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Neutron interferometric search for quaternions in quantum mechanics

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Following a proposal by Asher Peres we have carried out a neutron interferometer experiment searching for quaternion noncommutative contributions to the neutron-nuclear scattering amplitude. Using slabs of Ti and Al we find that the phase shift experienced by one of the beams in our interferometer is invariant under the interchange of the two slabs to an accuracy of 1 part in 30 000.

INTRODUCTION

About 20 years ago there were two major theoretical investigations on the possibilities of enlarging the number field underlying quantum mechanics from complex to quaternion numbers.[1,2] Quaternions are hypercomplex numbers of the form

$$Q = q_0 + q_1\hat{i} + q_2\hat{j} + q_3\hat{k} \quad , \tag{1}$$

where

$$\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = -1; \text{ and } \hat{i}\hat{j} = -\hat{j}\hat{i} = \hat{k}, \text{ etc.} \tag{2}$$

The algebra based upon these numbers is noncommutative. It was first studied by Hamilton,[3] in which he regarded them as the sum of a scalar (real) part and a vector (imaginary) part of the form

$$Q = q_0 + \vec{q} \cdot \vec{i} \quad . \tag{3}$$

Quaternions are the only generalization of complex numbers satisfying the associative and distributive laws, and for which division is possible and unique. Consequently, as a possibility for generalizing quantum mechanics, an algebra based upon them appears somewhat attractive. Deciding whether they have any real physical manifestitation is a matter for experiment.

In 1979 Peres[4] proposed two tests, involving thermal neutrons, of the possibility that neutron-nuclear scattering amplitudes have a small quaternion "component." Assuming that the scattering amplitudes are complex numbers, he derived a universal relationship between the six coherent scattering cross sections of any three scatterers, taken singly and then pairwise. A numerical violation of this relationship would indicate that the scattering amplitudes are quaternions. The second suggested test involves a search for the noncommutativity of phase shifts experienced by a neutron beam on passage through two dissimilar material slabs. We have carried out this second proposed experiment utilizing neutron interferometry.

The basic idea of the experiment is the following: consider a plane wave $e^{i\vec{k}\cdot\vec{r}}$ incident on a flat, highly polished, homogeneous plate of thickness D. Upon passage through the plate this wave is changed to $\alpha e^{i\phi} e^{i\vec{k}\cdot\vec{r}}$, where α is the transmission coefficient and ϕ is the phase shift given by

$$\phi = \frac{2\pi}{\lambda}(n-1)D \quad , \tag{4}$$

where n is the index of refraction, λ the wavelength, and D the plate thickness. For thermal neutrons the index of refraction is related to the coherent nuclear scattering length b by the formula[5]

$$n = 1 - \lambda^2 N b / 2\pi, \tag{5}$$

where N is the atom density. Thus, the phase shift is directly proportional to the scattering length. Both α and ϕ can be measured by interference with a reference beam. If two plates of different materials, say A and B, are introduced into the beam, the total transmission coefficient α_{AB} will not in general be precisely $\alpha_A \alpha_B$ because of small effects due to multiple reflections between the plates. However, if complex quantum theory is correct, the total phase shift ϕ_{AB} should precisely equal the phase shift ϕ_{BA} for the beam passing through the plates in the reverse order. Furthermore, ϕ_{AB} should be the sum of the phase shifts ϕ_A and ϕ_B experienced by the beam in traversing plates A and B separately. However, if the scattering lengths are quaternions, rather than simple coplanar complex numbers, the phase shifts ϕ_A and ϕ_B will not in general precisely commute. The reasoning here is based upon the results (4) and

(5) of complex quantum theory, which require the proportionality between the phase shifts and the scattering lengths. Of course, we must then reinterpret a phase shift to be a rotation of a vector in the four-dimensional space of quaternions rather than a rotation of a vector in the two-dimensional complex plane. To say this a different way, one must generalize the interpretation of the $i=\sqrt{-1}$ appearing in Schrödinger's equation. This question is discussed in the papers by Kaneno[1] and Finkelstein, Jauch, Schiminovich, and Speiser.[2]

Scattering lengths are typically not known to better than about 1%; in a few cases they have been measured with an accuracy approaching 0.1%. The addition of a small quaternion "component" of order 1% to the real part of the scattering length would not have been noticed in any neutron scattering experiment that we are aware of up to this time. Theoretical predictions of their values are much less precise.

EXPERIMENT

The experiment was performed at the 10-MW University of Missouri Research Reactor using a Bonse-Hart three-crystal Laue-Laue-Laue (LLL) interferometer[6] shown schematically in Fig. 1. This device was first shown to work for thermal neutrons by Rauch, Treimer, and Bonse.[7] The incoming neutron beam of wavelength $\lambda = 1.268$ Å is produced by a double copper (220) crystal monochromator. The incident beam is coherently split in the first silicon crystal slab near point A by Bragg reflection from the (220) lattice planes. The resulting two beams are again coherently split by the second Si crystal slab near points B and C. Two of these beams are directed toward point D in the third Si slab where they are mixed by the periodic crystal potential and interfere. The outgoing beams from point D are counted by two high-pressure ^{3}He detectors. The noninterfering beam directed along the line AB is counted with a beam monitoring fission chamber.

When a slab of material of thickness D is inserted into the interferometer between points B and C as shown in Fig. 1, a phase shift of the beam on path I relative to the beam on path II is induced, of magnitude $\phi = -\lambda NbD$. The intensities of the beams entering the detector vary sinusoidally with the phase angle. In order to search for a quaternion, noncommutative part to the scattering length, we place slabs of two different materials in the sample position, measure the phase shift, reverse the order of the two slabs, and again measure the phase shift. Very high sensitivity to phase-shift differences is achieved by using slabs of sufficient thickness to cause a total phase shift in traversing either slab of order 10 000 degrees. The phase shifts are then measured, modulo 360°, to an accuracy of about ± 0.3 degrees in the following way: A phase rotator, in our case made of high-purity aluminum, is also placed in the interferometer as shown in Fig. 1. Rotating this slab about an axis perpendicular to the scattering plane of the interferometer through small angles δ increases the optical path length on path I and decreases it on path II, thus causing a relative phase shift, and an oscillating counting rate to be registered in the detector. We then place the sample in beam I and

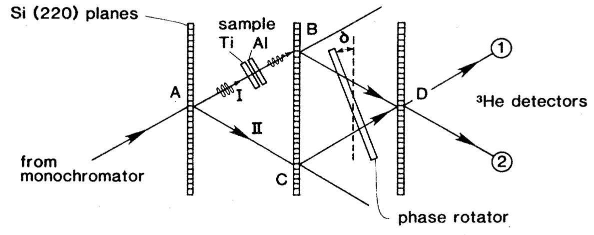

FIG. 1. Schematic of the Bonse-Hart three-crystal LLL interferometer used in the experiment.

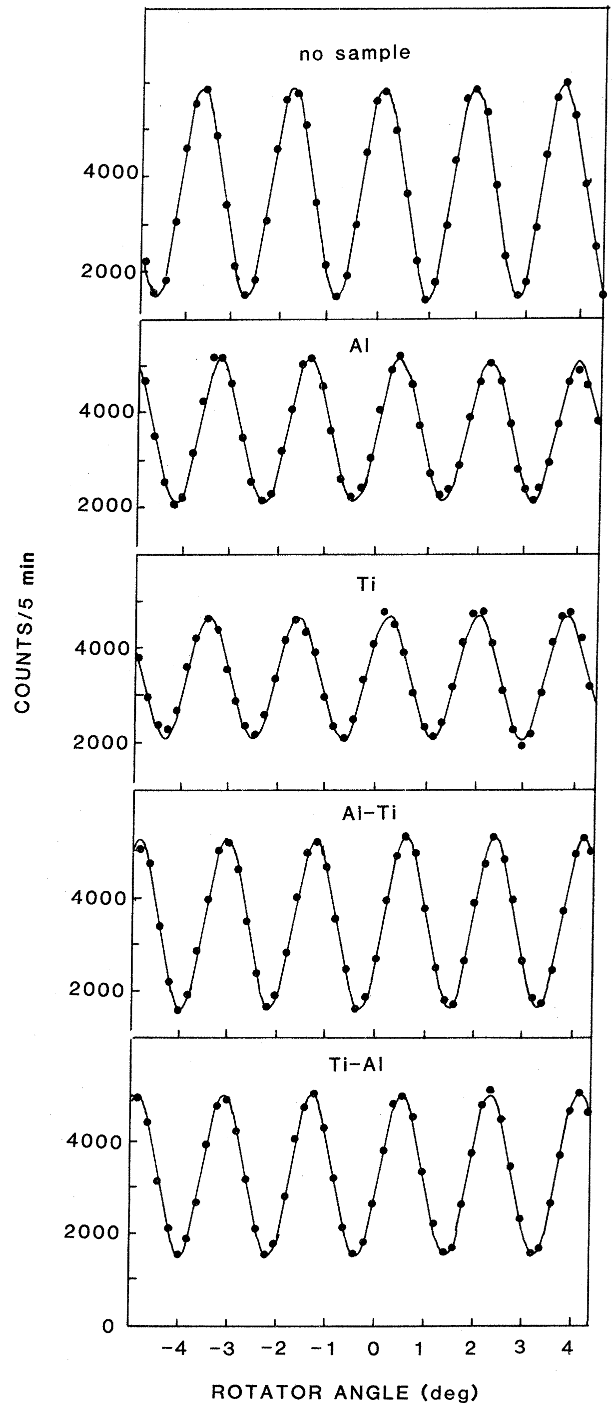

FIG. 2. Results of experiment.

TABLE I. Quaternion experiment slabs.

	Atom density ($10^{22}/\mathrm{cm}^3$)	Scattering length b (fm)	Thickness (cm)	Transmission α^2	Total phase shift ϕ (deg)
Ti	5.71	−3.4	0.699	0.719	+9860
Al	6.02	+3.4	0.661	0.942	−9980

again rotate the phase rotator observing another oscillating interferogram. The difference of the origins of these two sinusoids is the phase shift, induced by the sample, modulo 360°. In fact, these two interferograms are recorded point by point, i.e., sample-in–sample-out for each angle δ of the phase rotator. This procedure tends to eliminate effects of long-term drift of the zero phase of the interferometer. Since, in this experiment we were only interested in looking for small, noncommutative phase differences, the fact that the phase can only be determined modulo 360° by this technique is not important.

A priori, one has little theoretical idea or guidance on the selection of materials for this experiment. Peres suggested that materials which are known to have a significant imaginary part in their scattering lengths, and therefore a large absorption cross section, would be preferable. The idea is that one might suppose that rotations out of the complex plane into the four-dimensional quaternion space would be more likely for a vector not restricted to the complex real axis. Experimentally this would be necessary for the scattering experiment he proposes. However, in the interferometer experiment, the high absorption restricts one to very thin samples and therefore to small total phase shifts of the beam traversing the slab. We have taken an alternative logical approach. We have selected two very different materials, both from a chemical and a nuclear point of view, namely, aluminum and titanium. Both of these materials have rather small absorption cross sections, but the one has a positive (real) scattering amplitude and the other has a negative scattering amplitude. Characteristics of the samples used in these experiments are tabulated in Table I.

The results of our experiment are shown in Fig. 2. The top panel shows the intensity observed in detector 1, with no sample in the interferometer, as a function of the angle of the phase rotator δ. Interferograms taken first with the Al slab in the interferometer and then with the Ti slab in place are shown in the second and third panels. The bottom two panels show data obtained when both the Al and Ti slabs are in the interferometer, first in the order Al-Ti and then in the order Ti-Al. Mechanical care was exercised in assuring that the slabs could be inserted into the interferometer in a highly reproducible way, as to position and orientation with respect to the beam axis. Several runs were made, and each interferogram was fitted to a sinusoid using standard nonlinear least-squares analysis. Uncertainty in the fitted phases for a given run was typically less than 0.1°. The phase shift differences for the beam traversing the two-slab, composite sample in the two orders, Al-Ti, then Ti-Al was found to be less than ±0.3 degrees. Therefore, we find

$$|\phi_{\text{Al-Ti}} - \phi_{\text{Ti-Al}}| \leq 0.3° \quad . \tag{6}$$

Since the total phase shift in traversing either slab is of order 10 000 degrees, this means that the two phase shifts ϕ_{Ti} and ϕ_{Al} commute to better than 1 part in 30 000. Presumably, this implies that any noncommutative, quaternion contribution to the scattering lengths of these nuclei is less than 1 part in 30 000 of the real part.

We would like to point out an interesting feature of the data shown in Fig. 2. The observed contrast of the interferometer decreases when either the Al or Ti slabs are placed in the sample position. This is to be expected since both samples attenuate the wave function traversing path I (see Table I). However, adding the Ti slab after the Al slab is already in place, results in an enhanced contrast.[8] Our interpretation of this effect is the following: The positive optical potential of Al retards the transit time of the neutron wave packet on path I so that it arrives in the interference region near point D (Fig. 1) behind the wave packet on beam path II, thus decreasing the overlap, mutual coherence integral ($\int \psi_{\text{I}}\psi_{\text{II}}\,dx$). Since the optical potential of Ti is negative, it will advance the transit time. Thus, when both slabs are simultaneously placed in the sample position, the retardation of the wave packet by the Al slab is compensated by the advanced transit time due to its subsequent passage through the Ti slab, and the loss of contrast is partially restored. The interpretation of our recent longitudinal coherence length experiment[9] was based upon these ideas.[10]

In conclusion, we believe that these results represent the first direct search for the physical reality of quaternions in a quantum-mechanical system. At the level of 1 part in 30 000, complex quantum theory correctly predicts the results of this thermal neutron experiment.

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[1]T. Kaneno, Prog. Theor. Phys. 23, 17 (1960).

[2]D. Finkelstein, J. M. Jauch, S. Schiminovich, and D. Speiser, J. Math. Phys. 3, 207 (1962).

[3]W. R. Hamilton, *Elements of Quaternions* (Longmans, Green, &

Co., London, 1866). Hamilton's original treatise has been edited by C. J. Joly and published under the same title, in two volumes, by Chelsea Publishing Company, New York, NY 1969, 3rd ed.

[4]A. Peres, Phys. Rev. Lett. 42, 683 (1979).

[5]For a recent review of neutron optics, see A. G. Klein and S. A. Werner, Rep. Prog. Phys. 46, 259 (1983).

[6]U. Bonse and M. Hart, Appl. Phys. Lett. 6, 155 (1965).

[7]H. Rauch, W. Treimer, and U. Bonse, Phys. Lett. 47A, 369 (1974).

[8]This effect was first described as neutron phase echo principle. See G. Badurek, H. Rauch, and A. Zeilinger, *Neutron Spin Echo,* Lecture Notes in Physics, Vol. 128, edited by F. Mezei (Springer-Verlag, Berlin, 1980), p. 136.

[9]H. Kaiser, S. A. Werner, and E. A. George, Phys. Rev. Lett. 50, 560 (1983).

[10]A. G. Klein, G. I. Opat, and W. A. Hamilton, Phys. Rev. Lett. 50, 563 (1983).