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Mirrorless optical bistability with the use of the local-field correction

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A semiclassical model of two-level atoms driven by a local field is developed. It is shown that this leads to optical bistability without the presence of an optical cavity. The quantitative and qualitative character of this mirrorless bistability is different from cases considered earlier.

I. INTRODUCTION

Recently, there has been considerable interest in the problem of optical bistability.[1] Most experimental and theoretical work has concentrated on Fabry-Perot devices.[2] In addition, there has been an interest in devices using nonlinear surface concepts.[3] In this paper we consider the question of whether one can dispense with optical paraphenalia altogether, and make an optically bistable device out of the material system alone.

This idea of making "mirrorless" bistable devices,[4] including using the local-field correction[5] (LFC) to make the bistability, is not new. Earlier calculations have fallen into three categories. Some treat the medium as a single particle.[5] Aside from the inconsistency of using a many-body effect like the LFC in a single-particle calculation, this procedure leaves open serious questions as to whether the bistability is possible unless proper boundary conditions are imposed. Some of these calculations have yielded quite unrealistic results, predicting large nonlinearities at very weak powers.[5] Other calculations use the Dicke model,[4] which neglects the propagation of the electromagnetic field. Since propagation of optical fields can be important, this limits the results to qualitative interpretation. Finally, there are calculations in which nonunique molecular states are invoked to make mirrorless bistability, such as anharmonic oscillators[6] (e.g., HF). In such cases, bistability is exhibited by each molecule independently of each other. Since there are no strong intermolecular forces, each molecule fluctuates between its bistable states independently of all others. Under such circumstances, the memory of the initial state can decay (called a "glitch") rapidly in time due to fluctuations.

The first-principles aspects of the LFC is considered here, although not in detail. The point of a development from first principles is to indicate where the fundamental assumptions of the treatment lie. Our primary purpose in this development is not to demonstrate in any absolute sense that optical bistability is possible using the LFC, but rather to develop the problem in a systematic fashion so that the basic issues are at least clear. We thus consider this paper to be a preliminary exercise. We hope it will result in more attention to a problem that we believe is of both fundamental and practical interest.

The microscopic semiclassical formulation of optical interactions follows directly from quantum electrodynamics when the electromagnetic fields are described by Glauber states and the Heisenberg equations of the system are assumed to factorize. Our equations contain terms quadratic in elements of the density matrix, and it is by no means clear that such factorization is allowed. Hence factorization is a strong assumption in this work, and no justification other than the intractability of the fully quantum problem is intended. In addition, we find that the media are not truly bistable; rather there are a very large number of stationary solutions. We restrict the solutions we examine in detail by using the additional assumption that all atoms in a given locality are in the same superposition of atomic states. We refer to this solution as "cooperative," in accord with nomenclature of the Dicke model. This assumption is made for simplicity, and no effort to justify the validity of the assumption is made. The question of the stability of the solutions is beyond the scope of this work, and may require quantum electrodynamics for proper evaluation.

We eventually formulate the problem in terms of a self-consistent solution of the macroscopic, semiclassical density-matrix and Maxwell equations for two-level atoms interacting with a radiation field. In addition, we develop the problem of optical bistability in a microscopic two-atom model. This shows that the macroscopic and microscopic solutions differ only in geometric factors. We also show that bistable operation is restricted to situations in which slowly varying solutions are invalid. Hence we develop techniques for numerically evaluating the exact Maxwell equations.

Our development is, thus, far from complete. There are a number of problems we do not address. These need considerable research on their own, and must be answered before one can assert that bistable devices of this kind are possible. These questions are as follows. Is the LFC a cooperative phenomenon? This is of primary importance in discovering the expected glitch rate of such a device; does the answer to the previous question depend on the structure factor (see below)? Is it permissable to use two-level descriptions for atoms interacting as strongly as these, or do multilevel and/or band-structure effects play a role? What is the influence of medium inhomogeneities and anistropies?

II. THE LOCAL-FIELD CORRECTION

A conventional derivation of the LFC can be found in Jackson.[7] The LFC is introduced as part of the passage from microscopic to macroscopic electrodynamics. The derivation in Jackson suggests that the correction is due to all of the atoms in the system, i.e., it is some kind of a cooperative phenomenon. If this is the case, then bistability based on this phenomenon would seem to be potentially useful, since the device would have the low glitch rate of a cavity device, as opposed to the high rate of the single-molecule case. It is unclear, however, whether this cooperative interpretation suggested by the development in Jackson is correct. Moreover, there are two concepts embedded in the LFC which are discussed in detail below, and they may work differently with respect to cooperative operation.

In this overview of the LFC we are drawing heavily on the discussion of Sipe and Van Kranendonk.[8] Let us begin the discussion by emphasizing that there is no local-field correction in the microscopic theory of electromagnetism. In that case, the force on atom m comes from a field denoted by E_m, where

$$E_m = E_i + \sum_{j\,(\neq m)} G_{mj} p_j \; . \tag{1}$$

Here E_i is the input field, p_j is the polarization of atom j, and G_{mj} is the Green's function (i.e., the Lenard-Wiechart potential) that takes into account geometry. Retardation and vector properties are not denoted explicitly, but are understood to be part of the description of the fields. The absence of m in the sum comes from the removal of the self-field of atom m which appears in the theory as damping and spontaneous emission.

In the passage to macroscopic electromagnetism, two problems occur.

(1) Reintroduction of the self-field. The macroscopic Maxwell's equation in isotropic media reads

$$-\nabla^2 E + \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P}{\partial t^2} \; . \tag{2}$$

Here P is the macroscopic polarization defined as

$$P = \sum_j p_j \; , \tag{3}$$

so that P contains the contributions of all atoms. The solution of the total field E from Eq. (2) then results in a contribution to the total field from all atoms. When this field is used to drive atom m, it contains the self-field of atom m. As shown in Ref. 8, the extra contribution is removed by taking the atom to be driven by a field

$$E_{\text{loc}} = E + \frac{4\pi P}{3} \; . \tag{4a}$$

The correction in Eq. (4a) is proved only for homogeneous broadening, and in the interpretation of Ref. 8, the amplitude of P is given by the amplitude of the dipole of atom m, not by the surrounding atoms [note that homogeneous broadening implies that the semiclassical values of the macroscopic and microscopic polarizations are proportional to each other; hence Eq. (4a) can be written in terms of P rather than in terms of the off-diagonal element ρ_{ab} of the 2×2 single-atom density matrix ρ]. This interpretation suggests that the local-field correction is a single-atom phenomenon.

(2) Neglect of detailed correlations of the medium. In this case, one observes that in crystals, it is by no means clear that the field at atom m is accurately represented by smearing out the positions of atoms $j\neq m$. This can only be significant for atoms in the neighborhood of m, so the correction is again proportional to P in the static limit. In the dynamic limit, questions of the appropriate scale of locality (i.e., the wavelength λ) become important. If such issues can be neglected, one obtains the total LFC as

$$E_{\text{loc}} = E + \left[\frac{4\pi}{3} + s\right] P \; . \tag{4b}$$

The name "structure factor" is given to the constant s, which comes from the correlations of atomic position that occur in crystals.[7,8] The contribution to the local field from s appears to involve explicitly the forces from neighboring atoms.

III. FORMULATION OF THE PROBLEM

In this development, the slowly varying approximation is made only with respect to time, and not in the usual fashion with respect to space. This is exact in Maxwell's equations, and is exact in Schrodinger's equation for circular polarization (for linear polarization we take the rotating-wave approximation to be valid). The convention for defining amplitudes of macroscopic fields (all other fields have the same convention) reads

$$E = \tfrac{1}{2}[\mathscr{E}\exp(-i\nu t) + \text{c.c.}] \; , \tag{5}$$

$$P = \tfrac{1}{2}[\mathscr{P}\exp(-i\nu t) + \text{c.c.}] \; , \tag{6}$$

and the slowly varying amplitude of the off-diagonal density-matrix element is

$$\rho_{ab} = \tfrac{1}{2}[iR_{ab}\exp(-i\nu t) + \text{c.c.}] \; . \tag{7}$$

In each case, ν is the frequency, t denotes time, $i=\sqrt{-1}$, and c.c. denotes the complex conjugate. Assuming homogeneity of the medium, Eqs. (3), (5), and (6) give

$$\mathscr{P} = i\wp N R_{ab} \; , \tag{8}$$

where $\wp$ is the dipole matrix element, and N is the density of atoms. From Eqs. (4)–(8), one obtains the amplitude $\mathscr{E}_{\text{loc}}$ of the local field as

$$\mathscr{E}_{\text{loc}} = \mathscr{E} + i\zeta\wp N R_{ab} \; , \tag{9}$$

where $\zeta = 4\pi/3 + s$. The density-matrix equations of the two-level atoms whose upper (lower) states are labeled a (b) read

$$\frac{dR_{ab}}{dt} = -(i\delta + \gamma_{ab})R_{ab} + \frac{\wp}{2\hbar}\mathscr{E}_{\text{loc}} n \; , \tag{10}$$

$$\frac{dn}{dt}=-\gamma(1+n)+\frac{\wp}{\hbar}(\mathcal{E}^{*}_{\mathrm{loc}}R_{ab}+\mathrm{c.c.})\ , \tag{11}$$

where the inversion n is

$$n=\rho_{aa}-\rho_{bb}\ , \tag{12}$$

and ρ_{aa} and ρ_{bb} are the diagonal elements of the density matrix. In the above formulation, it is assumed that the lower-state decay is zero, so that $\gamma=\gamma_a$. The decay of the off-diagonal element is γ_{ab}, δ is the detuning of the atoms from the field, and $\hbar$ is Planck's constant divided by 2π.

Upon substitution of Eq. (9) into Eq. (11) one obtains

$$\frac{dn}{dt}=-\gamma(1+n)+\frac{\wp}{\hbar}(\mathcal{E}^{*}R_{ab}+\mathrm{c.c.})\ . \tag{13}$$

Thus the local field drops out of the equation for the inversion. The equation for the off-diagonal element can then be rearranged using Eq. (9) to read

$$\frac{dR_{ab}}{dt}=-[i(\delta-B\gamma_{ab}n)+\gamma_{ab}]R_{ab}+\frac{\wp}{2\hbar}\mathcal{E}n\ , \tag{14}$$

where

$$B=\frac{\zeta N\wp^2}{2\hbar\gamma_{ab}}\ . \tag{15}$$

One therefore sees that the local-field correction causes an inversion-dependent frequency shift. It can be shown that when taken off resonance, this shift leads to the Clausius-Mosatti[7] relationship. To obtain a magnitude for B, it is useful to recall the formula for the on-resonance absorption coefficent α which is[9]

$$\alpha=\frac{4\pi\nu N\wp^2}{c\hbar\gamma_{ab}}\ . \tag{16}$$

The absorption coefficient α can be expressed in terms of B as

$$\alpha=\frac{16B\pi^2}{\lambda\zeta}\ . \tag{17}$$

Here we have used $\nu/c=2\pi/\lambda$, where λ is the wavelength of light in vacuum. The numerical factors that relate B to α are appreciable. Since B is the shift in the central frequency relative to the linewidth, it follows that for any appreciable shift, the absorption coefficient is of the order of or much greater than $1/\lambda$. Since bistability occurs near resonance, the slowly varying approximation in space is not usable.

IV. TWO-ATOM PROBLEM

In the case of two two-level atoms we deal explicitly with the microscopic problem. We formulate the problem in a fashion paralleling that of Milonni and Knight,[10] who formulated the problem for spontaneous radiation of a two-atom system in vacuum. We assume a strong external field, and factorize their quantum-electrodynamic equations, leading to a semiclassical description of the problem. This is the simplest system in which bistability can be shown self-consistently. The microscopic Maxwell and density-matrix system contains no LFC, since none exists in microscopic electrodynamics. However, the same inversion-dependent shift occurs in the resonance. A simple geometry is chosen for this demonstration, namely, one in which the atoms, labeled 1 and 2, are separated by a distance r along an axis parallel to the wave normal of the incident field. Let A be defined as

$$A=\frac{k^2}{r}+\frac{ik}{r^2}-\frac{1}{r^3}\ , \tag{18}$$

where k is the wave vector of the incident field. Then

$$E_1=E_i+A\exp(ikr)p_2\ , \tag{19}$$

$$E_2=E_i\exp(ikr)+A\exp(ikr)p_1\ . \tag{20}$$

The slowly varying amplitudes of E_1 and E_2 then read

$$\mathcal{E}_1=\mathcal{E}_i+iA\wp\exp(ikr)R_{ab}(2)\ , \tag{21}$$

$$\mathcal{E}_2=\mathcal{E}_i\exp(ikr)+iA\wp\exp(ikr)R_{ab}(1)\ , \tag{22}$$

where we denote the density-matrix elements of atoms 1 and 2 by arguments (we are only interested in cases in which the dipole amplitudes are constant, so their retardation is not explicitly denoted). The development of the equations for the atoms in this case parallels the macroscopic case except that A is complex, and hence one cannot straightforwardly simplify the equation for the inversion. The polarizations in steady state read

$$R_{ab}(1)=\frac{\wp}{2\hbar}[\mathcal{E}_i+iA\wp\exp(ikr)R_{ab}(2)]\left[\frac{n(1)}{D(\delta)}\right]\ , \tag{23}$$

$$R_{ab}(2)=\frac{\wp}{2\hbar}[\mathcal{E}_i\exp(ikr)+iA\wp\exp(ikr)R_{ab}(1)]\left[\frac{n(2)}{D(\delta)}\right]\ , \tag{24}$$

where

$$D(\delta)=i\delta+\gamma_{ab}\ . \tag{25}$$

Equations (23) and (24) form a set of two equations and two unknowns (i.e., the R's) which is solved to give the R's as a function of the n's. At this point it is easiest to redefine the field amplitude to absorb the factor $\wp/\hbar$, and to redefine A by absorbing a factor $\wp^2/2\hbar$ to obtain

$$R_{ab}(1)=\frac{n(1)}{2D(\delta)}\mathcal{E}_i\frac{D(\delta)+iA\exp(ikr)n(2)}{D(\delta)^2+A^2\exp(i2kr)n(1)n(2)}\ , \tag{26}$$

$$R_{ab}(2)=\frac{n(2)}{2D(\delta)}\exp(ikr)\mathcal{E}_i\times\frac{D(\delta)+iAn(1)}{D(\delta)^2+A^2\exp(i2kr)n(1)n(2)}\ . \tag{27}$$

It is now straightforward to see the basic conclusions of the two-atom case. Let us consider the simplest limit, namely, $kr\ll\pi$, in which case A is real and the terms going as $\exp(ikr)$ and $\exp(i2kr)$ are all unity. One can see that, upon substitution into the equations for the inversion, there will be a nonlinear polynomial of the form of two coupled cubic equations with two unknowns. This may have more than one physical solution, and there is the possibility of bistability. Solutions to these equations exist for which $n(1)\neq n(2)$ and $n(1)=n(2)=n$. We restrict our attention to the latter cases. This restriction is not justified here, and is taken for the purpose of simplicity. As noted above, it is unclear whether quantum electrodynamics is needed to properly describe this problem, and may be necessary to determine the stability of solutions (we would, at least, need to include retardation explicitly, which leads to complicated expressions for the stability eigenvalues). Such an investigation is beyond the scope of this analysis. With this restriction, Eqs. (26) and (27) give

$$R_{ab}=\frac{n\mathscr{E}_i}{2D(\delta)[D(\delta)-iAn]} \; . \tag{28}$$

This is the same solution, except for differences in the geometric factors denoted A and B, that one obtains from Eq. (14) in steady state. Both cases give an inversion-dependent frequency shift. We show later that the same shift occurs with the LFC, and that this leads to bistable solutions. In the two-atom calculation we have shown that such shifts are reasonable based on a semiclassical electrodynamic model in which no LFC is involved.

V. SINGLE-ATOM PROBLEM

Treating an LFC problem as if one had a single atom entails obvious logical inconsistencies. Nonetheless, such techniques have been used in previous studies,[5] and it is a useful first step in the analysis. It helps, for example, to establish the magnitude of the parameters involved in the bistability.

It is straightforward to find the steady-state solutions of Eqs. (13) and (14) for the case where the external field is constant in time. This is given by a cubic equation

$$(1+n)\left[1+\left[\frac{\delta}{\gamma_{ab}}-Bn\right]^2\right]+Sn=0 \; , \tag{29}$$

where

$$S=\frac{\mathscr{E}^2}{\gamma\gamma_{ab}} \; . \tag{30}$$

The term containing the geometric factor B comes directly from the inversion-dependent frequency shift of Eq. (14). Equation (29) has three real roots in the parameter range $B\geq 6$, $S\geq 4$, $\delta/\gamma_{ab}\simeq -2$. These roots are always such that $|n|\leq 1$, so they are physical (in practice one finds $n<0$). These solutions have the property that the largest and smallest values of n are stable, and the middle value is unstable. We find that a complete solution of the problem does give bistability in this general range of parameters, and not in others. For $B\geq 6$ and $s\simeq 0$, Eq. (17) gives $\alpha\lambda\gg 1$. At the value of detuning given above, the medium is opaque.

VI. MAXWELL'S EQUATIONS

The single-atom solution, while useful, is nonetheless self-contradictory in the case of a many-body effect like the LFC. We therefore consider next the multiatom macroscopic solution. The single-atom equations are the equations of motion for the medium.

As noted previously, the large values of the absorption coefficient needed for bistable operation preclude the use of the slowly varying approximation in the spatial coordinate to compute answers. On the other hand, since the media are opaque, one is not interested in large propagation distances, and hence it is feasable to numerically integrate the exact equations over distances short compared to an optical wavelength. The distances involved in this calcultion are so short that there is no possibility of cavitylike resonances occurring between the surfaces of the medium, although small amounts of interference occasionally complicate the results. There has been no work, to our knowledge, on how to solve an exact, nonlinear propagation problem in this limit, so we discuss our technique in some detail.

In the steady-state, plane-wave case, Maxwell's equations reduce to

$$-\frac{d^2\mathscr{E}(z)}{dz^2}-\frac{\nu^2\mathscr{E}(z)}{c^2}=\frac{4\pi\nu^2\mathscr{P}(z)}{c^2} \; , \tag{31}$$

where the amplitudes are independent of time, and hence $\mathscr{E}$ is described by an ordinary differential equation. To obtain the solution to Eq. (31) it is necessary to apply boundary conditions on the amplitudes. We denote by z the position in a medium extending from $z=0$ to $z=L$. The total field to the left of the medium reads

$$E=\tfrac{1}{2}\{\mathscr{E}_i\exp[i(kz-\nu t)] + \mathscr{E}_R\exp[i(-kz-\nu t)]+\text{c.c.}\}, \quad z\leq 0 \tag{32}$$

from which one obtains the amplitudes and derivatives as

$$\mathscr{E}=\mathscr{E}_i\exp(ikz)+\mathscr{E}_R\exp(-ikz), \quad z\leq 0 \tag{33}$$

$$\frac{d\mathscr{E}}{dz}=ik[\mathscr{E}_i\exp(ikz)-\mathscr{E}_R\exp(-ikz)], \quad z\leq 0 \; . \tag{34}$$

For fields to the right of the medium we have

$$E=\tfrac{1}{2}\{\mathscr{E}_T\exp[i(kz-\nu t)+\text{c.c.}]\}, \quad z\geq L \tag{35}$$

from which one obtains the amplitudes and derivatives as

$$\mathscr{E}=\mathscr{E}_T\exp(ikz), \quad z\geq L \tag{36}$$

$$\frac{d\mathscr{E}}{dz}=ik\mathscr{E}_T\exp(ikz), \quad z\geq L \; . \tag{37}$$

Here the subscripts T and R denote the transmitted and reflected field. We assume normal incidence. In this case the important boundary conditions are the continuity of E and H (the magnetic field) at each boundary. The former requires that $\mathscr{E}$ is continuous at each boundary, the latter can be easily shown to require that the derivative of $\mathscr{E}$ must be continuous. At the boundary $z=0$, Eqs. (33) and (34) provide the conditions

$$\mathscr{E}=\mathscr{E}_R+\mathscr{E}_i, \quad z=0 \tag{38}$$

$$\frac{d\mathscr{E}}{dz}=ik(\mathscr{E}_i-\mathscr{E}_R), \quad z=0 \tag{39}$$

to integrate Eqs. (31), (13), and (14) across the medium, provided the reflected field is given. Equations (36) and (37) reduce to

$$\frac{d\mathcal{E}}{dz}=ik\mathcal{E}, \quad z=L \; . \tag{40}$$

Equation (40) is a criterion that must be met in order that the solution be valid.

Before discussing how the solution is obtained it is useful to define the following standard notation:

$$r=\frac{\mathcal{E}_R}{\mathcal{E}_i}, \quad R=|r|^2, \quad T=\frac{|\mathcal{E}_T|^2}{|\mathcal{E}_i|^2} \; , \tag{41}$$

$$A=1-T-R \; . \tag{42}$$

Here R and T are the usual reflection and transmission coefficients, A is the absorptance, i.e., the fraction of optical energy that is dissipated in the medium, and r is the reflection coefficient for the field amplitude.

VII. METHOD OF SOLUTION

The difficulty with the solution is that $\mathcal{E}_R$, i.e., r, is unknown. The solution method involves guessing r until one finds a result that meets the criterion of Eq. (40). When that is found, then one has a valid solution of the problem insofar as all of the boundary conditions are met.

As a practical matter, such a procedure is tedious, and it is necessary to find some method for systematizing the search for the proper r. The method used here is to assume that $\mathcal{E}(z=L)$ and its derivative are analytic functions of the complex number r, so that $d\mathcal{E}(z=L)/dr$ and $d^2\mathcal{E}(z=L)/dr\,dz$ are taken to exist. The measure of whether a solution is found is denoted $C(r)$, and reads

$$C(r)=\frac{d\mathcal{E}}{dz}-ik\mathcal{E}, \quad z=L \; . \tag{43}$$

The assumptions outlined above imply that $C(r)$ is an analytic function of r. The solution is found when $C(r)=0$, so the problem has been reduced to a complex Newton-Raphson procedure. The remaining difficulty is to define the derivative numerically. This is done by choosing some δr prior to the calculation, and then for each successive choice of r, the case $r+\delta r$ is computed, and the derivative of $C(r)$ is taken to be

$$\frac{dC(r)}{dr}=\frac{C(r+\delta r)-C(r)}{\delta r} \; .$$

The numerical criterion for a solution is taken to be $|C/\mathcal{E}_T|<0.0001$. There is an independent criterion on the solution that is simultaneously checked, and that is that the dissipated energy predicted by Eq. (42) must equal the dissipated energy computed by Eqs. (13) and (14). This criterion turns out to be met to within 2% except when dissipation is small, in which case it was within 5%. We take this as a measure of the expected accuracy of the solution. Criteria coming from the boundary condition are not as well suited to this purpose. The energy is an absolute criterion while the boundary condition is only measurable relative to a field which is itself guessed through the numerical procedure. The numerical accuracy of every computed point was independently checked (there are no test cases).

VIII. RESULTS

In the calculations we assume that the atoms are always locally in the same superposition of atomic states. To ensure this, we specify, in advance of the calculation, the state that the atom will be in (the "upper" or "lower") if a nonunique atomic state is encountered. The upper state is the one with the largest value of ρ_{aa}, and the lower state has the smallest value of ρ_{aa}. The former defines the upper bistable branch, and the latter defines the lower branch. These branches would be reached in practice by the usual methods; the lower branch by adiabatically increasing a weak field, and the upper by adiabatically decreasing a strong field. The upper and lower branches have the highest and lowest rates of dissipation of optical energy (i.e., as measured by the absorptance). By this measure, the terms upper and lower have the same significance as in a cavity. However, the transmission characteristics of the two states are reversed from the bistability in a cavity, in that the lower branch has the higher transmission.

In Fig. 1 the transmission function is shown as a function of detuning δ for three values of B, each with two or three values of $\mathcal{E}_i$. The case where $B=4$ and $\mathcal{E}_i=3$ was not finished due to extreme difficulties in numerical convergence, probably because it is approaching the condition of an optical transistor. For $B<6$, bistability is not expected, and is not found. Note the shift in the resonance frequency as the atoms become saturated. Hopefully, this shift could be measured experimentally, which would be an additional verification that the ideas presented here are valid. For $B=6$, $\delta\cong-2$, and $\mathcal{E}_i\cong2$, a nonunique absorption is seen, which is consistent with the values obtained in the single-atom calculation. Note that the curves for the upper and lower branches join continuous-

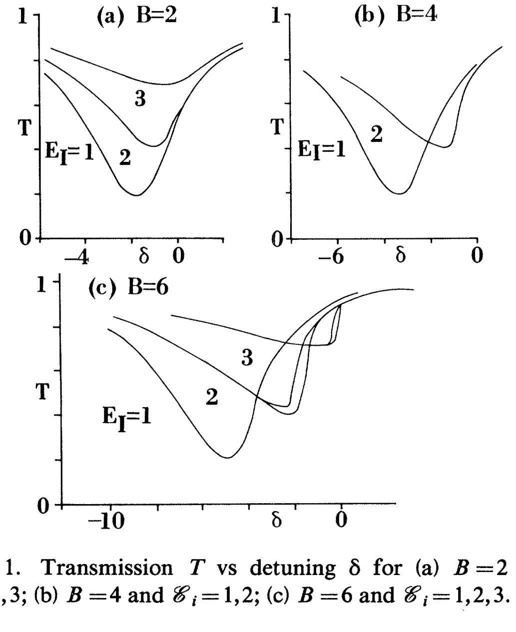

FIG. 1. Transmission T vs detuning δ for (a) $B=2$ and $\mathcal{E}_i=1,2,3$; (b) $B=4$ and $\mathcal{E}_i=1,2$; (c) $B=6$ and $\mathcal{E}_i=1,2,3$.

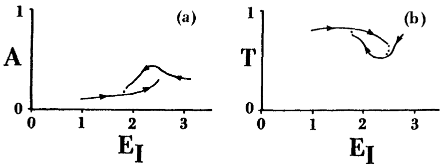

FIG. 2. (a) Absorptance A and (b) transmission T vs input field $\mathscr{E}_i$ for $\delta = -1.5$ and $B = 6$.

ly, so the states taken by the atoms as the tuning is varied are indeterminate in a steady-state analysis. Unlike bistability in a cavity, there is no method of systematically reaching bistable states by adiabatically tuning the frequency.

In Fig. 2, a bistable loop is shown for $B = 6$, $\delta = -1.5$, and $\mathscr{E}_i$ variable over a range from 1 to 3. Both the absorptance A and transmission T are shown. The absorptance is the same for this case and for bistability in a cavity, insofar as in both cases a larger amount of optical energy is dissipated if the atoms are in the upper branch than if they are in the lower branch. The condition on transmission then reverses the normal one. The lower branch has a higher transmission than the upper branch.

IX. CONCLUSION

In conclusion, it has been shown that the notion of using the LFC to generate optical bistability is a sensible one, insofar as the present understanding of semiclassical electromagnetism is concerned. Bistability is seen to be limited to cases in which the medium is so highly absorbing that it is opaque. In that case, we imagine using slabs of active media that are much thinner than an optical wavelength. The widths of these slabs are so short that no cavity resonances occur for the wavelengths of interest.

Our development is far from complete. There are a number of problems we have not addressed. These chiefly involve the stability of the solutions, and, in particular, whether semiclassical theory is capable of properly addressing the stability question. There is an additional important issue, and that is whether it is possible to devise experiments such that the answers to these theoretical questions can be obtained in conjunction with data.

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