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Classical spinning magnetic dipole in classical electrodynamics with classical electromagnetic zero-point radiation

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A classical spinning magnetic dipole is considered within classical electrodynamics with classical electromagnetic zero-point radiation. The stationary probability distribution for the angle of alignment between the spinning magnetic dipole and an external magnetic field is calculated when the system is located in an arbitrary spectrum of random classical radiation. It is found that for the Rayleigh-Jeans spectrum the probability distribution for alignment is just the Boltzmann distribution. However, in classical zero-point radiation the alignment probability distribution is independent of the magnetic field causing alignment. Moreover, for large classical spin angular momentum $S \gg \hbar$, the average component of the spin in the direction of alignment is given by $\langle S_z \rangle = S - \frac{1}{2}\hbar$, where $\hbar$ is Planck's constant (divided by 2π) used to set the scale of the classical zero-point radiation spectrum. The results seem suggestive of the idea of space quantization in quantum theory. The model discussed here was first considered by S. Sachidanandam (unpublished).

INTRODUCTION

In this article we consider a classical object which has an angular momentum from a spinning motion and which also carries a magnetic dipole moment parallel to this spin. A number of interesting results are found for the alignment of this spinning magnetic dipole in a magnetic field when the system is subjected to random classical electromagnetic radiation with Planck's spectrum including zero-point radiation.

These calculations show that if the spinning magnetic dipole is located in the Rayleigh-Jeans spectrum of classical radiation, then the alignment of the spin by a magnetic field follows the Boltzmann distribution of traditional classical statistical mechanics. If, however, the spinning dipole is in classical zero-point radiation, then the average component of the spin along an aligning magnetic field is independent of the strength of the magnetic field. This surprising result is reminiscent of the quantum notion of space quantization. Moreover, for large values S of the classical spin angular moment $S \gg \hbar$, the average component along the direction of the external field becomes $\langle S_z \rangle = S - \frac{1}{2}\hbar$ where the constant $\hbar$ is Planck's constant which was introduced to set the scale of the classical zero-point radiation spectrum. It is curious that this result associates the value $\frac{1}{2}\hbar$ with a spin angular momentum in contrast to the orbital angular momentum $\langle L_z \rangle = \pm\hbar$ found[1] for a free point charge in a magnetic field.

The present work represents another small step within classical electron theory including classical electromagnetic zero-point radiation. This theory[2] is often termed random or stochastic electrodynamics. It is a classical electromagnetic theory in which Planck's constant $\hbar$ is introduced as the scale factor in the Lorentz-invariant spectrum of random classical electromagnetic radiation which provides the homogeneous boundary condition on Maxwell's field equations. This classical theory has produced a number of results usually thought to be obtainable only from quantum theory. However, analysis of the theory has been hampered by the calculational difficulties of classical electromagnetism and of stochastic processes.

BASIC MODEL

The classical nature of our model can be emphasized by describing it as essentially a small spinning gyroscope with a permanent bar magnet mounted along the axis of the spinning rotor. We denote the spin angular momentum by $\vec{S}$ and the magnetic dipole moment by $\vec{\mu}$. The system is located in an external magnetic field $\vec{B}_0$ and also in random classical electromagnetic radiation.

Due to the magnetic field $\vec{B}_0$ the magnetic dipole experiences a torque $\vec{\Gamma} = \vec{\mu} \times \vec{B}_0$. Provided the rotor moment of inertia is small and the rate of spin large, any torque will act to change the direction of the spin $\vec{S}$ without ever changing the magnitude S of the spin. The gyroscope will precess just as does a toy gyroscope on a stand subject to a torque from the gravitational force.

However, our model requires more electromagnetic analysis. The precessing magnetic moment in our model will radiate electromagnetic waves and so lose energy. The magnetic dipole will tend to align itself with the magnetic field in the position of lowest magnetic energy. In this alignment it no longer precesses and, hence, no longer radiates away its energy. Now Sachidanandam pointed out that this tendency for the magnetic dipole to align itself with the external field will be counteracted by the random fluctuating torques due to the random classical electromagnetic radiation. The two opposing tendencies of alignment and antialignment will lead to a stationary probability distribution for the orientation of the system.[3] It is this stationary probability distribution which we calculate in this paper.

EQUATION OF MOTION

The equation of motion for our spinning magnetic dipole is that given by Bhabha[4]

$$\dot{\vec{S}}=\vec{\mu}\times\vec{B}_0-\tfrac{2}{3}c^{-3}\vec{\mu}\times\dddot{\vec{\mu}}+\vec{\mu}\times\vec{B}_R(\vec{0},t)\ , \quad (1)$$

where the random field $\vec{B}_R(\vec{r},t)$ is taken in the dipole approximation. The random classical radiation can be written as a sum over plane waves with magnetic field:

$$\vec{B}_R(\vec{r},t)=\sum_{\lambda=1}^{2}\int d^3k\ \vec{\epsilon}(\vec{k},\lambda)H(\vec{k},\lambda)\times\cos[\vec{k}\cdot\vec{r}-\omega t-\xi(\vec{k},\lambda)]\ , \quad (2)$$

where the radiation spectrum is set by $H^2(\vec{k},\lambda)$ and the random phases $\xi(\vec{k},\lambda)$ are distributed uniformly over $[0, 2\pi]$, independently distributed for each $\vec{k}$ and λ. The magnetic moment $\vec{\mu}$ is assumed parallel to the spin $\vec{S}$.

The equation of motion (1) is a nonlinear-stochastic differential equation. We will use a perturbative quasi-Markovian approximation to obtain a Fokker-Planck equation for the probability density $P(\theta)$ that the spin $\vec{S}$ makes an angle θ with respect to the z axis determined by the magnetic field direction. Thus, we assume that the radiation damping term $\frac{2}{3}c^{-3}\vec{\mu}\times\dddot{\vec{\mu}}$ and the fluctuating radiation torque $\vec{\mu}\times\vec{B}_R$ are small corrections to the unperturbed classical motion given by

$$\dot{\vec{S}}=\vec{\mu}\times\vec{B}_0\ . \quad (3)$$

We choose a right-handed coordinate system with $\vec{B}_0=\hat{k}B_0$ along the z axis. Then, writing

$$\vec{S}=\hat{n}S\ ,\quad \vec{\mu}=\hat{n}\mu \quad (4)$$

and

$$\dot{\vec{S}}=\frac{d\hat{n}}{dt}S=(\hat{\theta}\dot{\theta}+\hat{\varphi}\sin\theta\dot{\varphi})S\ , \quad (5)$$

Eq. (3) becomes

$$(\hat{\theta}\dot{\theta}+\hat{\varphi}\sin\theta\dot{\varphi})S=\mu\hat{n}\times(\hat{n}\cos\theta-\hat{\theta}\sin\theta)B_0 = -\hat{\varphi}\mu B_0\sin\theta\ . \quad (6)$$

We find $\dot{\theta}=0$ and

$$\dot{\varphi}=-\mu B_0/S\ . \quad (7)$$

Thus, when we ignore the loss of energy through radiation and the fluctuating torque due to random radiation, we have the spin oriented at a constant cone angle θ relative to the z axis and rotating at a constant rate $\dot{\varphi}$ around this axis. This is just the precessional motion anticipated.

FOKKER-PLANCK EQUATION

In order to evaluate the alignment probability distribution $P(\theta)$ through a Fokker-Planck equation, we need to find the first and second moments for the change $\Delta\theta$ in θ due to the perturbing influences during a small time τ. We first calculate the rate of change $(d\theta/dt)_{rr}$ due to radiation damping. Then we calculate the average change $\langle\Delta\theta\rangle$ and the average square change $\langle(\Delta\theta)^2\rangle$ in the angle θ due to random classical radiation acting during the time τ.

RADIATION DAMPING

The change of θ due to radiation damping can be found in lowest approximation by equating the rate of change of magnetic energy to the negative of the power radiated by the precessing dipole,

$$\frac{d}{dt}(-\vec{\mu}\cdot\vec{B}_0)_{rr}=-\frac{2}{3c^3}(\ddot{\vec{\mu}})^2\ , \quad (8)$$

where

$$\ddot{\vec{\mu}}=-\hat{r}\mu\sin\theta(\dot{\varphi})^2 \quad (9)$$

with

$$\hat{r}=\hat{i}\cos\varphi+\hat{j}\sin\varphi\ . \quad (10)$$

This gives

$$\left(\frac{d\theta}{dt}\right)_{rr}=-\frac{2}{3}\frac{\mu}{c^3B_0}\sin\theta(\dot{\varphi})^4\ . \quad (11)$$

This same result can also be obtained from the equation of motion (1) where we ignore the last term involving random radiation and where we compute an approximate value for $\dddot{\vec{\mu}}$ from the precessional motion alone as

$$\dddot{\vec{\mu}}=-\hat{\varphi}\mu\sin\theta(\dot{\varphi})^3\ . \quad (12)$$

Then, in this approximation, Eq. (1) becomes

$$[\hat{\theta}(\dot{\theta})_{rr}+\hat{\varphi}\sin\theta\dot{\varphi}]S=-\hat{\varphi}\mu B_0\sin\theta-\hat{\theta}\tfrac{2}{3}c^{-3}\mu^2\sin\theta(\dot{\varphi})^3\ . \quad (13)$$

Recalling Eq. (7), we see that (11) and (13) are in agreement.

CALCULATION OF $\langle(\Delta\theta)^2\rangle$

In the next part of our calculation we calculate the average effects of the random radiation while ignoring the effects of radiation damping. Thus, we use

$$\dot{\vec{S}}=\vec{\mu}\times\vec{B}_0+\vec{\mu}\times\vec{B}_R(\vec{0},t)\ , \quad (14)$$

where we write

$$\theta=\theta_0+\delta\ , \quad (15)$$

$$\varphi=\varphi_0+\eta t+\sigma\ , \quad (16)$$

with

$$\eta=-\mu B_0/S\ , \quad (17)$$

corresponding to the value of $\dot{\varphi}$ for the unperturbed motion as in (7). Substituting (4), (5), (15), and (16) into (14), we have

$$[\hat{\theta}\dot{\delta}+\hat{\varphi}\sin\theta(\eta+\dot{\sigma})]S$$
$$=-\hat{\varphi}\mu B_0\sin\theta+\mu\sum_{\lambda=1}^{2}\int d^3k\,H\cos(\omega t+\xi)\{\hat{\theta}(\epsilon_x\sin\varphi-\epsilon_y\cos\varphi)+\hat{\varphi}[(\epsilon_x\cos\varphi+\epsilon_y\sin\varphi)\cos\theta-\epsilon_z\sin\theta]\}\ . \tag{18}$$

This gives us the two equations

$$\dot{\delta}=(\mu/S)\sum_{\lambda=1}^{2}\int d^3k\,H\cos(\omega t+\xi)(\epsilon_x\sin\varphi-\epsilon_y\cos\varphi) \tag{19}$$

and

$$\dot{\sigma}=(\mu/S\sin\theta)\sum_{\lambda=1}^{2}\int d^3k\,H\cos(\omega t+\xi)[(\epsilon_x\cos\varphi+\epsilon_y\sin\varphi)\cos\theta-\epsilon_z\sin\theta]\ . \tag{20}$$

When calculating $\langle(\Delta\theta)^2\rangle$, we require the unperturbed expression $\varphi=\varphi_0+\eta t$ so that

$$\Delta\theta=\int_0^\tau dt\,\dot{\delta}=(\mu/S)\sum_{\lambda=1}^{2}\int d^3k\,H\int_0^\tau dt\cos(\omega t+\xi)[(\epsilon_x\sin(\varphi_0+\eta t)-\epsilon_y\cos(\varphi_0+\eta t)]\ . \tag{21}$$

We now carry out the time integration explicitly, form the product $(\Delta\theta)^2$, and then average over the random radiation phases $\xi(\vec{k},\lambda),\xi(\vec{k}',\lambda')$ and over the initial phase φ_0. Here we need

$$\langle\cos\xi(\vec{k},\lambda)\cos\xi(\vec{k}',\lambda')\rangle=\langle\sin\xi(\vec{k},\lambda)\sin\xi(\vec{k}',\lambda')\rangle=\tfrac{1}{2}\delta_{\lambda\lambda'}\delta^3(\vec{k}-\vec{k}')\ , \tag{22}$$

$$\langle\sin\xi(\vec{k},\lambda)\cos\xi(\vec{k}',\lambda')\rangle=0\ . \tag{23}$$

Then, integrating the prime variables over the δ functions $\delta_{\lambda\lambda'}\delta^3(\vec{k}-\vec{k}')$, we find

$$\langle(\Delta\theta)^2\rangle=\frac{1}{2}\left[\frac{\mu}{S}\right]^2\sum_{\lambda=1}^{2}\int d^3k\,H^2\left[\frac{\sin^2[\frac{1}{2}(\omega+\eta)\tau]}{(\omega+\eta)^2}+\frac{\sin^2[\frac{1}{2}(\omega-\eta)\tau]}{(\omega-\eta)^2}\right]\times(\epsilon_x^2+\epsilon_y^2)\ , \tag{24}$$

where we have written

$$1-\cos a=2\sin^2(a/2)\ .$$

The sum over polarization requires

$$\sum_{\lambda=1}^{2}\epsilon_i^2=1-k_i^2/k^2$$

and the angular integration gives

$$\int d\Omega\sum_{\lambda=1}^{2}(\epsilon_x^2+\epsilon_y^2)=16\pi/3\ . \tag{25}$$

Then Eq. (24) becomes

$$\langle(\Delta\theta)^2\rangle=\frac{8\pi}{3}\frac{\mu^2}{S^2}\int_0^\infty dk\,k^2H^2\left[\frac{\sin^2[\frac{1}{2}(\omega+\eta)\tau]}{(\omega+\eta)^2}+\frac{\sin^2[\frac{1}{2}(\omega-\eta)\tau]}{(\omega-\eta)^2}\right]\ . \tag{26}$$

Now ignoring the nonresonant term and evaluating the resonant term as a sharply peaked function approximated by the form

$$\int_{-\infty}^{\infty}dx\,(\sin^2x\tau)/x^2=\pi\tau \tag{27}$$

we have

$$\langle(\Delta\theta)^2\rangle=\tfrac{4}{3}\pi^2c^{-3}(\mu/S)^2\eta^2H^2(|\eta|)\tau\ . \tag{28}$$

CALCULATION OF $\langle\Delta\theta\rangle$

In order to evaluate $\langle\Delta\theta\rangle$ we first integrate Eq. (20) with respect to time. Here we use the unperturbed expressions for θ and φ on the right-hand side and recall $\sigma(0)=0$ so that

$$\sigma=\int_0^t dt'\dot{\sigma}=(\mu/S\sin\theta_0)\sum_{\lambda=1}^{2}\int d^3k\,H\int_0^t dt'\cos(\omega t'+\xi)\{[\epsilon_x\cos(\varphi_0+\eta t')+\epsilon_y\sin(\varphi_0+\eta t')]\cos\theta_0-\epsilon_z\sin\theta_0\}\ . \tag{29}$$

Then we go back and use this expression for σ in our equation (19) for $\dot{\delta}$ when we expand through first order in σ so that $\cos\sigma\cong1$ and $\sin\sigma\cong\sigma$. Thus, Eqs. (19) and (29) combine to give

$$\dot{\delta}=(\mu/S)\sum_{\lambda=1}^{2}\int d^3k\,H\cos(\omega t+\xi)$$
$$\times\Bigg\{[\epsilon_x\sin(\varphi_0+\eta t)-\epsilon_y\cos(\varphi_0+\eta t)]$$
$$+[\epsilon_x\cos(\varphi_0+\eta t)+\epsilon_y\sin(\varphi_0+\eta t)]\tfrac{1}{2}(\mu/S\sin\theta_0)$$
$$\times\sum_{\lambda'=1}^{2}\int d^3k'H'\left[\epsilon'_x\cos\theta_0\left[\frac{\sin[(\omega'+\eta)t+\xi'+\varphi_0]-\sin(\xi'+\varphi_0)}{(\omega'+\eta)}+(\eta\to-\eta,\varphi_0\to-\varphi_0)\right]\right.$$
$$+\epsilon'_y\cos\theta_0\left[\frac{\cos(\xi'+\varphi_0)-\cos[(\omega'+\eta)t+\xi'+\varphi_0]}{(\omega'+\eta)}\right.$$
$$\left.\left.-(\eta\to-\eta,\varphi_0\to-\varphi_0)\right]-\epsilon'_z\sin\theta_0\left[\frac{\sin(\omega't+\xi')-\sin\xi'}{\omega}\right]\right]\Bigg\}. \tag{30}$$

Next we carry out the averages (22) and (23) over $\xi(\vec{k},\lambda)$ and $\xi(\vec{k}',\lambda')$ and over φ_0, and then we integrate the prime variables over the δ functions $\delta_{\lambda\lambda'}\delta^3(\vec{k}-\vec{k}')$. This gives

$$\langle\dot{\delta}\rangle=\frac{\mu^2}{8S^2}\cot\theta_0\sum_{\lambda=1}^{2}\int d^3k\,H^2(\epsilon_x^2+\epsilon_y^2)\left[\frac{\sin(\omega+\eta)t}{\omega+\eta}+\frac{\sin(\omega-\eta)t}{\omega-\eta}\right]. \tag{31}$$

The sum over polarizations and integration over all solid angles lead again to Eq. (25). Integration with respect to time gives

$$\langle\Delta\theta\rangle=\int_0^\tau dt\langle\dot{\delta}\rangle=\frac{2}{3}\frac{\pi}{c^3}\frac{\mu^2}{S^2}\cot\theta_0\int_0^\infty d\omega\,\omega^2H^2\left[\frac{1-\cos(\omega+\eta)t}{(\omega+\eta)^2}+\frac{1-\cos(\omega-\eta)t}{(\omega-\eta)^2}\right]. \tag{32}$$

Finally, we use $1-\cos a=2\sin^2(a/2)$, retain only the resonant term as evaluated in (27), and arrive at

$$\langle\Delta\theta\rangle=(2\pi^2\mu^2/3c^3S^2)\cot\theta_0\eta^2H^2(|\eta|)\tau\,. \tag{33}$$

SOLUTION OF THE FOKKER-PLANCK EQUATION

The probability distribution $P(\theta)$ in steady state is given by the first integral of the time-independent Fokker-Planck equation where the integration constant is chosen to vanish corresponding to the absence of solutions which are singular at $\theta=0$ or π:

$$-P(\theta)\left[\left[\frac{d\theta}{dt}\right]_{rr}\tau+\langle\Delta\theta\rangle\right]+\frac{1}{2}\frac{\partial}{\partial\theta}[P(\theta)\langle(\Delta\theta)^2\rangle]=0\,. \tag{34}$$

Substituting the expressions (11), (28), and (33) calculated above, and noting that $\dot{\varphi}$ in (11) is exactly η of Eq. (17), we find that the Fokker-Planck equation reduces to

$$\frac{dP(\theta)}{d\theta}+\left[\frac{S^2}{\mu B_0}\frac{\eta^2}{\pi^2H^2(|\eta|)}\sin\theta-\cot\theta\right]P(\theta)=0\,. \tag{35}$$

This has the solution for the angular probability distribution

$$P(\theta)=\text{const}\times\sin\theta\exp\{[S|\eta|/\pi^2H^2(|\eta|)]\cos\theta\}\,, \tag{36}$$

where we have made use of Eq. (17).

AVERAGE z COMPONENT OF SPIN

The probability distribution $P(\theta)$ can be used to calculate the average component of the spin along the direction of the magnetic field. This is

$$\langle S_z\rangle=\langle S\cos\theta\rangle$$
$$=\left[\int_0^\pi d\theta\,S\cos\theta P(\theta)\right]\Big/\left[\int_0^\pi d\theta\,P(\theta)\right]. \tag{37}$$

Introducing the stationary distribution for $P(\theta)$ found in (36), we arrive at

$$\langle S_z\rangle=S\left[\coth\left[\frac{S|\eta|}{\pi^2H^2(|\eta|)}\right]-\left[\frac{\pi^2H^2(|\eta|)}{S|\eta|}\right]\right], \tag{38}$$

which is a Langevin function of argument

$$S|\eta|/[\pi^2H^2(|\eta|)]\,.$$

For a spectrum of random classical radiation given by the Planck spectrum, including zero-point radiation, we have

$$\pi^2H^2(\omega)=\tfrac{1}{2}\hbar\omega\coth(\hbar\omega/2kT)\,. \tag{39}$$

This gives the average value for the z component of the spin as

$$\langle S_z \rangle = S\left[\coth\left[\frac{2S}{\hbar\coth(\hbar\mu B_0/2SkT)}\right] - \frac{\hbar}{2S}\coth\left[\frac{\hbar\mu B_0}{2SkT}\right]\right]. \quad (40)$$

DISCUSSION OF THE LIMITS AT HIGH AND LOW TEMPERATURES

The expression (36) agrees with the qualitative ideas presented at the beginning of this paper. The spinning magnetic dipole is displaced from the aligned position due to the fluctuating random classical radiation. The tendency toward alignment along the direction of the external magnetic field is balanced by the random radiation impulses and thus produces a stationary probability distribution.

In the high-temperature limit where

$$kT \gg \hbar\mu B_0/2S$$

the spectrum of random classical radiation in (39) becomes the Rayleigh-Jeans limit

$$\pi^2 H^2(\omega) = kT . \quad (41)$$

The z component of the spin in (40) goes over to

$$\langle S_z \rangle = S\left[\coth\left[\frac{\mu B_0}{kT}\right] - \frac{kT}{\mu B_0}\right] \quad (42)$$

which is precisely the result given by traditional classical statistical mechanics[5] for a magnetic moment in a magnetic field. Indeed the probability distribution (36) becomes exactly the Boltzmann distribution

$$P(\theta) = \text{const} \times \sin\theta \exp[-(-\vec{\mu}\cdot\vec{B}_0)/kT] \quad (43)$$

associated with the magnetostatic energy.

At zero temperature $T=0$, the radiation spectrum (39) becomes the zero-point spectrum

$$\pi^2 H^2(\omega) = \tfrac{1}{2}\hbar\omega . \quad (44)$$

In this case the probability distribution $P(\theta)$ in (36) becomes

$$P(\theta) = \text{const} \times \sin\theta \exp[(2S/\hbar)\cos\theta] , \quad (45)$$

and the z component of spin in (40) is

$$\langle S_z \rangle = S[\coth(2S/\hbar) - \hbar/2S] . \quad (46)$$

This expression for $\langle S_z \rangle$ is independent of the magnetic field B_0 which causes the alignment in the first place. This seems a striking result which is reminiscent of the quantum notions of space quantization. From Eq. (38) we see that the zero-point spectrum (44) is, up to a multiplicative constant, the unique spectrum of random radiation which gives a value of $\langle S_z \rangle$ independent of magnetic field B_0. Also, when the classical spin S is large compared to $\hbar$, the zero-temperature result (46) goes over to

$$\langle S_z \rangle = S - \tfrac{1}{2}\hbar \quad (47)$$

(provided $S \gg \hbar$). It seems interesting that the value $\frac{1}{2}\hbar$ should appear here in connection with the spin. In the case of a free particle in a magnetic field and in zero-point radiation we saw[1] that the component of orbital angular momentum along the direction of an external magnetic field was

$$\langle L_z \rangle = \pm\hbar \quad (48)$$

with the + or − sign depending upon the sign of the charge.

Finally, we draw attention to an interesting symmetry appearing in this spinning dipole system in interaction with random radiation. The symmetry links the energy and the z component of angular momentum. The energy is entirely that of magnetostatic interaction $-\mu B_0\cos\theta$ while the z component of angular momentum is $S\cos\theta$. Thus, both involve constants multiplied by $\cos\theta$. The symmetry can be found in Eq. (40) in the functional dependence upon $2S/\hbar$ and $\mu B_0/kT$ but is most apparent in the limits $\hbar \to 0$ or $kT \to 0$. In the limit that $\hbar \to 0$ in (39) we find that $P(\theta)$ in (36) becomes the Boltzmann distribution

$$\exp(-E/RT) = \exp(\mu B_0\cos\theta/kT) ,$$

which is independent of the spin S. In the limit that $kT \to 0$ in (39) we find that $P(\theta)$ in (36) becomes the distribution

$$\exp(2S\cos\theta/\hbar) ,$$

which is independent of the magnetic moment and magnetic field B_0. This last distribution is presumably of the form $\exp(-2\tilde{J}/\hbar)$ with the action $\tilde{J}$ given here by $\tilde{J} = -S\cos\theta$. Thus, as noted in the past,[6] the Rayleigh-Jeans and zero-point laws are singled out as those spectra which give to mechanical systems without harmonics a phase space distribution depending upon either the system energy E or the system action $\tilde{J}$, respectively.

CLOSING SUMMARY

In 1979 Sachidanandam pointed out that a classical magnetic dipole carrying a spin parallel to the dipole moment does not align itself with an external magnetic field when immersed in random classical radiation. Although the precessing magnetic moment radiates and so loses energy and tends to align itself along the magnetic field, it is forced away from alignment by the random torque of the external field. In the present paper we have carried through the calculation for the stationary probability distribution for the alignment of the spinning magnetic moment.

When the spectrum of random radiation is given by the Rayleigh-Jeans law, we find that the probability distribution is just the Boltzmann result given by traditional classical statistical mechanics. However, when the spectrum is that given by zero-point radiation, and only for that spectrum, then we find that the component of the spin along the magnetic field is independent of the strength of the magnetic field. This seems suggestive of quantum space quantization. Also in the limit of large classical spin, the average z component of the spin differs from full alignment by just $\frac{1}{2}\hbar$, independent of the (large) magnitude of the spin.

The results obtained here fit well with those found earlier within the theory of classical electrodynamics with classical electromagnetic zero-point radiation. However, they do not seem to provide any insight which might allow a major advance within the theory.

Note added in proof. The result $\langle S_z \rangle \cong S - \frac{1}{2}\hbar$ found in Eq. (47) of this classical analysis for the limit of large S actually agrees with the quantum result for any large spin. Thus for a spin eigenstate within quantum theory the total spin angular momentum is $S = [s(s+1)\hbar^2]^{1/2}$ while the z component at $T=0$ takes the eigenvalue $S_z = s\hbar$. At large quantum number s, $S \cong s\hbar + \frac{1}{2}\hbar = S_z + \frac{1}{2}\hbar$. This can be rewritten as exactly the classical result above. Also one can show the agreement between the classical and quantum results at finite temperature and large spin.

ACKNOWLEDGMENT

The model treated in this manuscript was first discussed during 1979 by S. Sachidanandam in unpublished manuscripts. The present work was stimulated by Sachidanandam's reports.

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[1]T. H. Boyer, Phys. Rev. A 21, 66 (1980). Allusion to the result $\langle L_z \rangle = \pm\hbar$ was first made by Sachidanandam in an unpublished manuscript.

[2]See the brief review by T. H. Boyer, in *Foundations of Radiation Theory and Quantum Electrodynamics,* edited by A. O. Barut (Plenum, New York, 1980), p. 49, and the extensive review by L. de la Peña, in *Proceedings of the Latin American School of Physics, Cali, Columbia, 1982,* edited by B. Gómes *et al.* (World Scientific Publishers, Singapore, 1983).

[3]Our model could just as well have involved an electric dipole along the spin axis. All our calculations would go through with electric fields replacing the magnetic fields given here.

[4]H. J. Bhabha, Proc. Indian. Acad. Sci. A 11, 247 (1940), Eq. (51). The equation, with an error in sign, is given by R. Schiller and H. Tesser, Phys. Rev. A 3, 2035 (1971). These authors refer to the calculation of H. J. Bhabha, Proc. R. Soc. (London) A 178, 314 (1941). See also A. F. Rañ, J. Phys. A 12, 1419 (1979). We will use the radiation damping term as a small correction where, actually, it can be obtained from energy conservation alone.

[5]See, for example, L. Rosenfeld, *Theory of Electrons* (Dover, New York, 1965), p. 49.

[6]T. H. Boyer, Phys. Rev. A 18, 1228 (1978); 18, 1238 (1978).