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Measurement of the 1s Lamb shift in hydrogenlike chlorine

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The 1s Lamb shift in hydrogenlike chlorine has been determined from a precision measurement of the $2p_{3/2}$-$1s_{1/2}$(Lyα_1) and $2p_{1/2}$-$1s_{1/2}$(Lyα_2) x-ray transitions using beam-foil excitation. The x rays are emitted from high-velocity chlorine-ion beams at several ion velocities produced by a Van de Graaf accelerator. The 1s Lamb shifts obtained from the Lyα_1 and Lyα_2 measurements are 0.84(12) and 0.90(10) eV, respectively, compared with a calculated value of 0.9384(6) eV. The fine-structure splitting of the 2p level was also determined in this experiment and found to be 3.889(30) eV compared with a theoretical value of 3.827 18(2) eV. A precision measurement of the Ar $K\alpha$ x rays was made in order to establish the energy scale for Cl Lyα x rays. The newly determined energies are 2957.813(8), 2955.684(13), and 2977.51(6) eV for the $K\alpha_1$, $K\alpha_2$, and $K\alpha_4$ energies, respectively.

A quantity of fundamental interest in atomic spectroscopy is the Lamb shift of hydrogenlike heavy ions. The Lamb shift is the shift of an electron energy level due to nuclear size effects and radiative corrections predicted from quantum electrodynamics (QED). Historically the $2s_{1/2}$-$2p_{1/2}$ splitting is referred to as the Lamb shift and has been measured by various techniques in elements from $Z=1$ to $Z=18$.[1] One obvious reason to extend the measurements of the Lamb shift to higher Z is the increase in its value with Z; however, the methods used to measure the $2s_{1/2}$-$2p_{1/2}$ splitting, such as resonant laser excitation[2] and level quenching in an electric field,[3] become difficult as the Z of the system is increased. The 1s Lamb shift ($1s_{1/2}$ level shift) is approximately eight times larger than the 2s Lamb shift, and can be observed in the $2p_{3/2}$-$1s_{1/2}$(Lyα_1) and the $2p_{1/2}$-$1s_{1/2}$(Lyα_2) x-ray transitions in one-electron ions. This approach requires a very high precision on the absolute energy determination of the Lyα x rays. Only one such high-precisison measurement of the 1s Lamb shift in hydrogenlike heavy ions has been published to date,[4] although a few lower precision results have been reported.[5-8] This is the first high-precision Lamb-shift experiment using high-velocity ions in which the K x rays were measured at many projectile velocities. This allows one to make a systematic analysis of the Doppler shift and of the x-ray satellites near the Lyα_1 and Lyα_2 transitions. The 1s Lamb shift $S(1s)$ can be obtained from the observed Lyα_2 transition energy, relative to the Dirac energy (corrected for reduced mass and relativistic reduced mass),[9] as given below:

$$S(1s)\text{-}S(2p_{1/2})=\epsilon_D(\mathrm{Ly}\alpha_2)-\epsilon(\mathrm{Ly}\alpha_2)\ .$$

The last term is the actual Lyα_2 energy which is subtracted from the calculated Dirac energy ϵ_D(Lyα_2). In order to obtain the 1s Lamb shift $S(1s)$ this difference has to be further corrected for the small $2p_{1/2}$ Lamb shift. A corresponding expression exists for the $2p_{3/2}\rightarrow 1s_{1/2}$ (Lyα_1) radiation.

In the present experiment the Lyα_1 and Lyα_2 x rays of Cl are investigated. The calculated Lyα_2 energy[9] including QED and nuclear size effects is 2958.5493(6) eV (Ref. 10) of which 0.9384(6) eV is the 1s Lamb shift. This is 317 ppm of the Lyα_2 transition energy which sets an upper limit on the required experimental uncertainty for a measurement being sensitive to the 1s Lamb shift. The first consideration in the experimental accuracy of the x-ray measurements is the determination of an energy scale. The Ar $K\alpha$ x rays from singly and doubly ionized Ar are in close proximity to the Cl Lyα x rays and can be produced in adequate quantities by electron or photon excitation for study by high-precision low-efficiency spectrometers. This precision determination of the argon $K\alpha_1$, $K\alpha_2$, and $K\alpha_4$ x-ray energies was performed at the National Bureau of Standards. The argon spectrum was measured in fluorescence on a vacuum double crystal spectrometer[11] equipped with high quality germanium (111) crystals. The recorded argon $K\alpha_{1,2}$ profile was analyzed with a three-line model using approximate Voigt functions.[12] The temperature of the analyzing crystals during the measurements was 25.05 ±0.15 °C. The temperature correction to the line positions was made using a linear expansion coefficient of $\alpha_{\mathrm{Ge}}=(5.95\pm0.11)\times10^{-6}$ K^{-1} to reduce the measurements to 22.5 °C where $2d=6533\,075\,56$ for the Ge(111) crystals.[13] Before starting the measurement sequence and after the final measurement the encoder of the second axis of the spectrometer was calibrated against a 24-sided optical polygon which had been calibrated from first principle on numerous ocassions with the aid of an interferometric goniometer.[13] The resulting x-ray energies[10] after applying corrections to the measured Bragg angle,[14] are as follows: $K\alpha_1 2957.813 \pm 0.008(3\ \text{ppm})$; $K\alpha_2 2955.684 \pm 0.013(4\ \text{ppm})$; $K\alpha_4 2977.51 \pm 0.06(20\ \text{ppm})$.

The Cl Lyα spectra were produced by the excitation of a high-velocity Cl beam traversing a thin, transmission *C* foil and the prompt x rays were measured with a crystal spectrometer viewing the interaction region. The high velocities are required in order to produce sufficient quantities of the one-electron ions. At high velocities, the Doppler shift of the x rays is a significant contribution to the energy of the x rays observed in the laboratory frame of reference. Because of the Doppler effect and the relatively low x-ray intensities, several restrictions are placed on the geometry and on the x-ray spectrometer. Figure 1 is a schematic drawing of the actual experimental setup which meets these restrictions. The ion beams were collimated by two sets of four-jaw slits separated by 5.31 m in order to limit the horizontal divergence of the beam to $\sim 0.04°$. An efficient curved crystal spectrometer[15] with a Rowland circle of 1-m radius was equipped with a position sensitive backgammon detector[16] located on the Rowland circle which was positioned normal to the beam axis. The emitted x rays were collimated so as to be viewed very near to 90° from the beam axis. Two sets of two-jaw slits separated by 0.42 m and located between the *C* foil and the SI(111) curved crystal, were used to collimate the x rays. To establish the x-ray emission angle, the Faraday cup was removed and the beam line slits centered with a telescope from the rear end of the beam line. A pentaprism was then placed at the location of the C target which bent the telescope alignment axis by 90° with high accuracy $(90.000 \pm 0.003°)$ and the x-ray slits were centered on this axis. By this method the four sets of slits could be positioned to form 90° between the beam direction and the x-ray emission direction as depicted in Fig. 1. For calibration purposes the carbon target was replaced by a gas cell containing Ar gas. A beam of 6-keV electrons was produced with a commercial electron gun,[17] directed through a window-free entrance and exit hole in the gas cell and monitored in the Faraday cup. A large Mylar window allowed the x rays to exit towards the spectrometer.

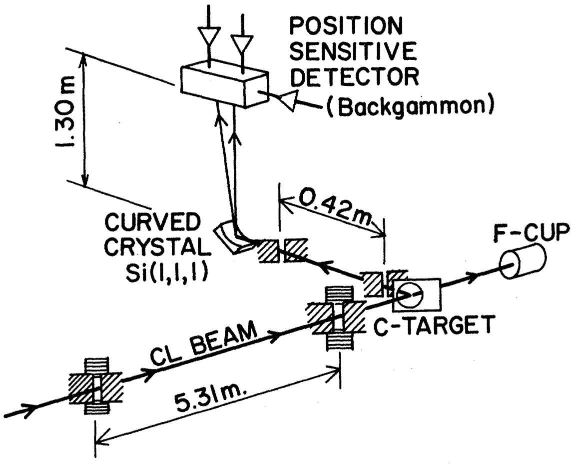

FIG. 1. Schematic of the experimental setup for the precision measurement of the beam-foil excited Lyα x rays in hydrogenlike chlorine.

An important element in the present experiment is the use of a position sensitive backgammon proportional counter which consists of a central anode wire held at 1350 V, two independent cathode plates with a backgammon design, and an enclosed gas mixture consisting of 90% xenon and 10% methane held at $\simeq 0.5$ atm. The gas was contained by a thin Be x-ray entrance window. The two signals from the cathode plates were amplified with equal gain. The position of an x-ray event in the detector was determined electronically by taking the signal from one-half of the backgammon cathode and dividing it by the sum of the two backgammon signals. These divided signals were gated by the linear pulse-height signal from the central wire, fed to an analog-to-digital converter (ADC) and stored in a computer to form the x-ray spectra without the need for scanning either the crystal or the detector.

A portion of the x-ray spectrum obtained with 157.5-MeV (4.5-MeV/amu) Cl is shown in Fig. 2. The Lyα_1 and Lyα_2 transitions are well resolved and considerably above the background. In each spectrum it was found that low-energy satellite lines were present on both the Lyα_1 and the Lyα_2 transitions. It is assumed that these satellites are due to spectator electrons (i.e., electrons in levels with $n \gtrsim 3$).

A careful step by step analysis was chosen to deduce the Lyman-α energies from the measured spectra in order to detect inconsistencies and to test each assumption separately with χ^2. First of all, the hydrogenlike chlorine spectra were fitted with a linear background and four lines (two for the main lines and two for the satellite lines) by the least-

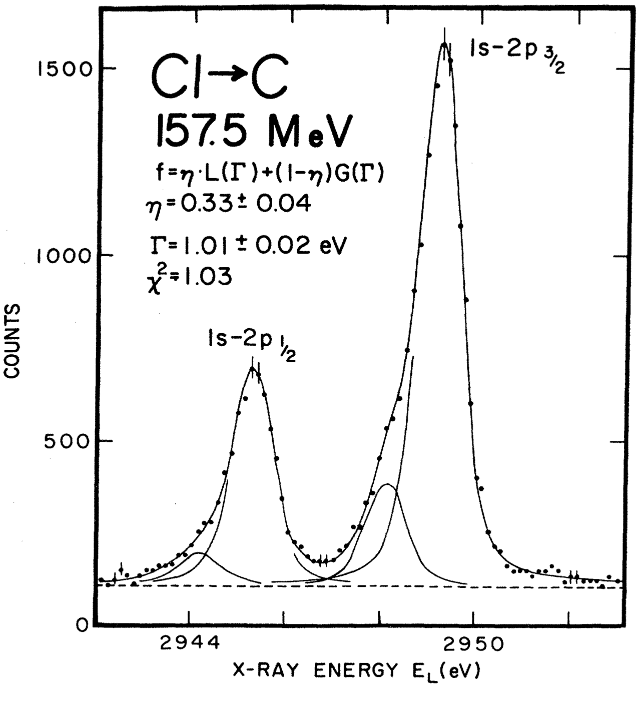

FIG. 2. Small section from the measured x-ray spectra obtained with 157.5-MeV Cl. The dots represent the accumulated counts per channel which are plotted vs the x-ray energy. The few vertical bars indicate the typical statistical errors. The broken curve shows the fitted background and the full curves show the superimposed individual x-ray line and the sum of all the above.

squares method, weighted according to the fitted values. For all four lines the same line shape was applied, determined by a linear combination of a Lorentzian (L) and a Gaussian (G) [$f=\eta L(\Gamma)+(1-\eta)G(\Gamma)$],[12] where the Voigt parameter η and the linewidth Γ [full width at half maximum (FWHM)] were fitted parameters. For all spectra the χ^2 per degree of freedom was, within its uncertainty, in agreement with 1.0, supporting the simple "four-line" model. The line shape is predominantly Gaussian ($\eta \sim 0.3$), caused by the instrumental resolution and the Doppler broadening due to the extended source location. The width is typically 1.0 eV (FWHM) and the satellites are quite consistently 1.05 eV below the main lines with a scatter of only 0.05 eV.

The same procedure was used to fit the Ar calibration spectra including only the $K\alpha_1$, $K\alpha_2$, and $K\alpha''$[18] peaks and subsequently the high-energy range including only the $K\alpha_3$ and $K\alpha_4$ peaks, yielding χ^2 consistent with 1.0. In these fits the lines are dominated by the Lorentzian shape ($\eta \sim 0.6$) caused by the fast Auger transitions, yielding a total linewidth of 1.7 eV (FWHM). The errors of the $K\alpha_2$ and $K\alpha_4$ positions were slightly increased because the positions depended slightly on the fitted background shape and data range. Because the $K\alpha_4$ line was out of the detector center, its fitted mean and its error were adjusted for the measured nonlinearity of the backgammon detector.

The fitted Ar peak positions (channel number) weighted with their uncertainites and the previously discussed Ar calibration energies $K\alpha_1$, $K\alpha_2$, and $K\alpha_4$, also weighted with their uncertainties, were used to determine the conversion from channel number to x-ray energy using the Bragg law with known $2d$ spacing. This least-squares regression yielded a reasonable χ^2 and a relative uncertainty of 0.5% for the evaluation of x-ray energies relative to Ar $K\alpha_1$. This determined conversion was used to calculate the x-ray energies and their errors for Lyα_1 and Lyα_2 at all beam energies. The uncertainty of the calibration conversion was neglected, in order to avoid correlations between the errors. It should be stated that all the errors quoted in parenthesis are standard errors for the right most digits.

In the next step the ion-velocity ($v=\beta\cdot c$)-dependent x-ray energies $E_{\rm L}$ were used to fit the emission angle, $\theta_{\rm L}$ according to the Doppler shift $E_{\rm L}=E_{\rm c.m.}(1-\beta^2)^{1/2}\times(1-\beta\cos\theta_{\rm L})^{-1}$, where the subscripts L or c.m. indicate the laboratory frame or center-of-mass frame. The data and the best fit curves are shown in Fig. 3 in linearized coordinates. The best fit angle is $\theta_{\rm L}=89.753°(17)$. The deviation of $\theta_{\rm L}$ from 90° is due to a misalignment of the x-ray slits relative to the crystal. The fitted angle was used to calculate the center-of-mass transition energies $E_{\rm c.m.}$ for both Lyα transitions at all beam energies. From these center-of-mass energies the weighted mean and its error were computed for both transitions. The resulting values are Lyα_1 $=2962.47(12)$ eV and Lyα_2 $=2958.59(10)$ eV where the individual error contributions are 0.01 for Lyα_1 and 0.02 eV for Lyα_2 due to line scattering and statistical uncertainties of the means, 0.08 eV due to the emission angle uncertainty, and 0.08 eV for Lyα_1 and 0.06 eV for Lyα_2 due to the calibration uncertainty. These experimental transition energies were used to obtain the experimental 1s Lamb shifts of 0.84(2) eV for Lyα_1 and 0.90(10) eV for Lyα_2. This result is in reasonable agreement with the theoretical values, namely, 0.948(4) eV,[10,19] 0.9383(6) eV,[10,20] and 0.9384(6) eV.[9,10]

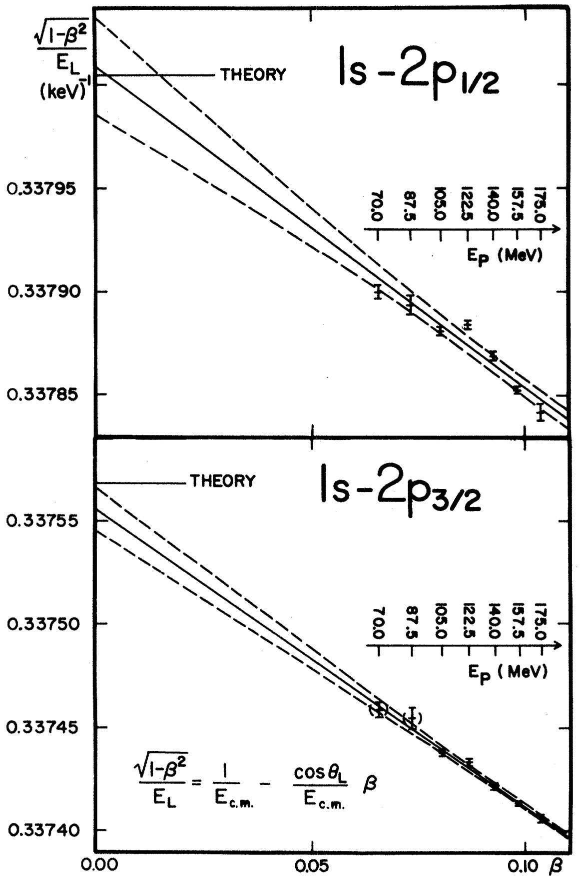

FIG. 3. Measured chlorine Lyα x-ray energies $E_{\rm L}$ as a function of the Cl-ion velocity $\beta=v/c$. In these special coordinates the Doppler shift is linear as shown by the inserted formula. The full curves give the best-straight line fit, whereas the broken curves show the standard deviations of the fit. The inverse of the intercepts correspond to the center-of-mass Lyα energies $E_{\rm c.m.}$ which are compared with the theoretical values. The experimental data put in parentheses were excluded from the fit.

Recently the energy difference between Lyα_1 and Lyα_2 (the fine structure splitting) gained increased interest in the literature.[4,7] We have evaluated this splitting in H-like chlorine to be 3.889(30) eV. This high accuracy in the splitting was obtained from the fitted difference of the peak positions and its error for each beam velocity. The channel differences were converted to x-ray energies and were further corrected for the Doppler shift. These center-of-mass energies were used to calculate the weighted mean energy and its error (0.022 eV). The error contribution due to calibration conversion is 0.020 eV which leads to a total error of 0.030 eV. The measured splitting seems to disagree with the theoretical values, namely, 3.8271(2) eV,[10,19] 3.8272 eV,[10,20] and 3.827 18(2) eV.[9,10] The discrepancy of about 2σ could be due to nearby satellite lines. The satellite intensities are expected to vary with beam velocity, and thereby, so will the measured splitting if it is indeed affected by satellite lines. To test such a possibility, the evaluated splittings were fitted as a power function of the ion velocity β yielding an exponent of -0.08(7). This analysis shows a splitting which decreases with increasing ion velocity but which is not statistically significant. However, if one used

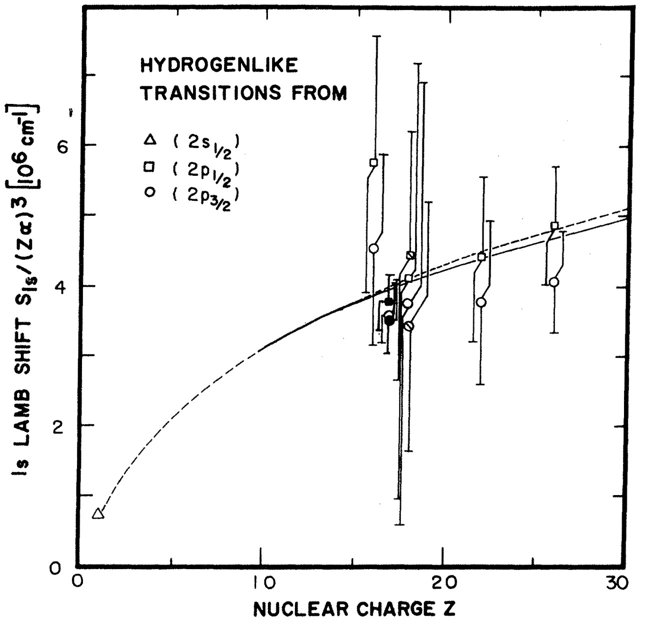

FIG. 4. Reduced 1*s* Lamb shift measured in 2*l*-1*s* transitions in hydrogenlike ions, where the initial state 2*l* is indicated by the symbol shape. The broken curve represents the calculated values by Erickson (Ref. 19) and the full curve shows the calculation by Mohr (Ref. 9). The measurement from this work ($Z = 17$, closed symbols) was done with beam-foil excitation as was most of the others; namely, $Z = 16$ (Ref. 5), $Z = 18$ [open symbols (Ref. 6), symbols with backslash (Ref. 8)], $Z = 22$ (Ref. 7) and $Z = 26$ (Ref. 4). Exceptions are $Z = 1$ with Doppler-free two-photon laser excitation (Ref. 21) and $Z = 17$ with excitation in a Tokamak hydrogen plasma (Ref. 22).

this power function to calculate the splitting for a beam energy of 157.5 MeV (where Fig. 2 indicates that the satellites are quite small) one obtains 3.867(34) eV which is in reasonable agreement with the theoretical values. Moreover, this careful analysis of the splitting indicates that the measured values of the 1*s* Lamb shift are not significantly affected by nearby satellite lines at the present stage of the experimental accuracy.

In conclusion, the 1*s* Lamb shift has been measured for hydrogenlike chlorine using beam-foil excitation. The measurements were made at several ion velocities which allowed us to make a systematic study of the Doppler shift and the effect of satellite transitions. The 1*s* Lamb shift obtained from the Lyα_1 measurements is 0.84(12) eV and from the Lyα_2 measurements is 0.90(10) eV, compared to the calculated value of 0.9384(6) eV.[9,10] At present this is the most accurate beam-foil measurement of the 1*s* Lamb shift to date as shown in Fig. 4 where all available results[4–8,21,22] are given. It is envisioned that the accuracy can be significantly increased by improved alignment, improved x-ray energy resolution, and by a more complete understanding of the satellite lines.

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