Effects of laser-field fluctuations on the intensity correlation of resonance fluorescence

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The effects of amplitude, phase, and frequency fluctuations in the exciting laser field on the intensity correlation of resonance fluorescence from a single two-level atom are investigated. Numerical results are presented for both weak and strong excitation with narrow-band and broadband laser fields having Lorentzian or Gaussian line shapes.

I. INTRODUCTION

Although the triplet spectrum of the light scattered by a two-level atom excited by a strong resonant radiation field has been predicted theoretically[1] and verified experimentally[2] for almost a decade, considerable activity persists at present in the investigation of the properties of resonance fluorescence. With refined experimental techniques, measurements of higher-order field correlations have demonstrated interesting features of the scattered radiation, e.g., the photon antibunching effect.[3] Recent theoretical investigations, on the other hand, have focused on the influence of different statistical properties of the exciting radiation on the scattered light.[4–15] Not only are such investigations important from the standpoint of understanding the effects of realistic, nonmonochromatic laser fields on the dynamics of the two-level atom, but they also pose a challenge in developing a theory that is applicable in the entire region of parameter space, i.e., for arbitrary strengths and bandwidths of the field. Exact results for the intensity and the spectrum of the scattered light have been obtained for the cases when the exciting field fluctuations have been described by the phase diffusion model (PDM) with Lorentzian line shape,[4,5] the extended phase diffusion model with a non-Lorentzian line shape,[15] the chaotic field,[9–11] and the real Gaussian field.[14]

For a complete description of the scattered light, knowledge of correlation functions of the field to all order is desired. While the spectrum yields information about the second-order correlation, higher-order correlations such as the intensity correlation demonstrate interesting features like the photon antibunching that are fundamental to the emission of light by an isolated two-level atom. It is well known that the intensity correlation of the light scattered by a two-level atom is proportional to the two-time atomic dipole correlation[16,17]

$$G^{(2)}(t,\tau)=\langle \hat{\sigma}_{21}(t)\hat{\sigma}_{21}(t+\tau)\hat{\sigma}_{12}(t+\tau)\hat{\sigma}_{12}(t)\rangle$$
$$=\langle \hat{\sigma}_{21}(t)\hat{\sigma}_{22}(t+\tau)\hat{\sigma}_{12}(t)\rangle \;, \tag{1}$$

where $\hat{\sigma}_{ij}=|i\rangle\langle j|$, $i,j=1,2$ are the slowly varying parts of the atomic density-matrix operator. Angular brackets in the above equation denote quantum averaging. Several authors have demonstrated the factorization property of the dipole correlation in (1) that enables us to rewrite $G^{(2)}(t,\tau)$ as[16,18–20]

$$G^{(2)}(t,\tau)=\langle \hat{\sigma}_{22}(t)\rangle\langle \hat{\sigma}_{22}(t+\tau\,|\,t,\ |1\rangle)\rangle \;, \tag{2}$$

where $\langle \hat{\sigma}_{22}(t+\tau\,|\,t,\ |1\rangle)\rangle$ denotes the population of the upper level $|2\rangle$ at time $t+\tau$ with the constraint that the atom was in state $|1\rangle$ at t. This particular form of $G^{(2)}(t,\tau)$ displays an important property of the fluorescence light from a single two-level atom: that of photon antibunching which is due to the fact that an isolated atom cannot emit two photons at any given time.

If the exciting field is monochromatic, Eq. (2) can be used to calculate the intensity correlation of the scattered field. However, if the atom interacts with a nonmonochromatic incoherent field, $G^{(2)}(t,\tau)$ in Eq. (2) becomes a stochastic function in time and hence it must be averaged over the fluctuations. Such averaging is nontrivial due to the nonlinear coupling of the atomic dynamics with the exciting electromagnetic field. Earlier treatments of the effects of field fluctuations (laser line-shape effects) on the intensity correlations have either been restricted to a limited range of parameter space or have been for the simplest model describing the fluctuations; the phase diffusion model.[5,7,12,13,21]

In this paper we present a theory that describes the behavior of the averaged intensity correlation

$$\langle G^{(2)}(t,\tau)\rangle=\langle\langle \hat{\sigma}_{22}(t)\rangle\langle \hat{\sigma}_{22}(t+\tau\,|\,t,\ |1\rangle)\rangle\rangle \;, \tag{3}$$

where the bold outer angular brackets denote the averaging with respect to the field fluctuations, which is valid for arbitrary excitation strengths (Rabi frequencies), bandwidths, and line shape. The fluctuation field will be treated either as a chaotic field, or as a field with fixed amplitude but the frequency undergoing fluctuations analogous to the velocity fluctuations of a particle performing Brownian motion. The phase diffusion model with Lorentzian line shape that has been widely used in discussing finite bandwidth effects will emerge as a special case of the latter model in the limit of extremely fast frequency fluctuations. Owing to the memory effects inherent in the dynamics of the two-level atom, the correlation function in Eq. (3) cannot, in general, be decorrelated into $\langle\langle\hat{\sigma}_{22}(t)\rangle\rangle\langle\langle\hat{\sigma}_{22}(t+\tau\,|\,t,\ |1\rangle)\rangle\rangle$ except in the case of the PDM with Lorentzian line shape. Thus, although the factorization of (1) holds true at the level of quantum averaging that leads to (2), factorization of (3) with respect to averaging over exciting field fluctuations will, in general, not be correct.

Starting from the Bloch equations describing the evolution of a two-level atom, we shall develop in the next section equations that describe the averaging procedure for chaotic field and the extended phase diffusion model. Results of numerical calculations are presented in Sec. III.

II. THEORY

The dynamics of a two-level atom with ground state $|1\rangle$ and excited state $|2\rangle$ that are separated by an energy ω_{21} and are coupled by an electric dipole transition (matrix element μ_{12}), are described by the following equations of motion for the quantum averaged slowly varying density-matrix elements $\sigma_{ij}=\langle\hat{\sigma}_{ij}\rangle$,[14]

$$\frac{d}{dt}\begin{bmatrix}\sigma_{22}(t)\\ \sigma_{11}(t)\\ \sigma_{12}(t)\\ \sigma_{21}(t)\end{bmatrix}=\begin{bmatrix}-\Gamma & 0 & -\frac{1}{2}i\omega_R^*(t) & \frac{1}{2}i\omega_R(t)\\ \Gamma & 0 & \frac{1}{2}i\omega_R^*(t) & -\frac{1}{2}i\omega_R(t)\\ -\frac{1}{2}i\omega_R(t) & \frac{1}{2}i\omega_R(t) & i\Delta-\frac{1}{2}\Gamma & 0\\ \frac{1}{2}i\omega_R^*(t) & -\frac{1}{2}i\omega_R^*(t) & 0 & -i\Delta-\frac{1}{2}\Gamma\end{bmatrix}\begin{bmatrix}\sigma_{22}(t)\\ \sigma_{11}(t)\\ \sigma_{12}(t)\\ \sigma_{21}(t)\end{bmatrix}. \tag{4}$$

In the above equation Γ denotes the spontaneous decay rate of state $|2\rangle$ and $\Delta=\omega_0-\omega_{21}$ the detuning of the center laser frequency ω_0 from resonance. The parameter $\omega_R(t)=2\hbar^{-1}\mu_{12}\epsilon(t)$ is the stochastic Rabi frequency with $\epsilon(t)=\sqrt{I}(t)e^{i\phi(t)}$ denoting the complex amplitude of the fluctuating exciting field. The root-mean-square value of $\omega_R(t)$, $\bar{\omega}_R=2\hbar^{-1}\mu_{12}\sqrt{I_0}$, will be referred to as the average Rabi frequency.

In order to analyze the behavior of the intensity correlation function $G^{(2)}(t,\tau)$ the equation of motion for two-time correlation functions $\sigma_{22}(t)\sigma_{22}(t+\tau\,|\,t,\ |1\rangle)$ is needed. This in turn is coupled to other two-time correlation functions of the type $\sigma_{22}(t)\sigma_{ij}(t+\tau\,|\,t,\ |1\rangle)$. These correlation functions obey, for $\tau>0$,

$$\frac{d}{d\tau}\begin{bmatrix}\sigma_{22}(t)\sigma_{22}(t+\tau\,|\,t,\ |1\rangle)\\ \sigma_{22}(t)\sigma_{11}(t+\tau\,|\,t,\ |1\rangle)\\ \sigma_{22}(t)\sigma_{12}(t+\tau\,|\,t,\ |1\rangle)\\ \sigma_{22}(t)\sigma_{24}(t+\tau\,|\,t,\ |1\rangle)\end{bmatrix}=\begin{bmatrix}-\Gamma & 0 & -\frac{1}{2}i\omega_R^*(\tau) & \frac{1}{2}i\omega_R(\tau)\\ \Gamma & 0 & \frac{1}{2}i\omega_R^*(\tau) & -\frac{1}{2}i\omega_R(\tau)\\ -\frac{1}{2}i\omega_R(\tau) & \frac{1}{2}i\omega_R(\tau) & i\Delta-\frac{1}{2}\Gamma & 0\\ \frac{1}{2}i\omega_R^*(\tau) & -\frac{1}{2}i\omega_R^*(\tau) & 0 & -i\Delta-\frac{1}{2}\Gamma\end{bmatrix}\begin{bmatrix}\sigma_{22}(t)\sigma_{22}(t+\tau\,|\,t,\ |1\rangle)\\ \sigma_{22}(t)\sigma_{11}(t+\tau\,|\,t,\ |1\rangle)\\ \sigma_{22}(t)\sigma_{12}(t+\tau\,|\,t,\ |1\rangle)\\ \sigma_{22}(t)\sigma_{21}(t+\tau\,|\,t,\ |1\rangle)\end{bmatrix}, \tag{5}$$

where the 4×4 matrix on the right-hand side is identical to that in Eq. (4).

It should be noted at this point that although Eq. (5) looks similar to that used in calculating the spectrum of the scattered light [see, e.g., (38) of Ref. 14] Eq. (5) differs from the latter. Firstly, quantum two-time correlation functions of the type $\langle\hat{\sigma}_{21}(t)\hat{\sigma}_{ij}(t+\tau)\rangle$ are required to calculate the spectrum of the scattered light while the products of quantum averaged density-matrix elements are needed to calculate the intensity correlations. Furthermore, the initial conditions in the two cases are different; in the former

$$\langle\hat{\sigma}_{21}(t)\hat{\sigma}_{ij}(t)\rangle=\langle\hat{\sigma}_{2j}(t)\rangle\delta_{i1}\,,$$

while in the present

$$\sigma_{22}(t)\sigma_{11}(t\,|\,t,\ |1\rangle)=\sigma_{22}(t)\,,$$

and all other product correlations are zero. These differences in the initial conditions lead to drastically different features in the solutions in the two cases [as, e.g., the existence of antibunching in the solutions of Eq. (5)].

Using the property $\sigma_{22}(t)+\sigma_{11}(t)=1$, one of the equations in (5) can be eliminated. Equation (5) can then be rewritten as

$$\frac{df}{d\tau}(t,\tau)=-\Gamma\sigma_{22}(t)-\Gamma f(t,\tau)-i[\omega_R^*g(t,\tau)-\omega_R h(t,\tau)]\,, \tag{6a}$$

$$\frac{dg(t,\tau)}{d\tau}=\left[i\Delta-\frac{\Gamma}{2}\right]g(t,\tau)-\frac{i\omega_R f(t,\tau)}{2}\,, \tag{6b}$$

$$\frac{dh(t,\tau)}{d\tau}=\left[-i\Delta-\frac{\Gamma}{2}\right]h(t,\tau)+\frac{i\omega_R^* f(t,\tau)}{2}\,, \tag{6c}$$

where

$$f(t,\tau)=\sigma_{22}(t)[\sigma_{22}(t+\tau\,|\,t,\ |1\rangle)-\sigma_{11}(t+\tau\,|\,t,\ |1\rangle)]\,, \tag{7a}$$

$$g(t,\tau)=\sigma_{22}(t)\sigma_{12}(t+\tau\,|\,t,\ |1\rangle)\,, \tag{7b}$$

and

$$h(t,\tau)=\sigma_{22}(t)\sigma_{21}(t+\tau\,|\,t,\ |1\rangle)\,. \tag{7c}$$

$G^{(2)}(t,\tau)$ is then calculated from the solutions of (6) with the initial conditions $f(t,0)=-\sigma_{22}(t)$, $g(t,0)=0=h(t,0)$, and using the relation

$$G^{(2)}(t,\tau)=\tfrac{1}{2}\sigma_{22}(t)+\tfrac{1}{2}f(t,\tau) \ . \qquad (8)$$

Note that Eqs. (6)–(8) are stochastic equations due to the fluctuating Rabi frequency and, as such, have to be averaged over their fluctuations in order to calculate observable quantities. In what follows this averaging procedure is briefly sketched for the case when the field fluctuations correspond to those of a chaotic field and for the case when the amplitude of the field remains fixed while its frequency undergoes fluctuations analogous to the velocity of a free particle performing Brownian motion. In both subsections we shall work within the Fokker-Plank formalism.

A. Chaotic field model

In the chaotic field model, the electric field $\epsilon(t)=\sqrt{I}(t)e^{i\phi(t)}$ described as a complex Gaussian process whose real and imaginary parts $[\epsilon(t)=\epsilon_1(t)+i\epsilon_2(t)]$ are independent and satisfy

$$\frac{d\epsilon_i}{dt}=-\tfrac{1}{2}\gamma\epsilon_i(t)+F_i(t), \quad i=1,2 \qquad (9a)$$

where the forces $F_i(t)$ are Gaussian δ-correlated stochastic variables with zero mean, i.e.,

$$\langle F_i(t)\rangle=0$$

and (9b)

$$\langle F_i(t)F_j(t')\rangle=\tfrac{1}{2}\delta_{ij}\gamma I_0\delta(t-t') \ .$$

I_0 in the above equation denotes the average intensity. The spectrum of the field described by the above model, which is the Fourier transform of the correlation function

$$\langle \epsilon^*(t)\epsilon(t')\rangle=I_0e^{-\gamma|t-t'|/2} \ ,$$

is a Lorentzian of full width at half maximum (FWHM) γ.

Calculation of the averages for the field undergoing fluctuations described by Eq. (9) has been described elsewhere.[11,22] Briefly, if the stochastic differential equations are written as

$$\left[\frac{d}{dt}+Q(I(t),\ \phi(t))\right]X(t)=0 \ , \qquad (10)$$

with $X(t)$ denoting a vector containing the dynamical variables of the system and $Q(I(t),\phi(t))$ describing the coefficient matrix, then the averaged quantities $\langle X(t)\rangle$ are calculated by first solving the partial differential equation

$$\left[\frac{\partial}{\partial t}+L+Q(I,\phi)\right](I,\phi,t)=0 \ , \qquad (11)$$

and then using the relation

$$\langle X(t)\rangle=\int_0^\infty dI\int_0^{2\pi}d\phi\, X(I,\phi,t) \ . \qquad (12)$$

L appearing in (11) is the Fokker-Plank operator for the model described by Eq. (9) and has been discussed in detail in Ref. 22 along with its eigenvalues and eigenfunctions. The initial condition for solving Eq. (11) is

$$X(I,\phi,\ t=0)=\langle X(t=0)\rangle P_{00}(I,\phi) \ , \qquad (13)$$

with $P_{00}(I,\phi)$ denoting the stationary probability distribution that satisfies $LP_{00}=0$.

Expanding $X(I,\phi,t)$ in the complete biorthonormal set of eigenfunctions $P_{\alpha n}$ and $\Phi_{\alpha n}$ of L ($LP_{\alpha n}=\Lambda_{\alpha n}P_{\alpha n}$, $L^\dagger\Phi_{\alpha n}=\Lambda_{\alpha n}\Phi_{\alpha n}$)

$$X(I,\phi,t)=\sum X_{\alpha n}(t)P_{\alpha n}(I,\phi) \ , \qquad (14)$$

the expansion coefficients

$$X_{\alpha n}(t)=\int \Phi_{\alpha n}(I,\phi)X(I,\phi,t)dI\,d\phi$$

satisfy the infinite set of coupled differential equations

$$\left[\frac{d}{dt}+\Lambda_{\alpha n}\right]X_{\alpha n}+\sum_{\alpha',m}\langle\Phi_{\alpha n}\,|\,Q(I,\phi)\,|\,P_{\alpha' m}\rangle X_{\alpha' m}=0 \ ,$$

$$n,m=0,1,2,\ldots, \quad \alpha,\alpha'=0,\pm1,\pm2,\ldots \ . \qquad (15)$$

Although the summation in the last term above is over the whole range of α' and m, only a few terms are nonzero due to the specific dependence of Q on I and ϕ and the recursion relationships involving various eigenfunctions. In Eq. (15)

$$\Lambda_{\alpha n}=\gamma\left[\frac{|\alpha|}{2}+n\right]$$

denotes the eigenvalue of the Fokker-Plank operator L. The averages $\langle X(t)\rangle$ are then given by the solution of (15) for $\alpha=n=0$, i.e., $\langle X(t)\rangle=X_{00}(t)$.

Applying the above technique to Eq. (6) one obtains, using the definition $\omega_R=2\hbar^{-1}\mu_{12}\sqrt{I}e^{i\phi(t)}$ and the recursion relation described in Ref. 22, the following equations for the evolution of the expansion coefficients $f_{\alpha n}(t,\tau)$, $g_{\alpha n}(t,\tau)$, and $h_{\alpha n}(t,\tau)$:

$$\frac{d}{d\tau}f_{0n}(t,\tau)=-\Gamma\sigma_{22}^{0n}(t)-(\Gamma+\Lambda_{0n})f_{0n}(t,\tau)-i\bar{\omega}_R(n+1)^{1/2}[g_{1n}(t,\tau)-h_{-1n}(t,\tau)]$$
$$+i\bar{\omega}_R n^{1/2}[g_{1(n-1)}(t,\tau)-h_{-1(n-1)}(t,\tau)] \ , \qquad (16a)$$

$$\frac{d}{d\tau}g_{1n}(t,\tau)=(i\Delta-\tfrac{1}{2}\Gamma-\Lambda_{1n})g_{1n}(t,\tau)-\tfrac{1}{2}i\bar{\omega}_R(n+1)^{1/2}[f_{0n}(t,\tau)-f_{0(n+1)}(t,\tau)] \ , \qquad (16b)$$

$$\frac{d}{d\tau}h_{-1n}(t,\tau)=(-i\Delta-\tfrac{1}{2}\Gamma-\Lambda_{-1n})h_{-1n}(t,\tau)+\tfrac{1}{2}i\bar{\omega}_R(n+1)^{1/2}[f_{0n}(t,\tau)-f_{0(n+1)}(t,\tau)], \quad n=0,1,2,\ldots \ . \qquad (16c)$$

The averaged intensity correlation is calculated using the relation

$$\langle G^{(2)}(t,\tau)\rangle = \tfrac{1}{2}\langle \sigma_{22}(t)\rangle + \tfrac{1}{2}\langle f(t,\tau)\rangle = \tfrac{1}{2}\sigma_{22}^{00}(t) + \tfrac{1}{2}f_{00}(t,\tau) \; . \tag{17}$$

Rewriting the infinite set of coupled Eq. (16) in a compact form as

$$\frac{dx(t,\tau)}{d\tau} = Ax(t,\tau) - \Gamma Y \; , \tag{18}$$

with

$$x = \mathrm{col}(f_{00}, g_{10}, h_{-10}, f_{01}, g_{11}, h_{-11}, \ldots) \; ,$$
$$Y = [\sigma_{22}^{00}(t), 0, 0, \sigma_{22}^{01}(t), 0, 0, \ldots] \; ,$$

and A denoting the coefficient matrix that follows from (16), a formal solution for $f_{00}(t,\tau)$ with the initial condition $f_{an}(t,0) = \sigma_{22}^{an}(t)$, $g_{an}(t,0) = h_{an}(t,0) = 0$ can be written in the form

$$f_{00}(t,\tau) = \sum_{k=0}^{\infty} B_{1,(3k+1)}(\tau)\sigma_{22}^{0k}(t) \; . \tag{19}$$

$B_{ij}(t)$ in the above equation is, in turn, expressed in terms of eigenvalues λ_m and eigenvectors $V_k(m)$ and reciprocal eigenvectors $W_k(m)$ as

$$B_{ij}(t) = \sum_m V_i(m)W_j(m)\left[\frac{\Gamma}{\lambda_m} - \left[1 + \frac{\Gamma}{\lambda_m}\right]e^{\lambda_m t}\right] \; . \tag{20}$$

At this point, knowledge of $\sigma_{22}^{0k}(t)$ is needed to complete the calculation of $f_{00}(t,\tau)$ and $\langle G^{(2)}(t,\tau)\rangle$. Realizing that $\sigma_z^{0n}(t) = \sigma_{22}^{0n}(t) - \sigma_{11}^{0n}(t)$, $\sigma_{12}^{1n}(t)$, and $\sigma_{21}^{-1n}(t)$ obey similar equations of motion as those obeyed by $f_{0n}(t,\tau)$, $g_{1n}(t,\tau)$, and $h_{-1n}(t,\tau)$, Eq. (16) with the inhomogeneous term in (16a) replaced by $-\Gamma\delta_{0n}$, $\sigma_{22}^{0k}(t)$ can be expressed as

$$\sigma_{22}^{0k}(t) = \tfrac{1}{2}\sigma_z^{0k}(t) + \tfrac{1}{2}\delta_{k0} = \tfrac{1}{2}B_{3k+1,1}(t) + \tfrac{1}{2}\delta_{k,0} \; . \tag{21}$$

Finally, combining (17), (19), and (21), $\langle G^{(2)}(t,\tau)\rangle$ is written in the form[23]

$$\langle G^{(2)}(t,\tau)\rangle = \tfrac{1}{2}\sigma_{22}^{00}(t) + \tfrac{1}{4}B_{1,1}(\tau) + \tfrac{1}{4}\sum_{k=0}^{\infty} B_{1,(3k+1)}(\tau)B_{(3k+1),1}(t) \; . \tag{22}$$

Note that the antibunching property of $\langle G^{(2)}(t,\tau)\rangle$ follows from (22) since $B_{ij}(t=0) = -\delta_{ij}$ and $\sigma_{22}^{00}(t) = \frac{1}{2} + \frac{1}{2}B_{11}(t)$. Numerical results are obtained by truncating the system of Eq. (16) up to a certain value of n and solving the resulting set of coupled linear equations using matrix methods.

B. Phase diffusion model

In this model the amplitude of the exciting field remains constant while its phase $\phi(t)$ undergoes random fluctuations that are described by the following stochastic differential equations[24,25,15]

$$\frac{d\phi}{dt} = \nu(t) \; , \qquad \frac{d\nu}{dt} = -\beta\nu(t) + F(t) \; , \tag{23}$$

where the force $F(t)$ is, again, a Gaussian random variable satisfying

$$\langle F(t)\rangle = 0, \quad \langle F(t)F(t')\rangle = \gamma_D\beta^2\delta(t-t') \; . \tag{24}$$

The spectrum of the field is given by the Fourier transform of the correlation function

$$\langle \epsilon^*(t)\epsilon(t+\tau)\rangle = I_0\langle e^{i[\phi(t+\tau)-\phi(t)]}\rangle = I_0\exp\left[-\frac{\gamma_D}{2}\left[|\tau| + \frac{e^{-\beta|\tau|}-1}{\beta}\right]\right] . \tag{25}$$

For $\gamma_D \ll \beta$, the spectrum is a Lorentzian with FWHM γ_D and has a cutoff at β, while in the other limit $\beta \to 0, \gamma_D \to \infty$ with the product $\beta\gamma_D$ remaining finite, the spectrum becomes a Gaussian with FWHM $2(\ln 2\beta\gamma_D)^{1/2}$. In the limit $\beta \to \infty$ the frequency fluctuations become δ correlated and the phase diffusion model widely used is recovered. In this paper we shall refer to both β finite and β infinite cases as the phase diffusion model with the understanding that earlier model is a special case of the present extended version ($\beta \to \infty$).

Performing the averages of $f(t,\tau)$, $g(t,\tau)$, and $h(t,\tau)$ in Eq. (6) proceeds along similar lines as done in Sec. II A for the chaotic field model. Defining $\tilde{g}(t,\tau) = e^{-i\phi}g(t,\tau)$ and $\tilde{h}(t,\tau) = e^{i\phi}h(t,\tau)$, Eq. (6) can be rewritten as

$$\frac{df(t,\tau)}{d\tau} = -\Gamma\sigma_{22}^{(t)} - \Gamma f(t,\tau) - i\bar{\omega}_R[\tilde{g}(t,\tau) - \tilde{h}(t,\tau)] \; , \tag{26a}$$

$$\frac{d\tilde{g}(t,\tau)}{d\tau} = (i\Delta - \tfrac{1}{2}\Gamma)\tilde{g}(t,\tau) - i\nu(\tau)\tilde{g}(t,\tau) - \tfrac{1}{2}i\bar{\omega}_R f(t,\tau) \; , \tag{26b}$$

and

$$\frac{d\tilde{h}(t,\tau)}{d\tau} = (-i\Delta - \tfrac{1}{2}\Gamma)\tilde{h}(t,\tau) + i\nu(\tau)\tilde{h}(t,\tau) + \tfrac{1}{2}i\bar{\omega}_R f(t,\tau) \; , \tag{26c}$$

where all the coefficients except $\nu(t)$ are constants as a function of time.

Following Ref. 25, we expand $f(t,\tau)$, $\tilde{g}(t,\tau)$, and $\tilde{h}(t,\tau)$ in terms of the complete biorthonormal eigenfunctions $\phi_n(\nu), P_n(\nu)$ of the Fokker-Plank operator that satisfy $LP_n = \Lambda_n P_n = n\beta P_n$ and $L^\dagger\phi_n = \Lambda_n\phi_n$ as

$$f(t,\tau) = \sum f_n(t,\tau)P_n(\nu) \; ,$$
$$\tilde{g}(t,\tau) = \sum \tilde{g}_n(t,\tau)P_n(\nu) \; , \tag{27}$$
$$\tilde{h}(t,\tau) = \sum \tilde{h}_n(t,\tau)P_n(\nu) \; .$$

The expansion coefficients in the above equation can be shown to satisfy

$$\frac{d}{d\tau}f_n(t,\tau)=-\Gamma\sigma_{22}^{n}-(\Gamma+n\beta)f_n(t,\tau)-i\bar{\omega}_R[\tilde{g}_n(t,\tau)-\tilde{h}_n(t,\tau)]\ , \tag{28a}$$

$$\frac{d}{d\tau}\tilde{g}_n(t,\tau)=(i\Delta-\tfrac{1}{2}\Gamma-n\beta)\tilde{g}_n(t,\tau)-\tfrac{1}{2}i\bar{\omega}_R f_n(t,\tau)-\tfrac{1}{2}i[\beta\gamma_D(n+1)]^{1/2}\tilde{g}_{n+1}^{(t,\tau)}-\tfrac{1}{2}i(\beta\gamma_D n)^{1/2}\tilde{g}_{n-1}(t,\tau)\ , \tag{28b}$$

$$\frac{d}{d\tau}\tilde{h}_n(t,\tau)=(-i\Delta-\tfrac{1}{2}\Gamma-n\beta)\tilde{h}_n(t,\tau)+\tfrac{1}{2}i\bar{\omega}_R f_n(t,\tau)+\tfrac{1}{2}i[\beta\gamma_D(n+1)]^{1/2}\tilde{h}_{n+1}(t,\tau)+\tfrac{1}{2}i(\beta\gamma_D n)^{1/2}\tilde{h}_{n-1}(t,\tau)\ . \tag{28c}$$

Since Eq. (28) can be cast in the form of (18) with

$$X=\mathrm{col}(f_0,\tilde{g}_0,\tilde{h}_0,f_1,\tilde{g}_1,\tilde{h}_1,\dots)\ ,$$

$$Y=(\sigma_{22}^{0},0,0,\sigma_{22}^{1},0,0,\dots)\ ,$$

and the coefficient matrix A being derived from the above equation, it follows from the analysis leading to Eq. (22) that the intensity correlation can again be expressed as

$$\langle G^{(2)}(t,\tau)\rangle=\tfrac{1}{2}\sigma_{22}^{0}(t)+\tfrac{1}{4}B_{11}(\tau)+\tfrac{1}{2}\sum_{k=0}^{\infty}B_{1,3k+1}(\tau)B_{3k+1,1}(t)\ , \tag{29}$$

where $B_{ij}(t)$ is given by Eq. (20) with the eigenvalues and eigenvectors of the matrix A that follows from Eq. (28).

Before closing this section, it should be pointed out that in the limit $\beta\to\infty$, $f_n,\tilde{g}_n,\tilde{h}_n\to 0$ for $n\neq 0$. However, the terms

$$-\tfrac{1}{2}i(\beta\gamma_D)^{1/2}\tilde{g}_1(t,\tau)\to-\tfrac{1}{2}\gamma_D\tilde{g}_0(t,\tau)$$

and

$$+\tfrac{1}{2}i(\beta\gamma_D)^{1/2}\tilde{h}_1(t,\tau)\to-\tfrac{1}{2}\gamma_D\tilde{h}_0(t,\tau)\ ,$$

and one recovers the PDM equations for Lorentzian line shape that has been discussed in several places in the literature.

III. NUMERICAL RESULTS AND DISCUSSION

To illustrate the effects of laser line shape as well as the effects of the amplitude fluctuations on the intensity correlation $\langle G^{(2)}(t,\tau)\rangle$, we present in this section results of numerical calculation of $\langle G^{(2)}(t,\tau)\rangle$ for the chaotic field (CF) and the extended diffusion model (EPDM) discussed in the preceding section. The results of each model will be compared with those for the phase diffusion model (PDM) with a Lorentzian line shape ($\beta=\infty$) for three values of bandwidths (FWHM) $\gamma=0.1\Gamma$, Γ, and 10Γ. The FWHM for CF is characterized by the parameter γ defined in Eq. (9) while γ_D defined in Eqs. (23)–(25) characterizes the FWHM for the PDM. Since the line shape in the EPDM depends on γ_D and β, we have determined, for a given β (finite) three values of γ_D such that the effective FWHM γ is 0.1Γ, Γ, and 10Γ. For $\beta=1$, these values of γ_D are, respectively, 0.1Γ, 1.153Γ, and 42.55Γ. Although, according to laser theory, $\beta>\gamma_D$, we shall discuss cases $\beta<\gamma_D$ as a mathematical model for Gaussian line shapes.

In Fig. 1 we have plotted the intensity correlation (IC) $\langle G^{(2)}(t,\tau)\rangle$ of the scattered light under steady-state conditions ($t\to\infty$) when the two-level atom is excited by a weak

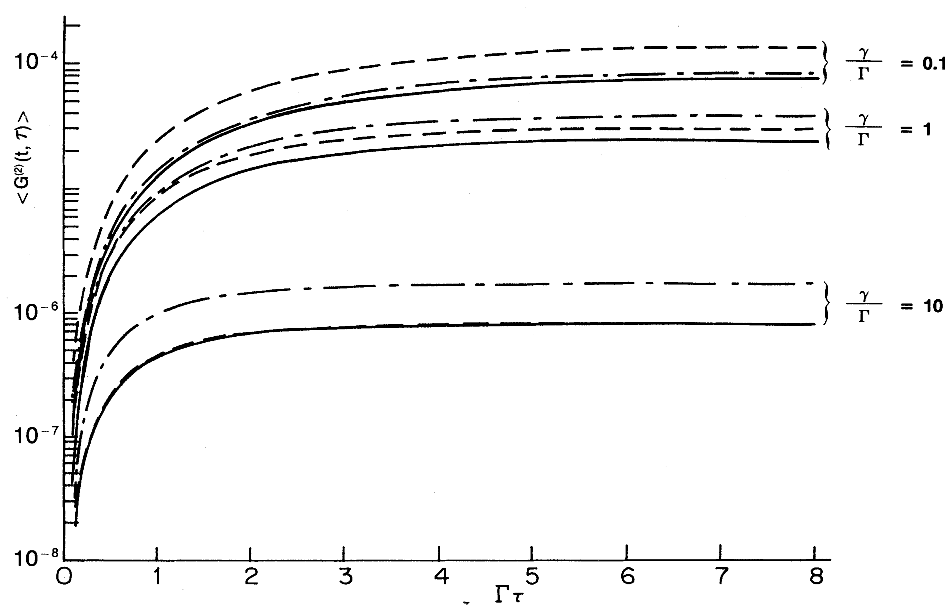

FIG. 1. Intensity correlation (IC) of the scattered light under weak excitation. The Rabi frequency is $\bar{\omega}_R=0.1\Gamma$ and, for all values of laser bandwidths, ——, phase diffusion model (PDM); –·–·–, extended phase diffusion model (EPDM); and — — —, the chaotic field (CF).

($\bar{\omega}_R = 0.1\Gamma$), resonant incoherent field. The IC is seen to decrease with increasing bandwidth irrespective of the type of fluctuations, amplitude, and/or phase. This is due to the fact that as the laser bandwidth increases, the excitation probability and, hence the emission probability decreases. Another remarkable feature common to all the three models is that the photon antibunching time, defined as the time delay τ required for the IC to reach $(1-e^{-1})$ of the maximum value, is independent of the nature of the fluctuations (which determine the absolute value of the maximum) and depends only on the bandwidth. Furthermore, this antibunching time ($\sim$2.9, 2.1, and 1.1 for $\gamma = 0.1, 1, 10$) decreases with increasing bandwidth. This does not mean that a broadband field speeds up the emission, but means simply that the two-photon counting probability reaches a reduced maximum value earlier as the increased laser bandwidth damps the transients and reduces the atomic response.

Comparing PDM and EPDM results, we see that the IC for the two models is the same for narrow bandwidth of the fluctuations while EPDM value is larger than the PDM one for larger γ. To understand this, note that the strength of a weak excitation depends on overlap of the field spectrum with the atomic line shape. For $\gamma = 0.1\Gamma$, the laser line is narrower than the atomic line and hence the atom responds to the total intensity of the field which is same in both models. For $\gamma \gg \Gamma$ the power at the center of the spectrum in EPDM is larger than that in PDM. This increase in the power gives rise to a larger two-photon counting probability as seen in the figure.

Several interesting features are seen by comparing the CF results with the PDM results. For narrow bandwidth fields ($\gamma = 0.1\Gamma$)

$$\langle G^{(2)}(t,\tau)\rangle_{CF} \approx 2\langle G^{(2)}(t,\tau)\rangle_{\rm PDM}$$

which is a manifestation of the 2! enhancement of the two-photon process implicit in the measurement of the intensity correlation. With increasing bandwidth, the difference between the CF results and the PDM results decreases. This is due to the decorrelation of the IC when the field fluctuations are sufficiently fast. It can be shown that, under such conditions, $\langle G^{(2)}(t,\tau)\rangle_{\rm CF} = \langle G^{(2)}(t,\tau)\rangle_{\rm PDM}$ for τ satisfying $\gamma\tau \gg 1$.

When the exciting field becomes strong, the IC becomes sensitive to the complete line shape as multiphoton transitions become important. In Fig. 2 we have the intensity correlation for strong excitation ($\bar{\omega}_R = 10\Gamma$) by a chaotic field and by a PDM field. It is seen that at all bandwidths, the Rabi oscillations in the IC for CF are washed out due to the amplitude fluctuations. Furthermore, the steady-state ($\Gamma\tau \gg 1$) value of $\langle G^{(2)}(t,\tau)\rangle$ for CF is smaller than that for PDM. This is a manifestation of the less effectiveness of CF in saturating a two-level atom. The clamping of oscillations in $\langle G^{(2)}(t,\tau)\rangle_{\rm PDM}$ as well as the reduction of the overshoot peak in $\langle G^{(2)}(t,\tau)\rangle_{\rm CF}$ with increasing bandwidth is a result of faster fluctuations of the field, the time scale of which is given by γ^{-1}. This observation raises the question of whether, if the fluctuations can be made slower while keeping the FWHM fixed, the Rabi oscillations in the broadband excitation case would become more pronounced. The EPDM, in which the fluctuations are characterized by two parameters γ_D and β, allows one to vary the rate of fluctuations by changing β, γ_D while keeping the FWHM constant. The results are illustrated in Fig. 3 where we have plotted $\langle G^{(2)}(t,\tau)\rangle$ for PDM ($\beta = \infty$) and EPDM ($\beta = 1$) for three values of effective FWHM $\gamma = 0.1\Gamma$, Γ, and 10Γ. For narrow-band excitation, the results of the two models are essentially identical implying the insensitivity of the dynamics to the line shape which appears monochromatic. With increasing bandwidths, however, the Rabi oscillations in $\langle G^{(2)}(t,\tau)\rangle$ are seen to remain more pronounced in the EPDM compared to those in PDM. Viewing the problem in the time domain, reduction of β slows down the fluctuations which in turn reduces the damping of the Rabi oscillations. In the complementary frequency domain, this effect can be understood by realizing the fact that reduction of β suppresses the wings of the spectrum compared to the

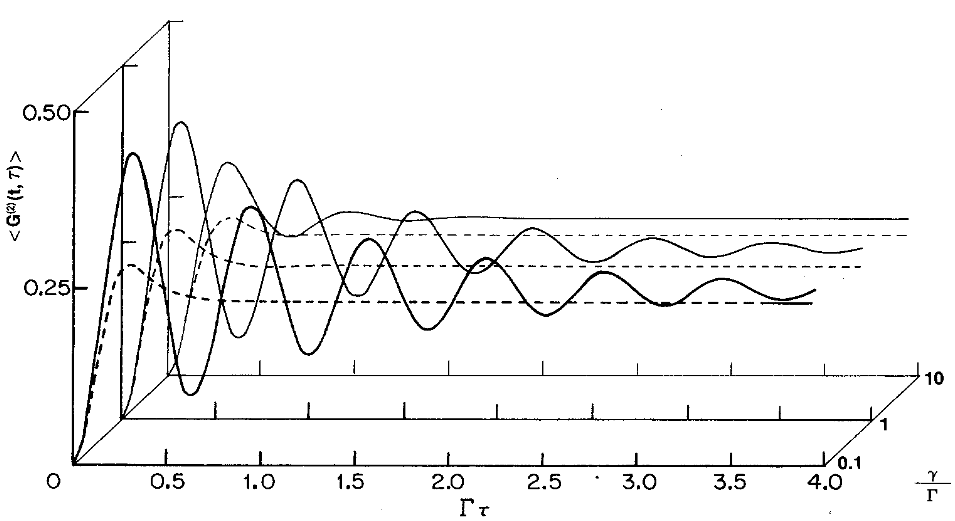

FIG. 2. IC under strong excitation by finite bandwidth chaotic field (dashed line) and the PDM field (solid line) mean Rabi frequency is $\bar{\omega}_R = 10\Gamma$. Effective FWHM ($\gamma$) of the field are indicated against each set of curves.

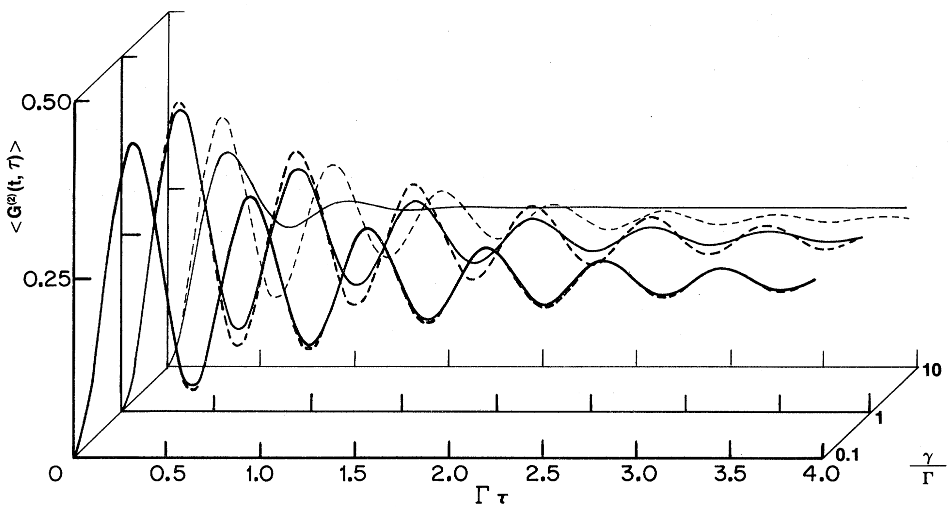

FIG. 3. IC under strong excitation by finite bandwidth EPDM field (dashed line) and the PDM field (solid line) mean Rabi frequency is $\bar{\omega}_R = 10\Gamma$. Effective FWHM ($\gamma$) of the field are indicated against each set of curves.

$\beta \rightarrow \infty$ case, thereby making the line shape appear more and more monochromatic. This in turn pushes $\langle G^{(2)}(t,\tau) \rangle$ towards the monochromatic excitation limit where the oscillations are damped on the time scale Γ^{-1}.

In conclusion, we have presented the theory of intensity correlations in resonance fluorescence in the presence of incoherent exciting fields. The analysis presented here is valid for arbitrary strengths and arbitrary bandwidths of the exciting field and, in a sense, supersedes earlier analyses that are applicable in a limited region of the parameters space. The sensitivity of the IC to the laser line shape is illustrated by comparing the results of the extended phase diffusion model that yields a non-Lorentzian line shape with the results for a Lorentzian line-shape phase diffusion model. Under broadband excitation condition, the slower fluctuations in the former are seen to reduce the damping of Rabi oscillations in $\langle G^{(2)}(t,\tau) \rangle$. The amplitude fluctuations in the exciting field wash out these oscillations completely, even for narrow bandwidth fields. In all these models, the antibunching property of the intensity correlation ($\langle G^{(2)}(t,\tau) \rangle = 0$) is preserved as this arises due to the nature of the emission process itself and does not depend on the fluctuation properties of the field.

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