

Instability of the critical surface of a laser-produced plasma in the presence of ion-acoustic turbulence

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The critical surface can be unstable to coherent rippling perturbations due to the action of negative pressure induced by the random magnetic field associated with ion-acoustic turbulence. The negative magnetic pressure occurs if there exists a preferential orientation of the random magnetic field (anisotropy of the ion-acoustic turbulence) when the nonpotential component of the magnetic pressure more than compensates the potential part.

I. INTRODUCTION

Resonant absorption is particularly significant in experiments where short-duration ($\approx 10^{-11}$ – 10^{-10} s) high-intensity laser pulses irradiate laser-produced plasmas. Inverse bremsstrahlung becomes negligible in this situation due to the small volume of underdense plasma, while parametric processes in the vicinity of the critical density are detuned due to plasma-density-profile modification induced by the ponderomotive force. The portion of laser energy absorbed by the target then depends on the geometry of both the focusing optics and the target and on the polarization of the incident wave. If for any reason rippling of the critical surface occurs, it favors resonance absorption and then even those "rays" which were normally incident upon the plasma without the ripples can be resonantly absorbed at the rippled critical surface. The aim of our paper is to describe one possible mechanism which can be responsible for rippling of the critical surface.

As early as 1956 Sagdeev¹ showed that a plasma vacuum boundary supported by an electromagnetic standing wave is unstable to perturbations of the surface-wave type. Extensive numerical simulations by Valeo and Estabrook^{2,3} show the effect of self-trapping of the radiation in media which have a nonlinear index of refraction, resulting in a sausage type of instability. More recently Wee Woo *et al.*⁴ investigated rippling of a very steep density profile ($\alpha \leq \lambda_0$ where α is the scale length of the plasma density gradient and λ_0 the vacuum wavelength of the incident radiation) due to four-wave processes when the incident wave is scattered into two electromagnetic waves generating ion-acoustic waves to enhance the rippling. The presence of ripples is claimed to be indirectly proved by Nishimura *et al.*⁵ They have observed a modulation of the angular dependence of the reflected laser light from a plasma for an *s*-polarized laser beam and applied Sagdeev's¹ approach to an instability of the critical surface which after being rippled is considered as a grating. The effect of resonant absorption at a rippled critical surface has been investigated by Kruer and Estabrook,⁶ Cairns,⁷ and David and Pellat.⁸ In the last work⁸ the effect is also discussed of generation of a spontaneous magnetic field

$\vec{B} \sim \vec{\nabla} n \times \vec{\nabla} T$ due to crossing of the density and temperature gradients where the modulation of the density gradient is due to rippling of the critical surface. David and Pellat⁸ have then combined the effects of rippling and of the magnetic field to compute the coefficient of resonant absorption by a magnetized plasma at a rippled critical surface.

In those experiments where the laser energy is mostly absorbed in a narrow region around the critical density, a \vec{B} -field-generating thermal instability can arise⁹ when the initially isothermal surface of the absorption region is coherently rippled giving rise to a magnetic field of thermoelectric origin $\vec{B} \sim \vec{\nabla} n \times \vec{\nabla} T$. Then, the local temperature T increases due to B dependence of the tensor of thermal conductivity.

In the case considered in this paper, rippling of the critical surface is investigated induced by a negative pressure¹⁰ (line-of-force stress) of an anisotropic random magnetic field associated with ion-acoustic turbulence. This treatment is thus partially complementary to that of Mora and Pellat¹¹ where an isotropic random magnetic field generated due to short-wavelength ion turbulence was discussed, resulting in a ponderomotive-force term which is purely of potential character.

II. THEORY

Let us consider the situation of one-dimensional laser-produced plasma flow where the intensity of laser radiation is above the threshold for a parametric decay instability so that an ion-acoustic turbulence results in random density fluctuations which can be a source of random magnetic fields. As for a characteristic frequency ω of the ion sound

$$\omega \tau_h \gg 1, \quad (1)$$

where τ_h is a characteristic hydrodynamic time, one can separate the fast and slow time scales and consider the time dependence due to slow hydrodynamic flow in the problem as a parameter.

We shall assume the plasma conductivity in the region investigated in its usual form,

$$\sigma = \frac{\omega_p^2 \nu}{\omega^2 + \nu^2} \gg \omega$$

and thus also $\sigma \tau_h \gg 1$, where ν is the electron collision frequency, $\omega_p = (4\pi e^2 n_{e0}/m_e)^{1/2}$ the electron plasma frequency, e and m_e the electron charge and mass, and n_{e0} the electron density. The last assumption neglects the displacement current and permits us to write the following equation for the magnetic field:¹²

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) - \frac{c}{en_e} \vec{\nabla} n_e \times \vec{\nabla} T_e - \vec{\nabla} \times (d \vec{\nabla} \times \vec{B}) + \mathcal{F}(B^2), \quad (2)$$

where

$$d = c^2 \nu / \omega_p^2 \quad (3)$$

is a diffusion coefficient, \vec{v} is the plasma flow velocity, c the speed of light, and T_e the electron temperature.

We assume that before the ion-acoustic turbulence was switched on the plasma flow had been one dimensional along the x axis, with plasma density

$$n_e = n_{e0}(x),$$

plasma temperature

$$T_e = T_e(x),$$

flow velocity

$$\vec{v} = \vec{v}_0(x),$$

and dc magnetic field (see Appendix)

$$\vec{B}_0 = \vec{0}.$$

If we assume that Δ denotes the size of the region around the critical density where the ion-acoustic turbulence is efficiently excited, then the effect of magnetic field diffusion can be neglected when

$$\Delta > (d/\omega)^{1/2}, \quad (4)$$

as well as the effect of plasma flow when

$$\omega \Delta > v_0. \quad (5)$$

Then assuming electron density n_e to consist of a regular part $n_{e0}(x)$ and a random part $n_1(x, y, z; t)$ so that

$$n_e(x, y, z; t) = n_{e0}(x) + n_1(x, y, z; t), \quad (6)$$

$$n_1(x, y, z; t) = n_c \mathcal{f}(x, y, z) \phi(y, z) e^{i\omega t}, \quad |\phi| = 1 \quad (7)$$

where \mathcal{f} is an envelope function of the fluctuations, ϕ a random function, and n_c stands for the critical density relevant to the laser light driving the ion-acoustic turbulence.

Assuming that the introduced function $\mathcal{f}(z, y, z)$ is slowly varying with y and z , in comparison to both the correlation length of the fluctuations and characteristic length of magnetic field diffusion, i.e.,

$$(\partial \ln \mathcal{f} / \partial y)^{-1} > (d/\omega)^{1/2}, \quad (\partial \ln \mathcal{f} / \partial z)^{-1} > (d/\omega)^{1/2},$$

$$\langle (\phi | \nabla_{y,z} \mathcal{f})^2 \rangle \ll \langle (\mathcal{f} | \nabla_{y,z} \phi)^2 \rangle,$$

and

$$\langle |B|^2 \rangle / 8\pi n_c T_e \ll 1,$$

where $\langle \dots \rangle$ denotes ensemble average, then the spontaneous magnetic field \vec{B} coherent with $\nabla_{y,z} \phi$ becomes

$$\vec{B} = \vec{B}_1 \exp(i\omega t), \quad \vec{B}_1 = (0, (\vec{B}_1)_y, (\vec{B}_1)_z),$$

where

$$(\vec{B}_1)_y = i \frac{cn_c}{en_{e0}\omega} \mathcal{f}(x, y, z) \frac{dT_e}{dx} \frac{\partial \phi}{\partial z},$$

$$(\vec{B}_1)_z = -i \frac{cn_c}{en_{e0}\omega} \mathcal{f}(x, y, z) \frac{dT_e}{dx} \frac{\partial \phi}{\partial y}. \quad (9)$$

Introducing a randomly orientated wave vector

$$\vec{k} = (0, k_y, k_z) \equiv \vec{\nabla} \phi / \phi \quad (10)$$

of the ion-acoustic waves, one can write the random magnetic field in a compact form

$$\vec{B}_1 = i \frac{cn_c}{en_{e0}\omega} \phi \mathcal{f}(x, y, z) \vec{k} \times \vec{\nabla} T_e, \quad (11)$$

and now to average means averaging over the ensemble of $\vec{k}_0 = \vec{k} / |\vec{k}|$.

As shown by Vainshtein,¹⁰ the random magnetic field gives rise to a net force acting on a plasma

$$\vec{F} = -\vec{\nabla} \cdot \vec{T}, \quad (12)$$

where the elements of the stress tensor \vec{T} are

$$T_{ij} = \frac{1}{4\pi} (\frac{1}{2} \mathcal{B}_{ij} \delta_{ij} - \mathcal{B}_{ij}), \quad i, j = x, y, z \quad (13)$$

and where \mathcal{B}_{ij} is a correlation function

$$\mathcal{B}_{ij} = \text{Re} \langle (\vec{B}_1)_i (\vec{B}_1^*)_j \rangle \quad (14)$$

taken at the same point.

Let us assume that

$$\vec{\kappa}_0 \equiv \vec{B}_1 / |\vec{B}_1| \quad (15)$$

possesses a preferred direction of orientation. A reason for that can be, for example, an anisotropy of the ion-acoustic turbulence due to inhomogeneity of the background plasma [$n_{e0} = n_{e0}(x)$]. Also, in the condition when the pump amplitude only just exceeds the threshold of the instability driving the ion waves, ion-acoustic turbulence is anisotropic. For example, if the instability was the parametric decay of the incident wave, the ion-acoustic waves are generated mainly with the wave vectors parallel to the \vec{E}_i vector of the incident wave.¹³ However, for laser fusion experiments at high irradiance intensities, the radiation pressure induces a plasma-density profile steepening in the vicinity of the critical density and, consequently, the threshold intensity for the parametric processes is increased. Therefore, there is a whole range of

the laser light intensities which are above but close to the threshold for the parametric instability.

In the plasma irradiation configuration considered here we set

$$\vec{\kappa}_0 = (0, \kappa_{0y}, \kappa_{0z})$$

and, specifically, assuming that the ion-acoustic turbulence was driven by a parametric decay of the normally incident wave with an electric vector $\vec{E}_i = (0, 0, E_i)$, then the introduced wave vector \vec{k} will be preferentially oriented in the z direction¹³ and, consequently, κ_0 preferentially in the y direction. Then, the correlation function \mathcal{B}_{ij} can be written as¹⁰

$$\mathcal{B}_{ij} = \frac{\langle |\tilde{B}_1|^2 \rangle}{3 + \mu} (\delta_{ij} + \mu \kappa_{0i} \kappa_{0j}), \quad (16)$$

with

$$1 + \mu \geq 0.$$

The case $\mu = 0$ corresponds to the isotropic case. Thus, the introduced constant μ represents a degree of preference of the direction $\vec{\kappa}_0$.

Then, as $\vec{k}_0 \perp \vec{\nabla} T_e$ and $k_0 = 1$, the self-correlation function becomes

$$\begin{aligned} \langle |\tilde{B}_1|^2 \rangle &= \frac{c^2 n_c^2 k^2}{e^2 n_{e0}^2 \omega^2} \mathcal{J}^2(x, y, z) \langle (\vec{k}_0 \times \vec{\nabla} T_e)^2 \rangle \\ &= \left[\frac{cn_c k}{en_{e0} \omega} \right]^2 \mathcal{J}^2(x, y, z) \left[\frac{dT_e}{dx} \right]^2. \end{aligned} \quad (17)$$

Consequently,

$$\begin{aligned} \langle (\tilde{B}_1)_i (\tilde{B}_1^*)_j \rangle &= \frac{1}{3 + \mu} \left[\frac{cn_c k}{en_{e0} \omega} \mathcal{J} \frac{dT_e}{dx} \right]^2 \\ &\quad \times (\delta_{ij} + \mu \kappa_{0i} \kappa_{0j}), \end{aligned} \quad (18)$$

keeping in mind that $\vec{\kappa}_0 \perp \vec{E}_i \parallel \vec{k}_0$.

Thus, the ponderomotive-type force \vec{F} arising due to the presence of the random magnetic field becomes

$$\begin{aligned} F_i &= -\frac{1}{8\pi} \frac{\partial}{\partial x_i} G(\mu, x, y, z) (1 + \mu \kappa_{0i}^2) \\ &\quad + \frac{1}{4\pi} \mu \kappa_{0i} \kappa_{0j} \frac{\partial}{\partial x_j} G(\mu, x, y, z), \end{aligned} \quad (19)$$

where

$$G(\mu, x, y, z) \equiv \frac{1}{3 + \mu} \left[\frac{cn_c k}{en_{e0} \omega} \mathcal{J}(x, y, z) \frac{dT_e}{dx} \right]^2.$$

Assuming no macroscopic coherent modulation (inhomogeneity) of the energy density of the random magnetic field in the y or z direction

$$G(\mu, z, y, t) = G(\mu, x),$$

the last term in (19) yields no contribution as long as $\vec{\kappa}_0$ is perpendicular to the x axis.

A completely different situation appears when the mag-

netic field has coherent modulation in the y or z direction. Then the last term in (19) which is of nonpotential character, is turned on and can even dominate the first potential term.

Introducing the coherent modulation of the plasma-density fluctuations in the direction of $\vec{\kappa}_0$ specifies the envelope function \mathcal{J} ,

$$\mathcal{J}(x, y, z) = \mathcal{J}(x, y),$$

where y is a slowly varying parameter if

$$\frac{\partial \mathcal{J}}{\partial y} \ll k_y \mathcal{J}.$$

Such a modulation gives rise to the negative pressure acting in the y direction

$$F_y = (\mu - 1) \frac{1}{8\pi} \frac{\partial}{\partial y} G(\mu, x, y) \quad (20)$$

if

$$\mu > 1/\kappa_{02}^2 = 1.$$

At the beginning of our analysis we introduced the envelope function \mathcal{J} of the plasma-density fluctuations localized in the vicinity of the critical density. For ion-acoustic turbulence driven by the parametric decay instability, this function \mathcal{J} expresses how accurately the three-wave matching conditions (Manley-Rowe) are fulfilled. Clearly, for a plasma-density profile monotonic in the x direction in the region investigated, this envelope function must have a single maximum somewhere below but close to the critical density n_c . Thus, omitting the modulation in the y direction, the dispersion properties of the waves involved in the three-wave decay process imply an implicit dependence of the function \mathcal{J} on plasma density

$$\mathcal{J}(x) = \mathcal{J}(n_e(x)).$$

For plasma density $n_e < n_{\text{opt}} \leq n_c$ below an optimum density for the three-wave process, the function \mathcal{J} is then an increasing function of the plasma density

$$\frac{d\mathcal{J}(n_e)}{dn_e} > 0, \quad n_e < n_{\text{opt}}.$$

Now, introducing a long scale modulation in the y direction one sets

$$\mathcal{J}(x, y) = \mathcal{J}(n_{e0}(x) + \tilde{n}(y)),$$

where we assume $|\tilde{n}| \ll n_c$.

In further calculations we shall approximate the function \mathcal{J} by

$$\mathcal{J}(x, y) = \eta \{ [n_{e0}(x) + \tilde{n}(y)] / n_c \}^\alpha, \quad (21)$$

where η is a dimensionless constant and

$$\alpha = \begin{cases} \alpha_- > 0 & \text{for } n_e < n_{\text{opt}} \\ \alpha_+ < 0 & \text{for } n_e > n_{\text{opt}} \end{cases} \quad (22)$$

$$n_e = n_{e0} + \tilde{n}.$$

In fact $\alpha = \alpha(n_e)$ with $\alpha \rightarrow 0$ as $n_e \rightarrow n_{\text{opt}}$. However, assuming $|\tilde{n}| \ll n_c$, approximation of α by the two con-

stants retains the basic features of the physics of the process investigated.

Inserting (21) into (19), the y component of the force density \vec{F} becomes

$$F_y = \beta \frac{\partial}{\partial y} \left[\left[\frac{n_{e0}(x) + \tilde{n}(y)}{n_c} \right]^{2\alpha_{\pm}} \right], \quad (23)$$

where

$$\beta \equiv \frac{\eta^2}{8\pi} \frac{\mu-1}{\mu+3} \left[\frac{ckn_c}{en_{e0}\omega} \frac{dT_e}{dx} \right]^2. \quad (24)$$

Further, let us examine $\tilde{n}(y)$ as a perturbation in the plasma hydrodynamics and investigate its time evolution assuming

$$\tilde{n}(y) = \tilde{n}(y, t) = \tilde{n} \exp[i(\tilde{\omega}t - \tilde{s}y)], \quad \tilde{s} \ll k.$$

We will restrict our treatment to the case

$$\tau_h^{-1} \ll |\tilde{\omega}| < \omega \quad (25)$$

when the coherent perturbation which has been introduced is a relatively slow process. Then, the linearized set of hydrodynamic equations for density and velocity perturbations \tilde{n}, \tilde{v} reads

$$\begin{aligned} \frac{\partial \tilde{n}}{\partial t} + n_{e0} \frac{\partial \tilde{v}}{\partial y} &= 0, \\ \frac{1}{Z} n_{e0} m_i \frac{\partial \tilde{v}}{\partial t} &= \left[2\beta\alpha \left[\frac{n_{e0}}{n_c} \right]^{2\alpha-1} n_c^{-1} \right. \\ &\quad \left. - C\gamma \left[\frac{n_{e0}}{n_c} \right]^{\gamma-1} n_c^{-1} \right] \frac{\partial \tilde{n}}{\partial y}, \end{aligned} \quad (26)$$

where Z is the degree of ionization, m_i the ion mass, and we have assumed that the plasma behaves as an adiabatic gas with thermokinetic pressure

$$p = C(n/n_c)^\gamma,$$

where C is a dimensional constant $C = p(n = n_c)$. The last assumption is consistent with the long-wavelength character of the coherent plasma-density modulation.

Setting

$$\tilde{v}(y, t) \sim \exp[i(\tilde{\omega}t - \tilde{s}y)],$$

one obtains the following dispersion relation $\tilde{\omega} = \tilde{\omega}(\tilde{s})$ for the perturbation being considered:

$$\tilde{\omega} = \left\{ \left[-2\beta\alpha \left[\frac{n_{e0}}{n_c} \right]^{2\alpha} + C\gamma \left[\frac{n_{e0}}{n_c} \right]^\gamma \right] \frac{Z}{n_{e0}m_i} \right\}^{1/2} \tilde{s}. \quad (27)$$

With the plasma-density fluctuations turned off ($\beta \equiv 0$), relation (27) represents ion-acoustic waves. On the other hand, in the cold plasma limit $C \rightarrow 0$, this relation (27) corresponds to an analog of Alfvén waves where the elastic force is due to random magnetic field ($\beta \sim \langle \tilde{B}_1^2 \rangle$).

As can be seen from (27), the oscillations under investigation can be unstable if

$$n < n_{\text{opt}},$$

i.e.,

$$\alpha = \alpha_- > 0,$$

and if

$$2\beta\alpha_- \left[\frac{n_{e0}}{n_c} \right]^{2\alpha_-} > C\gamma \left[\frac{n_{e0}}{n_c} \right]^\gamma, \quad (28)$$

which is not in contradiction with (8) since $\alpha_- \gg 1$ as shown later. The last relation (28) defines a threshold value for the random magnetic field energy density (or threshold value for amplitude of the plasma-density fluctuations η) for the instability to arise. Note that this instability saturates due to the fact that $\alpha \rightarrow 0$ as $n \rightarrow n_{\text{opt}}$. Thus, the maximum amplitude of the excited waves \tilde{n} is $\tilde{n}_m \simeq n_{\text{opt}} - n_{e0}(x_0)$ where $x = x_0$ is the plane where the perturbation has been initiated. As both ω and $\tilde{\omega}$ satisfy the ion sound-type dispersion relation, the last threshold condition (28) together with our assumption $|\tilde{\omega}| < \omega$ mean that our theory is applicable to the case $\tilde{s} \ll k_D = 2\pi/\lambda_D$ (λ_D the Debye length) which is consistent with our initial assumption $\tilde{s} \ll k < k_D$. The lower limit on \tilde{s} is imposed by the effect of the magnetic field diffusion, which is unimportant if

$$\Delta > (d/|\tilde{\omega}|)^{1/2} > (d/\omega)^{1/2}. \quad (29)$$

As shall be shown next, for plasma parameters in laser fusion experiments, the random magnetic field amplitude is usually well above the threshold and one can write

$$\tilde{\omega} \simeq -i \left[\frac{Z}{n_{e0}m_i} \beta\alpha_- \right]^{1/2} \tilde{s}, \quad (30)$$

$$k > \tilde{s} > d/[\Delta^2(Z\beta\alpha_-/n_{e0}m_i)^{1/2}]. \quad (31)$$

However, it still remains to evaluate the constants η , α_- , and Δ and the magnitude of electron temperature gradient in the region considered.

An estimate of the amplitude of the plasma-density fluctuations

$$\eta = \left\langle \left[\frac{\Delta n}{n_c} \right]^2 \right\rangle^{1/2}$$

can be made on the basis of ion-trapping arguments.¹⁴ If

$$W = e^2 n_c \phi^2 / 2T_e$$

is the energy density of ion waves in the regime where the instability is saturated and ϕ their potential, then¹⁴

$$\eta = e\phi/T_e \lesssim \frac{1}{4} [(1 + k^2 \lambda_D^2)^{-1/2} - 3T_i/T_e]^2 \equiv \eta_i, \quad (32)$$

where $T_i \ll T_e$ is the ion temperature and k the wave number of the ion-acoustic mode with the highest growth rate.

If, as assumed, the ion-acoustic turbulence was driven by the parametric decay instability, then the width Δ of the region where the three-wave process is effective can be estimated on the basis of matching conditions. As shown by Kaw and Dawson,¹⁵ the mismatch $\omega_0 - \omega_L$ between the frequency ω_0 of the pumping wave and that of excited Langmuir wave ω_L should not be out of range

$$\omega_0 - \omega_L \simeq (1.7 \pm 0.5) kc_s. \quad (33)$$

Otherwise the growth rates are negligibly small (c_s the speed of sound). For a given plasma-density gradient with a characteristic length

$$\alpha = \left[\left[\frac{1}{n_{e0}} \frac{dn_{e0}}{dx} \right]_{n_{e0}=n_c} \right]^{-1}$$

at the critical density, the relation (33) places restriction on the width Δ as follows

$$\Delta/\alpha \simeq 2(Zm_e/m_i)^{1/2} k/k_D, \quad k \leq k_D.$$

If Δ is assumed to be a half-width of the envelope function

$$\ell(x, y) = \ell(n) = \eta(n/n_c)^\alpha,$$

then, as $\Delta/\alpha \ll 1$, the constant α_- can be estimated as

$$\alpha_- \simeq \alpha/\Delta = (m_i/Zm_e)^{1/2} k_D/k,$$

where k corresponds to a mode with the highest growth rate¹⁶ (i.e., $k \simeq 0.1k_D$).

The last quantity that remains to be determined is a temperature gradient in the region considered. This can be done on the basis of simple energy balance arguments

$$|n_e v_T l_e dT_e/dx| \simeq \mathcal{A} I_0 \quad (34)$$

when the heat-flux density equals the absorbed radiation-flux density. Here $v_T = (T_e/m_e)^{1/2}$, l_e is the electron mean free path, \mathcal{A} the absorption coefficient, and I_0 the intensity of incident radiation. Such a procedure is possible if the major portion of the laser light is absorbed in a narrow region around the critical density, i.e., if either the short-pulse regime is considered where inverse bremsstrahlung as a volume effect is negligible compared to resonance absorption, or the long-pulse regime when I_0 is above the threshold for nonlinear mechanisms of absorption which then can dominate inverse bremsstrahlung in the bulk of the underdense plasma.

Finally, the growth rate $|\tilde{\omega}|$ of the instability being considered in its linear stage can be written as follows:

$$|\tilde{\omega}| = \frac{\eta}{(8\pi)^{1/2}} \left[\frac{Z}{n_{e0} m_i} \right]^{1/2} \frac{cm_e v}{ec_s n_c T_e} \times \mathcal{A} I_0 \left[\left[\frac{m_i}{Zm_e} \right]^{1/2} k_D/k \right]^{1/2} \tilde{s}, \quad (35)$$

where η is given by relation (32) and we have replaced ω by kc_s .

The range of wave numbers \tilde{s} of the long scale density modulations where the present theory is applicable is defined by conditions (25) and (31), i.e.,

$$\omega = kc_s > |\tilde{\omega}| \quad (36)$$

and

$$\tilde{s} > \tilde{s}_{\min} = d/[\Delta^2(Z\beta\alpha_-/n_{e0}m_i)^{1/2}].$$

Defining the upper boundary \tilde{s}'_{\max} of the wave number range of the validity of our theory by

$$0.1k_D c_s \simeq |\tilde{\omega}(\tilde{s} = \tilde{s}'_{\max})|,$$

one obtains (for $\alpha \simeq \lambda_0$, $\eta = 0.1$, $\mathcal{A} = 0.2$, and $\mu \gg 1$) in units of cm^{-1}

$$\begin{aligned} \tilde{s}'_{\max} &\simeq 10^4 T_e^3 (I_0 \lambda_0^2)^{-1}, \\ \tilde{s}_{\min} &\simeq 2 \times 10^3 T_e^{3/2} (I_0 \lambda_0^2)^{-1}, \end{aligned}$$

where $[T_e] = \text{keV}$ (i.e., T_e is given in units of keV), $[I_0 \lambda_0^2] = 10^{15} \text{ W } \mu\text{m}^2 \text{ cm}^{-2}$, and $[\lambda_0] = \mu\text{m}$. Thus for a Nd laser and $I_0 = 10^{14} \text{ W cm}^{-2}$, $T_e \simeq 1 \text{ keV}$, $\eta \simeq 0.1$, $\mu \gg 1$, and $\alpha \simeq \lambda_0$,

$$\tilde{S}'_{\max} \simeq 10^5,$$

$$\tilde{S}_{\min} \simeq 2 \times 10^4$$

in units of cm^{-1} , while for a CO_2 laser, $I_0 \simeq 10^{12} \text{ W/cm}^2$ and $T_e \simeq 1 \text{ keV}$, \tilde{s}'_{\max} remain the same, however,

$$\tilde{S}_{\min} \simeq 2 \times 10^3$$

in units of cm^{-1} due to a significant decrease of diffusion of the magnetic field. Simultaneously, in the framework of the present theory the maximum value of the growth rate $\tilde{\omega}$ of the instability becomes

$$\omega \simeq |\tilde{\omega}_{\max}| = |\tilde{\omega}(\tilde{s} = \tilde{s}'_{\max})| \simeq 10^{13} \lambda_0^{-1} \text{ s}^{-1}, \quad [\lambda_0] = \mu\text{m}$$

for the set of plasma and radiation parameters considered here.

As laser pulse durations of current interest are $\tau > 10^{-12} \text{ s}$, the initial assumptions $\omega\tau_h \gg 1$ and $\sigma \gg \omega$ were not violated as well as the assumption $\omega\Delta > v_0 \simeq 10^7 \text{ cm s}^{-1}$ of a negligible effect of plasma flow on the established magnetic field.

The upper limit \tilde{s}'_{\max} introduces a minimum characteristic length $\tilde{\lambda}_{\min} = 2\pi\tilde{s}'_{\max}^{-1}$ of the coherent modulation of the "critical" surface for which the results of our theory are still valid. However, to neglect the magnetic field diffusion also in the y direction, as we did, one must be sure (as mentioned previously) that

$$\tilde{\lambda}_{\min} > (d/|\tilde{\omega}_{\max}|)^{1/2}.$$

This condition implies a new value \tilde{s}''_{\max} which for the plasma and radiation parameters considered above is $\tilde{s}''_{\max} > \tilde{s}'_{\max}$. For lower radiation intensities, for example, it can be $\tilde{s}''_{\max} < \tilde{s}'_{\max}$ and thus the actual upper boundary of validity of the present theory is obviously

$$\tilde{s}_{\max} = \min(\tilde{s}'_{\max}, \tilde{s}''_{\max}).$$

The actual radiation intensity scaling of \tilde{s}_{\min} and \tilde{s}_{\max} depends on the intensity scaling of the electron temperature and this in turn can significantly differ from one experimental situation to another. Thus, while simple steady-state arguments of balance of a heat flux and absorbed intensity yield $T_e \sim I_0^{2/3}$, in our experiments,¹⁷ using short- (20-ps) duration Nd laser pulses, we observe a very weak dependence as $T_e \sim I_0^{0.1}$ (note that T_e is the temperature of the cold electron component).

III. CONCLUSIONS

Concluding, one can summarize the results together with the basic assumptions as follows: It has been shown

that the critical surface can be unstable to coherent rippling perturbations due to the action of negative pressure induced by the random magnetic field associated with the ion-acoustic turbulence. The necessary condition for the random magnetic field pressure to become negative is a dominance of the so-called line-of-force stress.¹⁰ This nonpotential component of the magnetic pressure can more than compensate the potential part and thus make the net magnetic pressure negative if there exists a preferential orientation of the random magnetic field. As the required anisotropy of the random magnetic field is directly related to anisotropy of its pump, i.e., of the ion-acoustic turbulence, one would expect to observe this effect in the linear stage of the ion-acoustic instability. This means that the pump of the ion-acoustic instability must operate above but close to the relevant threshold.

We have chosen the parametric decay instability as a source of the ion-acoustic turbulence in a laser-produced plasma. A major difference between this and the alternative return-current driven instability is that the turbulent region is spatially localized in the vicinity of the critical density which makes it easier to treat the problem analytically.

Because the ion-acoustic waves generated by the parametric decay instability in its linear stage are launched in the direction of the electric field $\vec{E}_i = (0, 0, E_i)$ of the incident laser radiation, the random magnetic field \vec{B}_1 is parallel to $\vec{B}_i = (0, B_i, 0)$. In that case, it is the coherent rippling in the y direction which is potentially unstable. While initially we have made an assumption of the presence of a coherent perturbation (noise) of the critical surface in the y direction, this can, in fact, be generated parallel to \vec{B}_i by the incident wave via another instability of the four-wave type.⁴ As shown in Ref. 4, the optimum (with lowest threshold) wave number \tilde{s} of the ripples due to this four-wave process on a steepened plasma-density profile is $\tilde{s} \simeq 0.85\tilde{s}_0$ (\tilde{s}_0 the wave number of the incident laser light) and is independent of the temperature T_e , at least in the range investigated. Consequently, the instability discussed in Ref. 4 and the one discussed here can reinforce each other.

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APPENDIX

Neglecting plasma flow and magnetic field diffusion [see (8)], Eq. (2) for a net magnetic field \vec{B} reads

$$\frac{\partial \vec{B}}{\partial t} = -\frac{c}{en_e} \vec{\nabla} n_e \times \vec{\nabla} T_e - \frac{c}{8\pi en_e^2} \vec{\nabla} n_e \times \vec{\nabla} B^2. \quad (\text{A1})$$

After introducing the plasma-density fluctuations $n_1(x, z; t)$ on its background $n_{e0}(x)$ so that

$$n_e = n_{e0} + n_1, \quad (\text{A2})$$

one has, formally, to write the net magnetic field as

$$\vec{B} = \vec{B}_0 + \vec{B}_1, \quad (\text{A3})$$

where \vec{B}_0 is the regular part and \vec{B}_1 represents the fluctuations.

Since from symmetry considerations

$$(\vec{B}_1)_x \equiv 0, \quad (\vec{B}_0)_x \equiv 0,$$

and

$$\vec{B}_0 = \vec{B}_0(x), \quad \langle |\vec{B}_1|^2 \rangle = \langle |\vec{B}_1(x)|^2 \rangle,$$

Eq. (A1) can be split into two equations for the regular and random parts of the magnetic field as follows:

$$\frac{\partial \vec{B}_0}{\partial t} = -\frac{c}{4\pi en_{e0}^2} \langle \vec{\nabla} n_1 \times \vec{\nabla} (\vec{B}_0 \cdot \vec{B}_1) \rangle \quad (\text{A4})$$

and

$$\frac{\partial \vec{B}_1}{\partial t} = -\frac{c}{en_{e0}} \vec{\nabla} n_1 \times \vec{\nabla} T_e - \frac{c}{8\pi en_{e0}^2} \vec{\nabla} n_1 \times \vec{\nabla} B_0^2, \quad (\text{A5})$$

where only the part of \vec{B}_1 being coherent with $\vec{\nabla} n_1$ has been considered. Further, since

$$n_1 \sim \exp(i\omega t),$$

then also

$$B_1 \sim \exp(i\omega t)$$

and \vec{B}_0 has a dc component plus higher harmonics $N\omega$ ($N \geq 2$) driving multiple harmonics of \vec{B}_1 through the nonlinear terms in (A4) and (A5). Further, we will restrict our discussion to a dc component of \vec{B}_0 and \vec{B}_1 oscillating at the fundamental frequency ω . In that case Eq. (A4) becomes

$$\langle \vec{\nabla} n_1 \times \vec{\nabla} (\vec{B}_0 \cdot \vec{B}_1) \rangle = 0, \quad (\text{A6})$$

where

$$\vec{B}_1 = \frac{ic}{en_{e0}\omega} \vec{\nabla} n_1 \times \vec{\nabla} T_e + \frac{ic}{8\pi en_{e0}^2\omega} \vec{\nabla} n_1 \times \vec{\nabla} B_0^2. \quad (\text{A7})$$

Also, since the source term in (A4) must be parallel to \vec{B}_0 , then $\vec{B}_0 = (0, B_0, 0)$. Thus, after inserting (A7) into (A6) and making use of the following property of the random fluctuations [see (7) and (10) for $\mathcal{f} = \mathcal{f}(x)$]

$$\frac{\partial n_1}{\partial x} \frac{\partial^2 n_1}{\partial z^2} = \frac{\partial n_1}{\partial z} \frac{\partial^2 n_1}{\partial x \partial z},$$

one obtains the following equation for the dc magnetic field B_0 :

$$\left\langle \left[\frac{\partial n_1}{\partial z} \right]^2 \right\rangle \left[\frac{dB_0}{dx} \frac{dT_e}{dx} + B_0 \frac{d^2 T_e}{dx^2} \right] = 0$$

which possesses an obvious solution

$$B_0 = a (dT_e/dx)^{-1},$$

where a is a dimensional constant. However, since \vec{B}_1 and hence also \vec{B}_0 should both vanish when $dT_e/dx \rightarrow 0$, the constant a must be $a \equiv 0$ and consequently $B_0 \equiv 0$ and therefore the nonlinear term $\mathcal{F}(B^2)$ in Eq. (3) can be omitted.

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