Auroral kilometric radiation due to a new plasma instability

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In this paper we show that a high-frequency plasma instability can lead to auroral kilometric radiation. In the presence of low-frequency ion-cyclotron turbulence and a high-frequency extraordinary-mode (X-mode) test wave, this instability occurs due to a nonlinear force which originates from the resonant interaction between electrons and modulated electric fields. The growth rate of the X-mode wave, in the form of auroral kilometric radiation, has been calculated and compared with observations.

I. INTRODUCTION

Several theories have been proposed to explain the auroral kilometric radiation (AKR) in recent years.[1,2] Very recently, two of the present authors have proposed[3] that AKR is produced by the enhanced extraordinary-mode (X-mode) radiation and occurs due to bremsstrahlung interaction[4] between aurora beam electrons and electrostatic ion-cyclotron turbulence or double layers. This mechanism has been found to have several interesting features including its large growth rate and the close correlation between AKR and double layers. In this paper, we present a slightly different method to explain AKR while keeping essentially the general treatment of the previous paper in Ref. 3. Specifically, it will be shown that the high-frequency X-mode instability, producing AKR in presence of ion-cyclotron turbulence or double-layer potentials, occurs due to a high-frequency nonlinear force which comes from the resonant interaction between electrons and a modulated electric field. Thus electrons suffer acceleration (or deceleration) due to the nonlinear force and the accelerated electrons can radiate X-mode waves in the form of auroral kilometric radiation.

In contrast to the parametric interaction process, where a low-frequency wave becomes unstable in presence of a pump wave which is a high-frequency wave, in our mechanism the high-frequency wave grows in the presence of low-frequency wave turbulences.

The nonlinear force can be calculated within the framework of the linear-response theory[5] and is obtained in Sec. II. In Sec. III, with the help of the fluid equations, a new high-frequency instability due to the nonlinear forces is predicted and the growth rate of this instability is calculated in Sec. IV. Comparison with the linear theory[1] and discussions are contained in Sec. V.

II. CALCULATION OF THE NONLINEAR FORCE

In order to calculate the nonlinear force, we need an expression for the modulated electric field and the electron distribution function. For this purpose, we consider a homogeneous magnetized plasma in the presence of an enhanced turbulence due to electrostatic ion-cyclotron waves, with the external magnetic field $\vec{B}_0$ in the z direction. The basic equations governing the interaction of the turbulent fields with an electron which leads to X-mode radiation are the set of Vlasov-Maxwell equations

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}-\frac{e}{m}\left[\vec{E}+\frac{\vec{v}\times\vec{B}}{c}\right]\cdot\frac{\partial}{\partial\vec{v}}\right]f_e(\vec{r},\vec{v},t)=0\ , \qquad (1)$$

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}\ , \qquad (2)$$

$$\vec{\nabla}\times\vec{B}=\frac{1}{c}\frac{\partial\vec{E}}{\partial t}-\frac{4\pi}{c}ne\int\vec{v}f_e(\vec{r},\vec{v},t)d\vec{v}\ . \qquad (3)$$

Since the ion-cyclotron wave turbulence which is assumed to propagate almost perpendicular to the magnetic field with propagation vector $\vec{k}=(k_\perp,0,k_{\parallel})$ is already present in our system (see Fig. 1), the various physical quantities can be written as

$$f_e=f_{0e}+\epsilon f_{1e}+\epsilon^2 f_{2e}+\delta f\ , \qquad (4)$$

$$\vec{E}=\epsilon\vec{E}_l+\delta\vec{E}\ , \qquad (5)$$

$$\vec{B}=\vec{B}_0+\delta\vec{B}\ , \qquad (6)$$

where f_{0e} is the space- and time-averaged part, f_{1e} and f_{2e} are the low-frequency fluctuating parts of the electron distribution function, $\vec{E}_l$ is the electrostatic ion-cyclotron wave field which is assumed to be in the z and x directions, ϵ is a small parameter, δf is the perturbed distribution function, $\delta\vec{E}(\vec{r},t)$ and $\delta\vec{B}(\vec{r},t)$ are the perturbed electric and magnetic fields of the high-frequency X-mode test wave which is introduced to our system *a priori*. The X-mode wave is assumed to propagate perpendicular to the magnetic field with propagation vector $\vec{K}=(K,0,0)$. According to the linear-response theory of a turbulent plasma, we have

$$\delta f=\mu\,\delta f_h+\mu\epsilon\delta f_{lh}+\mu\epsilon^2\Delta f\ , \qquad (7)$$

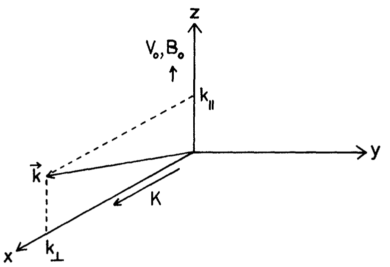

FIG. 1. Geometry of model; K is the propagation vector of the X-mode radiation, and $\vec{k}$ is the propagation vector for the electrostatic ion-cyclotron waves or double layers.

$$\delta\vec{E}=\mu\,\delta\vec{E}_h+\mu\epsilon\,\delta\vec{E}_{lh}+\mu\epsilon^2\Delta\vec{E}\ , \tag{8}$$

$$\delta\vec{B}=\mu\,\delta\vec{B}_h+\mu\epsilon\,\delta\vec{B}_{lh}+\mu\epsilon^2\Delta\vec{B}\ , \tag{9}$$

where μ is another small parameter ($\mu \ll \epsilon$) and $\delta\vec{E}_{lh}$, $\Delta\vec{E}$, $\delta\vec{B}_{lh}$, $\Delta\vec{B}$, δf_{lh}, and Δf come from the mixed mode perturbation. Linearizing the Vlasov equation we obtain, to the order ϵ,

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right]f_{1e}=\frac{e}{m}\vec{E}_l\cdot\frac{\partial}{\partial\vec{v}}f_{0e}\ . \tag{10}$$

To order μ,

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\frac{\partial}{\partial\vec{r}}-\frac{e}{mc}\vec{v}\times\vec{B}_0\cdot\frac{\partial}{\partial\vec{v}}\right]\delta f_h-\frac{e}{m}\left[\delta\vec{E}_h+\frac{1}{c}\vec{v}\times\delta\vec{B}_h\right]\cdot\frac{\partial}{\partial\vec{v}}f_{0e}=0\ . \tag{11}$$

To order $\mu\epsilon$,

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\frac{\partial}{\partial\vec{r}}-\frac{e}{mc}\vec{v}\times\vec{B}_0\cdot\frac{\partial}{\partial\vec{v}}\right]\delta f_{lh}-\frac{e}{m}\vec{E}_l\cdot\frac{\partial}{\partial\vec{v}}\delta f_h-\frac{e}{m}\left[\delta\vec{E}_h+\frac{1}{c}\vec{v}\times\delta\vec{B}_h\right]\cdot\frac{\partial}{\partial\vec{v}}f_{1e}-\frac{e}{m}\left[\delta\vec{E}_{lh}+\frac{1}{c}\vec{v}\times\delta\vec{B}_{lh}\right]\cdot\frac{\partial}{\partial\vec{v}}f_{0e}=0\ . \tag{12}$$

And to order $\mu\epsilon^2$,

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\frac{\partial}{\partial\vec{r}}-\frac{e}{mc}\vec{v}\times\vec{B}_0\cdot\frac{\partial}{\partial\vec{v}}\right]\Delta f-\frac{e}{m}\left\langle\vec{E}_l\cdot\frac{\partial}{\partial\vec{v}}\delta f_{lh}+\left[\delta\vec{E}_{lh}+\frac{1}{c}\vec{v}\times\delta\vec{B}_{lh}\right]\cdot\frac{\partial}{\partial\vec{v}}f_{1e}\right\rangle=0\ , \tag{13}$$

where $\langle\cdots\rangle$ means the phase average over the low-frequency fluctuations. It should be noted here that the phase of $\vec{E}_l$ field is random while $\delta\vec{E}_h$ field is coherent. Thus, it is necessary to average over the phase of $\vec{E}_l$ field fluctuations. Equations (11) to (13) are the basic equations for the induced bremsstrahlung interaction which comes from electron acceleration due to nonlinear forces.

For low-frequency electrostatic waves, the electron motion along the magnetic field is important. The Fourier component of the corresponding distribution function f_{1e} is, from Eq. (10), given by

$$f_{1e}(\vec{k},\omega)=-\frac{\dfrac{e}{m}E_{l\|}(\vec{k},\omega)\dfrac{\partial}{\partial v_\|}f_{0e}}{i(\omega-k_\|v_\|)}\ , \tag{14}$$

where $\vec{k}$ and ω are, respectively, the wave vector and the frequency of the electrostatic waves and $\|$ means parallel to the magnetic field.

The Vlasov equations (11)–(13) are now solved by integrating along the orbits of the particles in the unperturbed fields. In cylindrical coordinates, $v_x=v_\perp\cos\phi$, $v_y=v_\perp\sin\phi$, $v_z=v_\|$, the particle orbits $r'(\tau)$ are given by

$$\begin{aligned}&v_x'=v_\perp\cos(\phi-\Omega_e\tau),\quad v_y'=v_\perp\sin(\phi-\Omega_e\tau),\quad v_z'=v_\|\ ,\\&x'=x-\frac{v_\perp}{\Omega_e}\sin(\phi-\Omega_e\tau)+\frac{v_\perp}{\Omega_e}\sin\phi\ ,\\&y'=y+\frac{v_\perp}{\Omega_e}\cos(\phi-\Omega_e\tau)-\frac{v_\perp}{\Omega_e}\cos\phi\ ,\\&z'=z+v_\|\tau,\quad \tau=t'-t\ .\end{aligned} \tag{15}$$

Here, $\Omega_e=eB_0/mc$ is the electron cyclotron frequency and the other symbols have their usual meaning.

From Eq. (13), retaining the most dominant nonlinear term in the high-frequency perturbation, the high-frequency nonlinear force $\vec{F}_N$ caused by the electron acceleration through the modulation electric fields ($\delta\vec{E}_{lh}$) can be written in the Fourier space as

$$\vec{F}_N(\vec{K},\Omega)=en_0\int\sum_{\vec{k}',\omega'}\left\langle\delta\vec{E}_{lh}(\vec{K}-\vec{k}',\Omega-\omega')\cdot\frac{\partial}{\partial\vec{v}}f_{1e}(\vec{k}',\omega')\right\rangle\vec{v}\,d\vec{v}\ . \tag{16}$$

Substituting Eqs. (A1) and (A2) for $\delta\vec{E}_{lh}(\vec{K}-\vec{k}',\Omega-\omega')$ and Eq. (14) for $f_{1e}(\vec{k}',\omega')$ we obtain, after a long but straightforward calculation,

$$F_{Nx}(\vec{K},\Omega)=-iA'B'\Omega_e\delta E_{hx}(\vec{K},\Omega) \tag{17}$$

and

$$F_{Ny}(\vec{K},\Omega) = -A'C'\Omega\,\delta E_{hy}(\vec{K},\Omega)\ , \tag{18}$$

where

$$A' = \sum_{k'} \frac{4\omega_{pe}^2 e n_0 \Omega}{\Omega_e^2 - \Omega^2} \mid E_{l\parallel}(k') \mid^2 \left[\frac{e}{m}\right]^2 \frac{[1+\xi z(\xi)]}{k_{\parallel}'^2 v_e^4} \left[\frac{k_\perp'}{k_\parallel'}\right],$$

$$B' = \frac{1}{MN[c^2 k_\parallel'^2 - (\Omega-\omega')^2]} \times \left[1 + \frac{\omega_{pe}^2 \Omega^2}{\Omega_e^2 - \Omega^2} \frac{1}{H[c^2(K'^2 + k_\parallel'^2) - (\Omega-\omega')^2]}\right],$$

$$C' = \frac{1}{HP[c^2(K'^2 + k_\parallel'^2) - (\Omega-\omega')^2]} \times \left[1 + \frac{\omega_{pe}^2 \Omega_e^2}{\Omega_e^2 - \Omega^2} \frac{1}{M[c^2 k_\parallel'^2 - (\Omega-\omega')^2]}\right]. \tag{19}$$

Here $z(\xi)$ is the plasma dispersion function, $\xi = (\omega' - k_\parallel' v_0)/k_\parallel' v_e$ and $v_e = (2T/m)^{1/2}$ is the electron thermal velocity. In deriving Eqs. (17) and (18), we have taken the space- and time-averaged distribution function to be

$$f_{0e} = \left[\frac{m}{2\pi T}\right]^{3/2} \exp\left[-\frac{m(v_\parallel - v_0)^2}{2T}\right] \exp\left[-\frac{m v_\perp^2}{2T}\right],$$

where v_0 is the electron drift velocity along z direction.

III. DISPERSION RELATION OF THE X MODE

To obtain the dispersion relation of the X mode, we follow closely the method of Chen[6] and write the equation of motion for the electrons with the nonlinear force term as

$$mn\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -en\left[\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right] + \vec{F}_N\ . \tag{20}$$

Maxwell's equations are

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\ ,$$
$$\vec{\nabla}\times\vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}\ , \tag{21}$$
$$\vec{J} = -en\vec{v}\ ,$$

where the symbols have their usual meaning. Linearizing and Fourier analyzing we get from Eqs. (20) and (21)

$$\vec{v} = \frac{e}{im\Omega}\left[\delta\vec{E}_h(\vec{K}) + \frac{\vec{v}\times\vec{B}_0}{c}\right] + \frac{i\vec{F}_N}{mn\Omega}\ , \tag{22}$$

$$\Omega^2 \delta E_{hx}(\vec{K}) = 4\pi i e n_0 \Omega v_x\ , \tag{23}$$

$$(\Omega^2 - c^2K^2)\delta E_{hy}(\vec{K}) = 4\pi i e n_0 \Omega v_y\ . \tag{24}$$

Now, substituting the values of $\vec{F}_N$ from Eqs. (19), we obtain the values of v_x and v_y from Eq. (22) as

$$v_x = \left[-\frac{ie}{m\Omega}\delta E_{hx}(\vec{K}) - \left[\frac{e}{m\Omega} + \frac{A'C'}{mn_0} - \frac{A'B'}{mn_0}\right]\frac{\Omega_e}{\Omega}\delta E_{hy}(\vec{K})\right]\left[1 - \frac{\Omega_e^2}{\Omega^2}\right]^{-1} \tag{25}$$

and

$$v_y = \left[\frac{e}{m\Omega}\frac{\Omega_e}{\Omega}\delta E_{hx}(\vec{K}) - i\left[\frac{e}{m\Omega} - \frac{1}{mn_0}\frac{\Omega_e^2}{\Omega^2}A'B' + \frac{1}{mn_0}A'C'\right]\delta E_{hy}(\vec{K})\right]\left[1 - \frac{\Omega_e^2}{\Omega^2}\right]^{-1}. \tag{26}$$

Substituting Eq. (26) into Eq. (24) and noting that only the contribution of C' is important, we get

$$(\Omega^2 - c^2K^2)\delta E_{hy}(\vec{K}) = \frac{4\pi i e n_0 \Omega^3}{(\Omega^2 - \Omega_e^2)}\left\{\frac{e}{m\Omega}\frac{\Omega_e}{\Omega}\delta E_{hx}(\vec{K}) - i\left[\frac{e}{m\Omega} + \sum_{k'}\frac{4\omega_{pe}^2 e\Omega}{m(\Omega_e^2 - \Omega^2)}\left[\frac{e}{m}\right]^2 \mid E_{l\parallel}(k')\mid^2 \frac{[1+\xi z(\xi)]}{k_\parallel'^2 v_e^4}\frac{1}{HP[c^2(K'^2 + k_\parallel'^2) - (\Omega-\omega')^2]} \times \left[\frac{k_\perp'}{k_\parallel'}\right]\left[1 + \frac{\omega_{pe}^2\Omega_e^2}{(\Omega_e^2 - \Omega^2)}\frac{1}{M[c^2k_\parallel'^2 - (\Omega-\omega')^2]}\right]\right]\delta E_{hy}(\vec{K})\right\}. \tag{27}$$

Here we note that for the simple case of $KC > \omega_{pe}$ considered in this problem, the coefficient of $\delta E_{hx}(\vec{K})$ is much smaller than that in $\delta E_{hy}(\vec{K})$.[7] Therefore, dropping the first term in the right-hand side of Eq. (27) and rearranging, we get the dispersion relation for the X-mode wave given by

$$(\Omega^2-\Omega_e^2)\left[1-\frac{c^2K^2}{\Omega^2}\right]-\omega_{pe}^2=\sum_{k'}\frac{4\omega_{pe}^4\Omega^2}{\Omega_e^2-\Omega^2}\left[\frac{e}{m}\right]^2|E_{l\parallel}(k')|^2\frac{[1+\xi z(\xi)]}{k_\parallel'^2v_e^4}\frac{1}{HP[c^2(K'^2+k_\parallel'^2)-(\Omega-\omega')^2]}\left[\frac{k_\perp'}{k_\parallel'}\right]$$
$$\times\left[1+\frac{\omega_{pe}^2\Omega_e^2}{(\Omega_e^2-\Omega^2)}\frac{1}{M[c^2k_\parallel'^2-(\Omega-\omega')^2]}\right]. \quad (28)$$

IV. GROWTH RATE OF THE X-MODE RADIATION

The X-mode wave instability can be calculated by setting $\Omega=\Omega_r+i\gamma$ in Eq. (28), where Ω_r is the real frequency and γ is the growth rate. Now, neglecting the nonlinear frequency shift, the real frequency Ω_r of the X-mode radiation can be calculated, by equating the real terms of Eq. (28) to zero, as

$$1-\frac{c^2K^2}{\Omega_r^2}-\frac{2\omega_{pe}^2}{\Omega_r^2-\Omega_e^2}=0\ . \quad (29)$$

For $\Omega_r>cK$, Eq. (29) reduces to

$$\Omega-\Omega_e=\frac{\omega_{pe}^2}{\Omega_e}\ll\Omega_e,\ \ \omega_{pe}\ll\Omega_e\ . \quad (30)$$

Equating the imaginary terms in Eq. (28) we get the growth rate for the X-mode radiation for $cK\ll\Omega$, given by

$$\gamma=(2\pi)^{1/2}\sum_{k'}\frac{|E_{l\parallel}(k')|^2}{4\pi n_0T}\frac{\omega_{pe}^6}{\Omega_e}\frac{1}{k_\parallel'^2v_e^2}\left[\frac{\Omega_e}{\omega_{pe}}\right]^2\left[\frac{v_0}{v_e}\right]\frac{1}{HP[c^2(K'^2+k_\parallel'^2)-(\Omega-\omega')^2]}\left[1-\frac{\Omega_e^2}{M[c^2k_\parallel'^2-(\Omega-\omega')^2]}\right]\left[\frac{k_\perp'}{k_\parallel'}\right]. \quad (31)$$

In deriving Eq. (31), we have used relation (30). Now, calculating the values of H, P, and M, we get the final expression for growth rate of the X-mode radiation given by

$$\gamma=(2\pi)^{1/2}\omega_{pe}^4\sum_{k'}\frac{1}{\{c^2[k_\perp'(k_\perp'-2K)]+2\Omega\omega'\}}\left[1-\frac{\Omega_e^2}{c^2(k_\parallel'^2-K^2)+2\Omega\omega'}\right]\frac{1}{\Omega_e}\frac{\omega_{pe}^2}{k_\parallel'^2v_e^2}\frac{|E_{l\parallel}(k')|^2}{4\pi n_0T}\left[\frac{v_0}{v_e}\right]\left[\frac{\Omega_e}{\omega_{pe}}\right]^2\left[\frac{k_\perp'}{k_\parallel'}\right]. \quad (32)$$

V. APPLICATION AND DISCUSSION

As an illustration, we apply the result of our investigation to AKR. Accordingly, we take the typical plasma parameters;[8] $K=8.3\times10^{-6}$ cm^{-1} ($\lambda=7.5$ km, λ is the wavelength of AKR), $k_\perp'=2K$, $k_\perp'=10k_\parallel'$, $n_0=10$ cm^{-3}, $T=400$ ev, $\Omega_e=10\omega_{pe}$, $v_0=0.5v_e$, $\omega'\simeq\Omega_i=140$ Hz, and $\Omega\simeq\Omega_e=250$ kHz. We use these parameters in Eq. (32), then we get

$$\frac{\gamma}{\Omega_e}=(2\pi)^{1/2}(\Omega_e/cK)^2(v_0/v_e)\sum_{k'}(\omega_{pe}^2/2\Omega\omega')W\ , \quad (33)$$

here $W=[\,|E_{l\parallel}(k')|^2/4\pi n_0T](k_e^2/k_\parallel'^2)(k_\perp'/k_\parallel')$ is the normalized ion-cyclotron wave energy, k_e is the electron Debye wave number. According to the observations[8] $E_{l\parallel}=10$ mV/m and $W=1.98\times10^{-2}$, and then Eq. (33) reduces to $\gamma/\Omega_e\simeq10^{-1}$ which is large enough to generate AKR.

It is now clear that, in addition to the conventional three-wave decay[9] (the matching conditions are $K-K'=\pm k$, $\Omega-\Omega'=\pm\omega$) and nonlinear scattering[10] [the condition is $\Omega\pm\omega=(K\pm k)v$], the third mechanism as discussed here originates from the induced bremsstrahlung of X-mode waves and is caused by electrons which resonate with the modulation waves (the condition is $\omega=kv$). The readers should note that the linear resonance between the ion-cyclotron wave and resonant electrons has nothing to do with the new plasma instability pointed out by us. As is shown in Sec. III, the high-frequency nonlinear forces ($\vec{F}_N$) are the origin of the induced bremsstrahlung instability.

The scenario in this paper is shown in Fig. 2. During a magnetospheric substorm, the energetic electrons (~1 keV) are injected from the plasma sheet. The interaction between the high-energy electrons and the low-energy (~1 eV) background electrons generates ion-cyclotron waves (double layers). Next, the strong AKR occurs due to a new maser effect (turbulent bremsstrahlung instability).

Here, we compare our theory with a linear theory.[1] It is widely thought that AKR arises from a relativistic effect (gyrotron[11]). In Table I, we show the difference between the linear theory and the turbulent bremsstrahlung instability. We think that these two classes of theory are complementary to each other.

Next, we show a detailed comparison of the growth rates of the two instability mechanisms. Since the observed size of the auroral arcs is about 100 km, one might estimate the size of the AKR source region to be about 100 km.[1] The effective growth length (L) of the AKR is

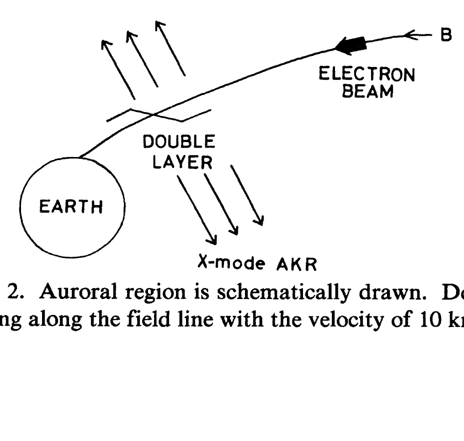

FIG. 2. Auroral region is schematically drawn. Double layer is moving along the field line with the velocity of 10 km/s.

TABLE I. Shown below are the differences in the linear and nonlinear theories.

	Linear theory (Ref. 1)	Nonlinear theory
Resonance condition	$K_{\|\|}v_{\|\|}-\Omega+\Omega_e/\gamma=0$	$\omega-k_{\|\|}v_{\|\|}=0$
Type of instability	Gyrotron	Turbulent bremsstrahlung instability
Phase bunching mechanism	Relativistic electron mass change	Resonant electron
Energy source	Loss cone+relativistic electron	Low-frequency turbulence (double layer)+electron beam

given by $L=v_g/\gamma$, here v_g and γ are the group velocity of the X mode and the growth rate, respectively. By using an average group velocity v_g of 8.45×10^2 km/s and electron gyrofrequency (Ω_e) of 250 kHz, the normalized growth rate $\gamma/\Omega_e>3\times10^{-5}$ is necessary for the growth of AKR. The maximum linear growth rate $\gamma/\Omega_e\simeq5\times10^{-2}$ is obtained by Omidi and Gurnett for the S3-3 electron distribution function. The growth rate predicted by Wu *et al.* is $5\times10^{-5}<\gamma/\Omega_e<5\times10^{-3}$. On the other hand, the growth rate obtained by the nonlinear theory is $\gamma/\Omega_e\simeq10^{-1}$. This large growth rate is overestimated, because we have assumed only the hot component for the electron distribution function. The inclusion of the cold component in the nonlinear theory is left for future study. However, judging from the result of this paper, we can say that the nonlinear theory will be an important component of the AKR production in various situations.

The fine structure of AKR emission spectra have been reported by Gurnett *et al.*[12] How these observation reconcile with our theory is a very important point. In Fig. 3, the thickness of X-mode trajectory is proportional to the amplitude of the X mode. Note that if waves propagate in directions down, coming toward the Earth from the source (S), they will eventually reach the right-hand cutoff frequency point and reflect back toward lower magnetic field region. Thus, X-mode radiation also reaches the escaping point (E). If the source point (S) moves at a velocity about 10 km/s, then the dynamic spectrum would show a drift pattern as is observed by Gurnett *et al.*[12]

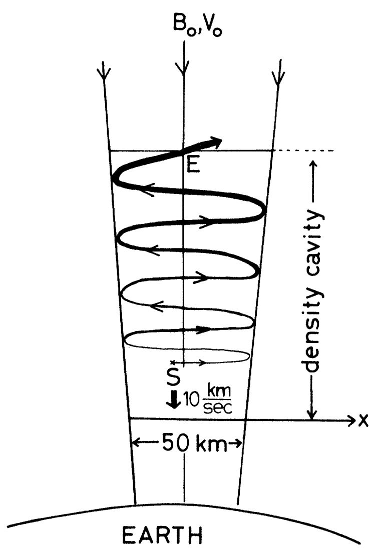

FIG. 3. Path of X-mode radiation in density cavity region ($\omega_{pe}\ll\Omega_e$, here ω_{pe} and Ω_e are the electron plasma frequency and electron gyrofrequency). Multiple reflections off density cavity boundaries provide long paths for growth. The X mode eventually reaches a weak field point (E) where it is accessible to free space. Source point (S) and the escaping point (E) are assumed to be situated at altitudes of one Earth radius and two Earth radii, respectively. Source point (S) is assumed to move downward at the velocity 10 km/s.

The turbulent bremsstrahlung instability predicts a particular phase relation between a pump field and the high-frequency radiation. To clarify the physics, we take a low-frequency pump field as an electrostatic wave without external magnetic field. According to Chen,[6] we turn on a coherent low-frequency electrostatic wave $E_l(x,t)$ and consider the electron dynamics. The linearized fluid equation for the beam is

$$m\left[\frac{\partial v_1}{\partial t}+u\frac{\partial v_1}{\partial x}\right]=-eE_l\sin(kx-\omega t)\,, \qquad (34)$$

where u is the beam velocity. A possible solution for the nonresonant electron is

$$v_{\text{nonres}}=-\frac{eE_0}{m}\frac{\cos(kx-\omega t)}{\omega-ku}\,. \qquad (35)$$

The linearized continuity equation is

$$\frac{\partial n_1}{\partial t}+u\frac{\partial n_1}{\partial x}=-n_u\frac{\partial v_1}{\partial x}\,, \qquad (36)$$

here n_u is the beam density with velocity u. Substituting Eq. (35) into Eq. (36) yields

$$n_{\text{nonres}}=-n_u\frac{eE_lk}{m}\frac{\cos(kx-\omega t)}{(\omega-ku)^2}\,. \qquad (37)$$

Equation (37) shows the number density perturbation for the nonresonant electrons ($\omega\neq ku$).

Next, we consider the fluid equation for resonant electrons ($\omega\approx ku$). For resonant electrons, we must consider the initial condition. Then we have, instead of Eq. (35),

$$v_{\text{res}}=-\frac{eE_l}{m}\frac{\cos(kx-\omega t)-\cos(kx-kut)}{\omega-ku}\,. \qquad (38)$$

We must solve the equation of continuity for n_1 [Eq. (36)] again subject to the initial condition $n_1=0$ at $t=0$. Thus we get for resonant electrons

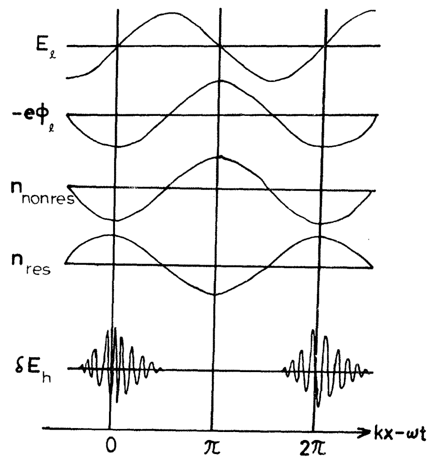

FIG. 4. Phase relation of density for electrons in a low-frequency electrostatic wave. E_l and ϕ_l are the electric field and the potential of the low-frequency wave. n_{nonres} and n_{res} show the number density for the nonresonant electrons and resonant electrons. δE_h is the high-frequency radiation field. High-frequency radiation occurs at the particular phase due to the turbulent bremsstrahlung instability.

$$n_{\mathrm{res}} = -n_u \frac{eE_l k}{m} \frac{1}{(\omega-ku)^2} \times [\cos(kx-\omega t)-\cos(kx-kut) - (\omega-ku)t\sin(kx-kut)] \ . \tag{39}$$

We expand Eq. (39) around $u \approx \omega/k$, then Eq. (39) reduces to

$$n_{\mathrm{res}} \approx \frac{n_u}{2} \frac{eE_l k}{m} t^2 \cos(kx-\omega t) \ . \tag{40}$$

Figure 4 shows what Eqs. (37) and (40) mean. The first two curves show the electric field E_l and the potential $-e\phi_l$ seen by the electrons. The third and fourth curves are a plot of Eqs. (37) and (40) for the nonresonant and the resonant electron density perturbation due to a coherent low-frequency wave (E_l). We see easily that resonant electrons are rich in density for the potential energy minimum. The resonant electrons are necessary to transfer energy from a pump field to the radiation field. Accordingly, the expected high-frequency bursts due to the turbulent bremsstrahlung instability have a close correlation in phase with that of resonant electrons number density. The bottom curve in Fig. 4 represents the high-frequency radiation field versus phase ($kx-\omega t$). We must note that the recent experiments[13] also reported such a phase relation between a low-frequency pump field and the radiation field (δE_h). In view of this discussion, we suggest that experimentalists should carefully look for such a phase relation between AKR and low-frequency ion density fluctuations.

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APPENDIX

After a long calculation, we can calculate δf_{lh} from Eqs. (11)—(13) and, finally, with the help of Maxwell's equations. The electric fields of the mixed mode perturbation are calculated to be

$$\begin{aligned}
&\delta E_{lhx}(\vec{K}-\vec{k}',\Omega-\omega') \\
&= -\frac{4\pi i e(\Omega-\omega')}{MN[c^2 k_{\parallel}'^2-(\Omega-\omega')^2]} \\
&\quad \times \int d\vec{v}\, v_x \frac{e}{m} \sum_{\vec{k}''',j,l} \frac{J_j \exp[i(j-l)\phi]}{i[l\Omega_e - k_{\parallel}' v_{\parallel} - (\Omega-\omega')]} \\
&\qquad \times \left\{ J_l E_{l\parallel}(\vec{k}''') \frac{\partial}{\partial v_{\parallel}} \delta f_h(\vec{K}-\vec{k}'-\vec{k}''') \right. \\
&\qquad + \tfrac{1}{2} \left[E_{l\perp}(\vec{k}''') \frac{\partial}{\partial v_{\perp}} \delta f_h(\vec{K}-\vec{k}'-\vec{k}''') + F' \delta E_{hx}(\vec{K}-\vec{k}'-\vec{k}''') \right] (J_{l+1}+J_{l-1}) \\
&\qquad \left. - \frac{i}{2} F' \delta E_{hy}(\vec{K}-\vec{k}'-\vec{k}''')(J_{l+1}-J_{l-1}) \right. \\
&\qquad - \frac{2\pi e(\Omega-\omega')A(J_{l+1}-J_{l-1})}{H[c^2(K'^2+k_{\parallel}'^2)-(\Omega-\omega')^2]} \\
&\qquad \times \int d\vec{v}\, v_y \frac{e}{m} \sum_{\vec{k}^{\mathrm{IV}},m,n} \frac{J_m \exp[i(m-n)\phi]}{i[n\Omega_e - k_{\parallel}' v_{\parallel} - (\Omega-\omega')]}
\end{aligned}$$

$$\times\Bigg[J_n E_{l\parallel}(\vec{k}^{\,\mathrm{IV}})\frac{\partial}{\partial v_\parallel}\delta f_h(\vec{K}-\vec{k}\,'-\vec{k}^{\,\mathrm{IV}}) + \tfrac{1}{2}\Bigg\{E_{l\perp}(\vec{k}^{\,\mathrm{IV}})\frac{\partial}{\partial v_\perp}\delta f_h(\vec{K}-\vec{k}\,'-\vec{k}^{\,\mathrm{IV}}) + F''\delta E_{hy}(\vec{K}-\vec{k}\,'-\vec{k}^{\,\mathrm{IV}})\Bigg\}(J_{n+1}+J_{n-1}) - \frac{i}{2}F''\delta E_{hy}(\vec{K}-\vec{k}\,'-\vec{k}^{\,\mathrm{IV}})(J_{n+1}-J_{n-1})\Bigg]\Bigg\}, \quad \text{(A1)}$$

$$\delta E_{lhy}(\vec{K}-\vec{k}\,',\Omega-\omega') = -\frac{4\pi i e(\Omega-\omega')}{HP[c^2(K'^2+k_\parallel'^2)-(\Omega-\omega')^2]} \times\int d\vec{v}\, v_y \frac{e}{m}\sum_{\vec{k}\,''',j,l}\frac{J_j\exp[i(j-l)\phi]}{i[l\Omega_e-k_\parallel' v_\parallel-(\Omega-\omega')]}$$
$$\times\Bigg\{J_l E_{l\parallel}(\vec{k}\,''')\frac{\partial}{\partial v_\parallel}\delta f_h(\vec{K}-\vec{k}\,'-\vec{k}\,''') + \tfrac{1}{2}\Bigg[E_{l\perp}(\vec{k}\,''')\frac{\partial}{\partial v_\perp}\delta f_h(\vec{K}-\vec{k}\,'-\vec{k}\,''') + F'\delta E_{hx}(\vec{K}-\vec{k}\,'-\vec{k}\,''')\Bigg](J_{l+1}+J_{l-1}) - \frac{i}{2}F'\delta E_{hy}(\vec{K}-\vec{k}\,'-\vec{k}\,''')(J_{l+1}-J_{l-1})$$
$$-\frac{2\pi i e(\Omega-\omega')A(J_{l+1}+J_{l-1})}{M[c^2k_\parallel'^2-(\Omega-\omega')^2]} \times\int d\vec{v}\, v_x\frac{e}{m}\sum_{\vec{k}^{\,\mathrm{IV}},m,n}\frac{J_m\exp[i(m-n)\phi]}{i[n\Omega_e-k_\parallel' v_\parallel-(\Omega-\omega')]}$$
$$\times\Bigg[J_n E_{l\parallel}(\vec{k}^{\,\mathrm{IV}})\frac{\partial}{\partial v_\parallel}\delta f_h(\vec{K}-\vec{k}\,'-\vec{k}^{\,\mathrm{IV}}) - \frac{i}{2}F''(\vec{K}-\vec{k}\,'-\vec{k}^{\,\mathrm{IV}})(J_{n+1}-J_{n-1}) + \tfrac{1}{2}\Bigg\{E_{l\perp}(\vec{k}^{\,\mathrm{IV}})\frac{\partial}{\partial v_\perp}\delta f_h(\vec{K}-\vec{k}\,'-\vec{k}^{\,\mathrm{IV}}) + F''\delta E_{hx}(\vec{K}-\vec{k}\,'-\vec{k}^{\,\mathrm{IV}})\Bigg\} \times (J_{n+1}+J_{n-1})\Bigg]\Bigg\}, \quad \text{(A2)}$$

where J_n is the Bessel function,

$$F'=\frac{\partial}{\partial v_\perp}f_{1e}(\vec{k}\,''')+\frac{k_\parallel'+k_\parallel'''}{\Omega-\omega'-\omega'''}\left[v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right]f_{1e}(\vec{k}\,'''),$$

$$F''=\frac{\partial}{\partial v_\perp}f_{1e}(\vec{k}^{\,\mathrm{IV}})+\frac{k_\parallel'+k_\parallel^{\mathrm{IV}}}{\Omega-\omega'-\omega^{\mathrm{IV}}}\left[v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right]f_{1e}(\vec{k}^{\,\mathrm{IV}}),$$

$$H=1+\frac{4\pi e^2(\Omega-\omega')}{m[c^2(K'^2+k_{||}'^2)-(\Omega-\omega')^2]}\int d\vec{v}\, v_y \sum_{j,l}\frac{J_j A(J_{l+1}-J_{l-1})\exp[i(j-l)\phi]}{2i[l\Omega_e-k_{||}'v_{||}-(\Omega-\omega')]}\ ,$$

$$M=1+\frac{4\pi ie(\Omega-\omega')}{c^2k_{||}'^2-(\Omega-\omega')^2}\int d\vec{v}\, v_x\frac{e}{m}\sum_{j,l}\frac{J_j A(J_{l+1}+J_{l-1})\exp[i(j-l)\phi]}{2i[l\Omega_e-k_{||}'v_{||}-(\Omega-\omega')]}\ ,$$

$$N=1-\frac{4\pi ie(\Omega-\omega')}{M[c^2k_{||}'^2-(\Omega-\omega')^2]}$$

$$\times\int d\vec{v}\, v_x\frac{e}{m}\sum_{j,l}\frac{J_j\exp[i(j-l)\phi]}{i[l\Omega_e-k_{||}'v_{||}-(\Omega-\omega')]}\frac{2\pi e(\Omega-\omega')A(J_{l+1}-J_{l-1})}{H[c^2(K'^2+k_{||}'^2)-(\Omega-\omega')^2]}$$

$$\times\int d\vec{v}\, v_y\frac{e}{m}\sum_{m,n}\frac{J_m A(J_{n+1}+J_{n-1})\exp[i(m-n)\phi]}{2i[n\Omega_e-k_{||}'v_{||}-(\Omega-\omega')]}\ ,$$

$$P=1-\frac{4\pi ie(\Omega-\omega')}{H[c^2(K'^2+k_{||}'^2)-(\Omega-\omega')^2]}$$

$$\times\int d\vec{v}\, v_y\frac{e}{m}\sum_{\vec{K}''',j,l}\frac{J_j\exp[i(j-l)\phi]}{i[l\Omega_e-k_{||}'v_{||}-(\Omega-\omega')]}\frac{2\pi e(\Omega-\omega')A(J_{l+1}+J_{l-1})}{M[c^2k_{||}'^2-(\Omega-\omega')^2]}$$

$$\times\int d\vec{v}\, v_x\frac{e}{m}\sum_{m,n}\frac{J_m A(J_{n+1}-J_{n-1})\exp[i(m-n)\phi]}{2i[n\Omega_e-k_{||}'v_{||}-(\Omega-\omega')]}\ ,$$

$$A=\left[1+\frac{k_{||}'v_{||}}{\Omega-\omega'}\right]\frac{\partial f_{0e}}{\partial v_\perp}-\frac{k_{||}'v_{||}}{(\Omega-\omega')}\frac{\partial f_{0e}}{\partial v_{||}}\ ,$$

$$K'=K-k_\perp'\ .$$

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