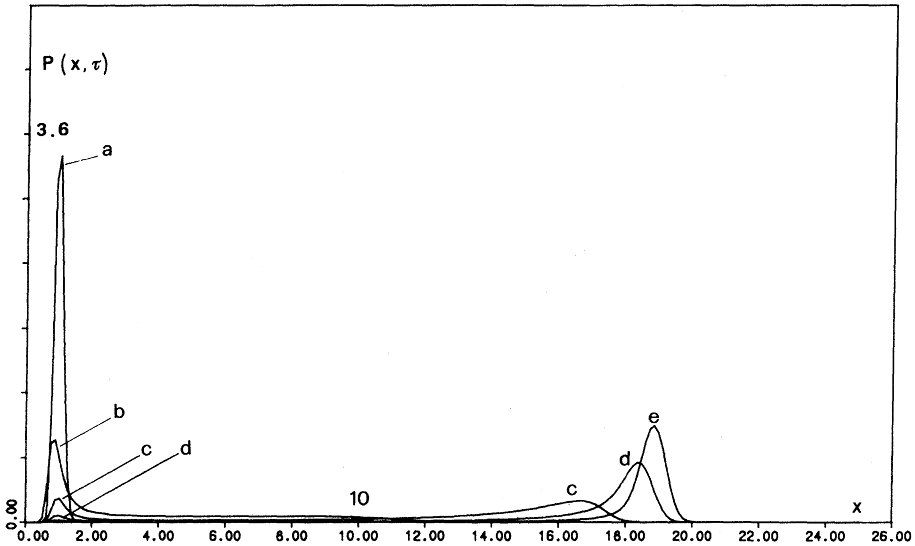

FIG. 3. Probability distribution $P(x,\tau)$ is shown for $q=0.1$ and five different vaues of time. (a) $\tau=0.2$, (b) $\tau=1.8$, (c) $\tau=3.6$, (d) $\tau=5.4$, (e) $\tau=7.2$. In this and in the following figures $C=20$, $y=21.04$.

whenever the evolution of the system involves a long induction period followed by an abrupt switching to a final stable attractor. Thus, one can easily envisage a huge variety of physical, chemical, biological, etc., systems which may exhibit this phenomenon. The specific interest of OB in this framework lies in the fact that it lends itself as a promising candidate for the observations of this effect.

We consider the case of purely absorbitive OB. The simplest model which describes amplitude fluctuations in this system[1,5] is given by the Fokker-Plank equation (FPE):

$$\frac{\partial P(x,\tau)}{\partial \tau}=\left[\frac{\partial}{\partial x}\left(x-y+\frac{2Cx}{1+x^2}\right)+q\frac{\partial^2}{\partial x^2}\right]P(x,\tau)\ , \qquad (1)$$

in which $x(y)$ is the normalized amplitude of the transmitted (incident) field, $P(x,\tau)$ the probability distribution of the variable x at the time τ, where τ is normalized to the cavity buildup time; $C=\alpha L/(2T)$ is the bistability parameter, where α is the absorption coefficient per unit length, L the length of the atomic sample, and T the mirror transmissivity coefficient. The form of the diffusion term in Eq. (1) corresponds to the situation of Gaussian white noise and the diffusion coefficient q measures the noise level. If we drop the diffusion term, we recover the semiclassical Eq. (3):

$$\frac{dx}{d\tau}=y-x-\frac{2Cx}{1+x^2}=-\frac{dU_y(x)}{dx}\ , \qquad (2)$$

$$U_y(x)=-yx+\tfrac{1}{2}x^2+C\ln(1+x^2)\ . \qquad (3)$$

When y is slightly larger than y_M (Fig. 1), the potential U_y presents a very flat part which produces the critical slowing down, and a well with a minimum at the semiclassical stationary state. By solving Eq. (2) with $x(0)=0$ we obtain the curve a of Fig. 2. More in general, the length of the plateau in the lethargic stage is the larger the smaller is the difference between the operating value y and the switch-up threshold y_M, and diverges for $y\rightarrow y_M$. The slope of the steep part of this time evolution is proportional to C.

We integrated numerically the FPE[1] using the Crank-Nicolson discretization method. The delta-function initial condition $P(x,0)=\delta(x)$ was approximated by a normalized rectangular function. In Figs. 3 and 4 we see the time evolution of $P(x,\tau)$ when y is slightly larger than y_M and $q=0.1$ or $q=0.005$, respectively. Figure 3 shows that at $\tau=0.2$ the distribution is one peaked, but soon $(\tau-1.8)$ it develops a long tail and subsequently $(\tau=3.6,5.4)$ becomes double peaked. Finally the left-hand peak disappears $(\tau=7.2)$ and the distribution approaches the steady-state one-peaked configuration. This transient bistability is particularly evident in the three-dimensional plot of Fig. 4. On decreasing q the peaks become higher and narrower. These

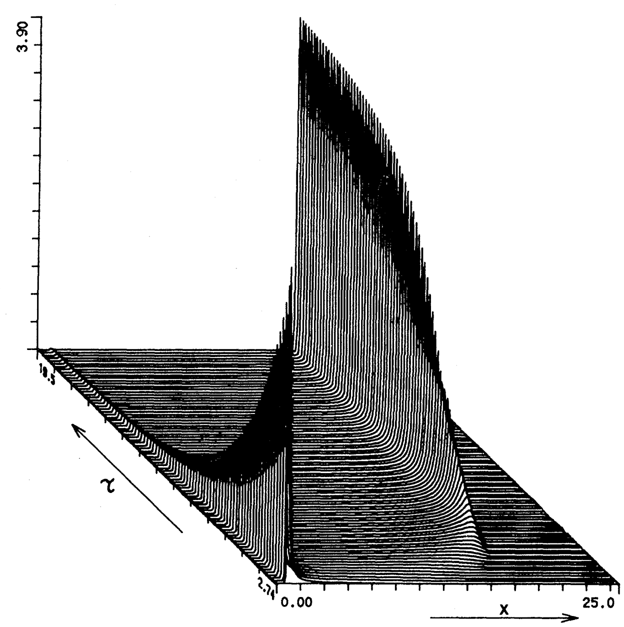

FIG. 4. Time evolution of the probability distribution $P(x,\tau)$ for $q=0.005$.

results fully agree with the picture of the process of *internal differentiation in time* given in Ref. 4. This phenomenon arises from the fact that, due to the critical slowing down, the probability distribution sits for a long time in the flat part of the potential. As a consequence of diffusion and of the asymmetry of the potential, it broadens and develops a long tail in the direction of the potential well. Due to the rapidity of the switching process, once the boundary of the well is reached, the leading edge of the tail is quickly transferred to the bottom, thereby giving rise to the second peak. Therefore this phenomenon of transient bistability is the more pronounced the longer is the semiclassical critical slowing down (i.e., the nearer the operating value y is to y_M) and the steeper the switching process (i.e., the larger is C). Figure 2 shows also the time evolution of the mean value $\langle x \rangle$ for three different values of the noise parameter. For $q=0.1$ the plateau of the lethargic stage is nearly absent, whereas it increases when q is lowered. Figure 5 shows the time evolution of the transition velocity, defined as $v(\tau)=dP_2/d\tau$, where $P_2(\tau)$ is the area of the second peak at time τ. Hence the quantity $v(\tau)\Delta\tau$ gives the probability that the system switches between time τ and time $\tau+\Delta\tau$. Accordingly, the distribution $v(\tau)$ is normalized to unity. The switching time distribution shown in Fig. 5 is broad and its mean value is definitely smaller than the semiclassical switching time. By decreasing q from 0.1 to 0.005 the v distribution broadens and the average switching time increases.

These results show that in the range of values of the noise parameter q considered in our calculations the switching process is diffusion (i.e., noise) dominated. The duration of the transient bistability phenomenon increases when q is decreased. On this basis, we expect that the phenomenon should persist when q is lowered to values as 10^{-3} or 10^{-4}, keeping the other parameters unchanged. However, when the noise level becomes so low that the length of the plateau in the curve of $\langle x \rangle$ vs τ approaches that of the semiclassical plateau, the switching process becomes drift dominated and therefore we expect the transient bistability to decrease and finally disappear. In this situation the probability distribution $P(x,\tau)$ is a narrow single peak drifting in such a way that the mean value $\langle x \rangle$ follows the semiclassical evolution. According to this descritpion, as y is taken nearer and nearer to y_M the transient bistability phenomenon persists for lower and lower values of q.

Our analysis is based on the simple model Eq. (1), which neglects several facts such as, e.g., phase fluctuations, the possibility of multiplicative noise, etc. These additional features will be considered in future work. However, we expect that the main results presented here should remain qualitatively unchanged. The message of our analysis is that the transient bistaility phenomenon can persist for low noise levels, hopefully comparable to those one has in real experiments. We suggest to reconsider the experiments on critical slowing down so far reported[6,7] by repeating the same run several times in order to obtain a statistics. The switching time distribution can be obtained easily; in this connection we note that large fluctuations of the switching time have already been observed.[7] Also, one can obtain histograms of the transmitted intensity for a number of selected times, to be compared with the behavior of our probability distribution.

In connection with this suggestion, a remark is necessary. With the value of $y=21.04$ that we used, one has $(y-y_M)/y_M \sim 6.5\times 10^{-4}$, and it seems hard to obtain such a level of reproducibility. Actually, this difficulty is *per se* one of the possible manifestations of noise. In fact it arises on the one hand from the finite resolution of physical measurements (observational noise), on the other hand from the incident field fluctuations (external noise). In this connection, we should like to propose the following interpretation of Fig. 5 in the paper by Grant and Kimble:[6] the horizontal error bar is an estimate of the variance of the incident intensity fluctuations (observational and external noise); the vertical bar is an estimate of the variance of the switching time distribution. The data agree qualitatively with our paper, because the vertical bar increases when $y-y_M$ decreases.

Our analysis shows that even very small noise (e.g., in the value of the incident intensity) produces dramatic fluctuations in the behavior of the system, when critical slowing down is involved. Hence, since the critical slowing down situation is so sensitive to noise, it can even be used to estimate the noise level in the system.

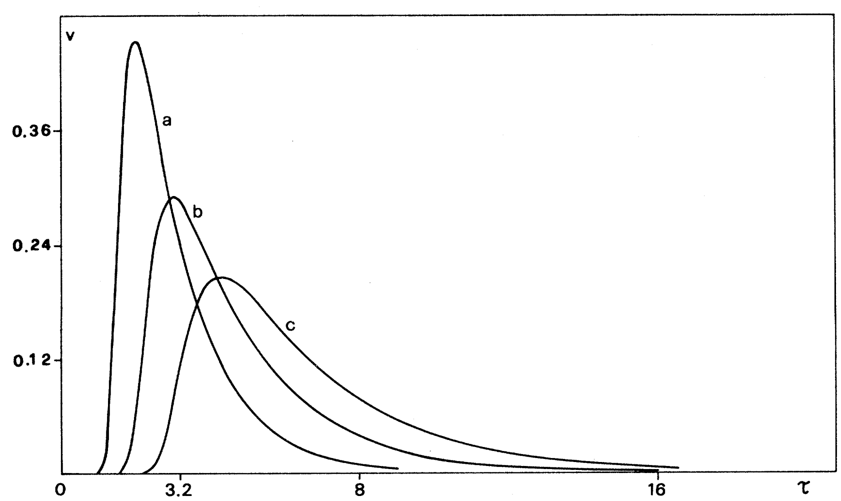

FIG. 5. Switching time distribution for (a) $q=0.1$, (b) $q=0.02$, and (c) $q=0.005$ (see text).

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[1]R. Bonifacio and L. A. Lugicato, Phy. Rev. Lett. 40, 1023 (1978).

[2]S. L. McCall, H. M. Gibbs, F. A. Hopf. D. L. Kaplan, and S. Ovadia, Appl. Phys. B 28, 99 (1982).

[3]R. Bonifacio and P. Meystre, Opt. Commun. 29, 131 (1978); V. Benza and L. A. Lugiato, Lett. Nuovo Cimento 26, 405 (1979).

[4]F. Baras, G. Nicolis, M. Malek Mansour, and J. W. Turner, J. Stat. Phys. 32, 1 (1983); M. Frankowicz and G. Nicolis, *ibid.* (to be published).

[5]R. Bonifacio, M. Gronchi, and L. A. Lugiato, Phys. Rev. A 18, 2266 (1978); A. Schenzle and H. Brand, Opt. Commun. 27, 85 (1978).

[6]E. Garmire, J. H. Marburger, S. D. Allen, and H. G. Winful, Appl. Phys. Lett. 34, 374 (1979); D. E. Grant and H. J. Kimble, Opt. Commun. 44, 415 (1983); F. Mitschke, R. Deserno, J. Mlynek, and W. Lange, *ibid.* 46, 135 (1983).

[7]S. Barbarino, A. Gozzini, F. Maccarrone, I. Longo, and R. Stampacchia, Nuovo Cimento 71B, 183 (1982).