Macroscopic theory of electromagnetic fluctuations and stationary radiative heat transfer

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For the macroscopic description of thermal electromagnetic fluctuations two methods are frequently used: (i) linear-response theory and the fluctuation-dissipation theorem (FDT of the first kind) and (ii) Langevin theory, i.e., stochastic electrodynamics (FDT of the second kind). The two methods are compared. The identity of both theories in global thermal equilibrium is proven. This identity is expressed by relations between the imaginary and real parts of the Green functions on the one side and volume integrations over products of two Green functions on the other side. The correct treatment of infinitely extended vacuum regions with respect to these integrations is discussed. The local interpretation of the two methods in open systems leads to different results: The Poynting vector calculated by the FDT of the first kind vanishes identically, whereas the FDT of the second kind results in a nonvanishing radiative heat transfer. The basic assumptions which are responsible for the different results are extensively discussed. Simple examples are dealt with to make contact with the phenomenological theory of radiative heat transfer.

I. INTRODUCTION

The basic ansatz for the macroscopic description of thermal electromagnetic (EM) fluctuations was developed by Leontovich and Rytov.[1] In this approach it is assumed that the macroscopic and classically interpreted EM fields $(\vec{E},\vec{H})$ are driven by external stochastic forces, i.e., by a fluctuating current distribution $(\vec{j})$ or, alternatively, by fluctuating polarization and magnetization fields $(\vec{P},\vec{M})$. Therefore, the Maxwell equations become Langevin equations.

It is obvious that the spectra of these forces must be determined by the thermal motion of the atoms or molecules out of which the condensed matter system is built up (temperature T). Consequently, on a macroscopic scale the correlation length of the force correlation functions may be set equal to zero $[\sim\delta(\vec{r}-\vec{r}\,')]$ if the dominant atomic interaction range is of the order of the atomic distances. The fluctuation strength is found by the requirement that the condensed matter system should radiate in accordance with Kirchhoff's radiation law.

The above sketched theory is a macroscopic (the matter is characterized by the conductivity σ or the electric and magnetic permeability ϵ and μ) semiclassical theory: The EM fields are described by the classical Maxwell equations, whereas the spectra of the fluctuating forces are considered as ensemble averages with respect to the equilibrium density operator of the condensed matter system alone.

Consequently, in the original theory of Rytov[2] there are no quantum electrodynamical (QED) vacuum fluctuation parts in the correlation functions of the random forces. This interpretation is only possible if the quantities σ, ϵ, and μ are mainly determined by the short-wave components of the EM field, i.e., the components for which the retardation may be neglected (i.e., $\lambda \lesssim \alpha$, where α denotes the interatomic distance) and which cause the formation of the condensed matter system out of its atoms (see Ref. 3, Secs. 75 and 80).

Starting from Rytov's theory Polder and Van Hove[4] and Caren[5] calculated the radiative heat transfer between closely spaced media (metals) of different temperatures. In these works the assumptions on which the local equilibrium interpretation of Rytov's theory is based were not fully discussed.

A quantum electrodynamical theory must consider the EM fields, the matter, and their interaction quantum mechanically; therefore, it is obvious that in thermal equilibrium such a theory cannot produce any radiative heat transfer. The macroscopic QED theory for the description of thermal EM fluctuations was given by Case and Chiu[6] in the special case of cavities with perfectly reflecting walls and, more generally, by Agarwal[7] and Landau and Lifschitz (Ref. 3, Sec. 76). The main point in this procedure which avoids an explicit quantization of the EM fields is the interpretation of the Maxwell equations as linear-response equations: The expectation values of the field operators $(\langle\hat{\vec{E}}\rangle,\langle\hat{\vec{H}}\rangle)$ respond to the external forces $\vec{P}$ and $\vec{M}$. The perturbation Hamiltonian is given by $\hat{H}_1=\int d^3r\{\hat{\vec{E}}\cdot\vec{P}+\hat{\vec{H}}\cdot\vec{M}\}$.

Therefore, the Green functions (tensors) of the Maxwell equations are interpreted as the commutators of the EM field which are averaged with respect to the equilibrium ensemble. This interpretation is possible due to the c-number properties of the commutators in the linear regime. Via the fluctuation-dissipation theorem[8–10] (FDT) the averaged commutators are connected with the averaged anticommutators, i.e., with the correlation functions. The thermal equilibrium is characterized by the density operator $\hat{\rho}_0\sim\exp(-\hat{H}_0/k_BT)$. The Hamiltonian $\hat{H}_0$ consists of three parts: The condensed-matter part $(\hat{H}_M)$, the long-wave $(\lambda \gg \alpha)$ radiation part $(\hat{H}_R)$, and the interaction part $(\hat{H}_{\rm int})$. Detailed balance exists everywhere in the system and, consequently, there is no radiative heat transfer. (Applications of this theory can be found, for example, in Refs. 7 and 11—20.)

In this macroscopic theory the averaged EM-field commutators are expressed by the quantities ϵ, μ, or σ. If we think of the microscopic (e.g., graph theoretical) calculation of these quantities,[3] in principle all field components and not only the nonretarded short-wave ($\lambda \lesssim \alpha$) components must be taken into account.

In a earlier edition of the textbook of Landau and Lifschitz[21] the zero-point vacuum fluctuations are included in Rytov's theory (neoclassical theory,[22–25] FDT of the second kind) and this extended theory is connected with the inverse formulation of the FDT (FDT of the first kind). It is stated that both formulations—in our notation we refer to Kubo's definition of the first and second FDT (Ref. 26)—lead to identical results.

In a recent paper[27] we already emphasized that identical results are only obtained if the correct succession of operations is observed. The identity of both methods excludes the possibility of calculating any radiative heat transfer via the second FDT.

So far as we know there exists no detailed analysis of the interrelations between both methods and of the assumptions on which they are based. Therefore, it may be useful to discuss in this paper these interrelations and to stress the involved different interpretations of the macroscopic Maxwell equations. In our discussion spatial dispersion and anisotropy of the electric (magnetic) permeability will be included.

Our paper is organized as follows. After the formal definition of the Green functions and after the discussion of their symmetries and their completeness relations we will sketch in the third section the FDT of the first kind. In Secs. IV and V the FDT of the second kind is formulated and the identity of both theorems in inhomogeneous bounded systems is proven. In open inhomogeneous systems this equivalence implies the correct succession of limiting processes and the demand for global thermal equilibrium. In Sec. VI the local meaning of both theorems in inhomogeneous systems is discussed. "Local" means that the temperature is only defined inside the dissipative part of the system. The reason of different results which are obtained in open systems is dealt with. In Sec. VII a theory of stationary radiative heat transfer is proposed and the basic assumptions are discussed. By this, the calculations made by Polder and Van Hove[4] and Caren[5] get their reasoning. As a simple example we consider in Sec. VIII the dielectric half-space. We will especially stress the question if Kirchhoff's radiation law is also valid for freely radiating bodies, i.e., for bodies which are not in thermal equilibrium with the surrounding radiation.[28–30]

II. MAXWELL EQUATIONS AND GREEN FUNCTIONS

The Fourier-transformed Maxwell equations are basic for both theorems:

$$\vec{\nabla}\times\vec{E}(\vec{r},\omega)=\frac{i\omega}{c}\vec{B}(\vec{r},\omega)\ , \tag{2.1}$$

$$\vec{\nabla}\times\vec{H}(\vec{r},\omega)=-\frac{i\omega}{c}\vec{D}(\vec{r},\omega)\ , \tag{2.2}$$

$$\vec{\nabla}\cdot\vec{D}(\vec{r},\omega)=0\ , \tag{2.3}$$

$$\vec{\nabla}\cdot\vec{B}(\vec{r},\omega)=0\ . \tag{2.4}$$

We include temporal and spatial dispersion and we assume that the external forces are represented by polarization and magnetization fields.

In the linear regime, to which we will restrict ourselves in this paper, the constitutive equations take the following form:

$$\vec{D}(\vec{r},\omega)=\int_V d^3r'\overleftrightarrow{\epsilon}(\vec{r},\vec{r}\,',\omega)\cdot\vec{E}(\vec{r}\,',\omega)+4\pi\vec{P}(\vec{r},\omega)\ , \tag{2.5}$$

$$\vec{B}(\vec{r},\omega)=\int_V d^3r'\overleftrightarrow{\mu}(\vec{r},\vec{r}\,',\omega)\cdot\vec{H}(\vec{r}\,',\omega)+4\pi\vec{M}(\vec{r},\omega)\ . \tag{2.6}$$

The particular solutions of (2.1)—(2.6) with respect to the appropriate boundary conditions are determined by the Green functions:

$$\vec{E}(\vec{r},\omega)=\int_V d^3r'[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r},\vec{r}\,',\omega)\cdot\vec{P}(\vec{r}\,',\omega)+\overleftrightarrow{\mathcal{G}}^{EH}(\vec{r},\vec{r}\,',\omega)\cdot\vec{M}(\vec{r}\,',\omega)]\ , \tag{2.7}$$

$$\vec{H}(\vec{r},\omega)=\int_V d^3r'[\overleftrightarrow{\mathcal{G}}^{HE}(\vec{r},\vec{r}\,',\omega)\cdot\vec{P}(\vec{r}\,',\omega)+\overleftrightarrow{\mathcal{G}}^{HH}(\vec{r},\vec{r}\,',\omega)\cdot\vec{M}(\vec{r}\,',\omega)]\ . \tag{2.8}$$

The inversion of (2.7) and (2.8) yields

$$\vec{P}(\vec{r},\omega)=\int_V d^3r'\{[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r},\vec{r}\,',\omega)]^{-1}\cdot\vec{E}(\vec{r}\,',\omega)+[\overleftrightarrow{\mathcal{G}}^{EH}(\vec{r},\vec{r}\,',\omega)]^{-1}\cdot\vec{H}(\vec{r}\,',\omega)\}\ , \tag{2.9}$$

$$\vec{M}(\vec{r},\omega)=\int_V d^3r'\{[\overleftrightarrow{\mathcal{G}}^{HE}(\vec{r},\vec{r}\,',\omega)]^{-1}\cdot\vec{E}(\vec{r}\,',\omega)+[\overleftrightarrow{\mathcal{G}}^{HH}(\vec{r},\vec{r}\,',\omega)]^{-1}\cdot\vec{H}(\vec{r}\,',\omega)\}\ . \tag{2.10}$$

In consequence of the principle of microscopic reversibility the linear-response theory postulates the symmetry relations:

$$\mathcal{G}_{ik}^{\{EE\atop HH\}}(\vec{r},\vec{r}\,',\omega)=\mathcal{G}_{ki}^{\{EE\atop HH\}}(\vec{r}\,',\vec{r},\omega)\ , \tag{2.11}$$

$$\mathcal{G}_{ik}^{\{EH\atop HE\}}(\vec{r},\vec{r}\,',\omega)=-\mathcal{G}_{ki}^{\{HE\atop EH\}}(\vec{r}\,',\vec{r},\omega)\ , \tag{2.12}$$

$$\{[\overleftrightarrow{\mathcal{G}}^{\{EE\atop HH\}}(\vec{r},\vec{r}\,',\omega)]^{-1}\}_{ik}=\{[\overleftrightarrow{\mathcal{G}}^{\{EE\atop HH\}}(\vec{r}\,',\vec{r},\omega)]^{-1}\}_{ki}\ , \tag{2.13}$$

$$\{[\overleftrightarrow{\mathcal{G}}^{\{EH\atop HE\}}(\vec{r},\vec{r}\,',\omega)]^{-1}\}_{ik}=-\{[\overleftrightarrow{\mathcal{G}}^{\{HE\atop EH\}}(\vec{r}\,',\vec{r},\omega)]^{-1}\}_{ki}\ . \tag{2.14}$$

Furthermore, for all Green functions (2.11)—(2.14) the relation

$$\overleftrightarrow{\mathcal{G}}(\vec{r},\vec{r}\,',\omega)=\overleftrightarrow{\mathcal{G}}^{*}(\vec{r},\vec{r}\,',-\omega) \tag{2.15}$$

is valid (the time-dependent fields are real). The insertion of (2.9) and (2.10) in (2.7) and (2.8) leads to completeness relations of the Green functions.

By the comparison of (2.1) and (2.2) with (2.9) and (2.10) we can read off the inverse Green functions directly:

$$[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r},\vec{r}\,',\omega)]^{-1}=-\frac{1}{4\pi}\overleftrightarrow{\epsilon}(\vec{r},\vec{r}\,',\omega)\ , \tag{2.16}$$

$$[\overleftrightarrow{\mathcal{G}}^{HH}(\vec{r},\vec{r}\,',\omega)]^{-1}=-\frac{1}{4\pi}\overleftrightarrow{\mu}(\vec{r},\vec{r}\,',\omega)\ , \tag{2.17}$$

$$\{[\overleftrightarrow{\mathcal{G}}^{EH}(\vec{r},\vec{r}\,',\omega)]^{-1}\}_{ik}=\frac{ic}{4\pi\omega}\epsilon_{ilk}\frac{\partial}{\partial x_l}\delta(\vec{r}-\vec{r}\,')\ , \tag{2.18}$$

$$\{[\overleftrightarrow{\mathcal{G}}^{HE}(\vec{r},\vec{r}\,',\omega)]^{-1}\}_{ik}=-\frac{ic}{4\pi\omega}\epsilon_{ilk}\frac{\partial}{\partial x_l}\delta(\vec{r}-\vec{r}\,')\ . \tag{2.19}$$

The relations (2.11)–(2.14) may be interpreted in a different manner. The symmetries of ϵ and μ—$\epsilon_{ik}(\vec{r},\vec{r}\,',\omega)=\epsilon_{ki}(\vec{r}\,',\vec{r},\omega)$ and $\mu_{ik}(\vec{r},\vec{r}\,',\omega)=\mu_{ki}(\vec{r}\,',\vec{r},\omega)$ (generalized susceptibilities)—and the structure of the Maxwell equations cause the relations (2.13) and (2.14) and, consequently, the relations (2.11) and (2.12) are also valid.

The expressions (2.18) and (2.19) are purely imaginary while (2.16) and (2.17) are complex. Therefore, we find

$$\frac{1}{2i}(\{[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r},\vec{r}\,',\omega)]^{-1}\}^*_{ik}-\{[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r}\,',\vec{r},\omega)]^{-1}\}_{ki})=\frac{1}{4\pi}\epsilon''_{ik}(\vec{r},\vec{r}\,',\omega)\ , \tag{2.20}$$

$$\frac{1}{2i}(\{[\overleftrightarrow{\mathcal{G}}^{HH}(\vec{r},\vec{r}\,',\omega)]^{-1}\}^*_{ik}-\{[\overleftrightarrow{\mathcal{G}}^{HH}(\vec{r}\,',\vec{r},\omega)]^{-1}\}_{ki})=\frac{1}{4\pi}\mu''_{ik}(\vec{r},\vec{r}\,',\omega)\ , \tag{2.21}$$

$$\frac{1}{2i}(\{[\overleftrightarrow{\mathcal{G}}^{EH}(\vec{r},\vec{r}\,',\omega)]^{-1}\}^*_{ik}-\{[\overleftrightarrow{\mathcal{G}}^{HE}(\vec{r}\,',\vec{r},\omega)]^{-1}\}_{ki})=0\ . \tag{2.22}$$

In the following we will assume that the considered system consists of spatially dispersive dielectric (magnetic) regions (volumes, V_{1i}) which are separated by sharp boundaries from vacuum regions (V_2):

$$\epsilon_{ik}(\vec{r},\vec{r}\,',\omega)=\begin{cases}\epsilon^{(1i)}_{ik}(\vec{r},\vec{r}\,',\omega) & \text{for } \vec{r},\vec{r}\,'\in V_{1i}\ ,\ \ i=1,2,\ldots,N\\ \delta(\vec{r}-\vec{r}\,') & \text{for } \vec{r},\vec{r}\,'\in V_2\\ 0, & \text{otherwise}\end{cases} \tag{2.23}$$

(we omit the analogous expression for the magnetic susceptibility).

III. FLUCTUATION-DISSIPATION THEOREM OF THE FIRST KIND

The electric and magnetic fields in (2.7) and (2.8) are interpreted as mean values. These mean values vanish if the external forces $\vec{P}$ and $\vec{M}$ are absent $[\vec{E}\to\langle\hat{\vec{E}}\rangle_0=\mathrm{tr}(\hat{\rho}_0\hat{\vec{E}}),\ \hat{\rho}_0\sim\exp(-\hat{H}_0/k_BT)]$. For nonvanishing forces we obtain the linear response equations (2.7) and (2.8) $(\langle\hat{\vec{E}}\rangle=\mathrm{tr}(\hat{\rho}\hat{\vec{E}}),\ \hat{\rho}\sim\exp[-(\hat{H}_0+\hat{H}_1)/k_BT],\ \hat{H}_1=\int_V d^3r(\hat{\vec{E}}\cdot\vec{P}+\hat{\vec{H}}\cdot\vec{M}))$.

A comparison with the linear-response theory reveals that the Green functions are considered as commutators of the EM field which are averaged with respect to the undisturbed density operator $\hat{\rho}_0$. In our case the linear response is the exact response. Therefore, the commutators are c numbers and the averaged commutators are independent of the temperature.

The FDT connects the symmetrized correlation functions with the averaged commutators and we obtain[7]

$$\langle\tfrac{1}{2}\{\hat{E}_i(\vec{r},t),\ \hat{E}_j(\vec{r}\,',t')\}\rangle\equiv E_{ij}(\vec{r},\vec{r}\,',\tau=t-t')=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\mathrm{Im}\mathcal{G}^{EE}_{ij}(\vec{r},\vec{r}\,',\omega)\ , \tag{3.1}$$

$$\langle\tfrac{1}{2}\{\hat{H}_i(\vec{r},t),\ \hat{H}_j(\vec{r}\,',t')\}\rangle\equiv H_{ij}(\vec{r},\vec{r}\,',\tau)=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\mathrm{Im}\mathcal{G}^{HH}_{ij}(\vec{r},\vec{r}\,',\omega)\ , \tag{3.2}$$

$$\langle\tfrac{1}{2}\{\hat{E}_i(\vec{r},t),\ \hat{H}_j(\vec{r}\,',t')\}\rangle\equiv M_{ij}(\vec{r},\vec{r}\,',\omega)=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right][-i\,\mathrm{Re}\mathcal{G}^{EH}_{ij}(\vec{r},\vec{r}\,',\omega)]\ . \tag{3.3}$$

All higher correlation functions can be reduced to the quadratic correlation functions (3.1)–(3.3).[14] This decomposition is characteristic for a stochastic Gaussian process and is caused by the c-number property of the field commutators.

The relation (2.15) allows us to define the spectra in (3.1)–(3.3) with respect to the positive frequency part:

$$E_{ij}(\vec{r},\vec{r}\,',\tau)=\int_0^{\infty}\frac{d\omega}{\pi}\cos(\omega\tau)\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\times\mathrm{Im}\mathcal{G}^{EE}_{ij}(\vec{r},\vec{r}\,',\omega)\ , \tag{3.4}$$

$$H_{ij}(\vec{r},\vec{r}\,',\tau)=\int_0^{\infty}\frac{d\omega}{\pi}\cos(\omega\tau)\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\times\mathrm{Im}\mathcal{G}^{HH}_{ij}(\vec{r},\vec{r}\,',\omega)\ , \tag{3.5}$$

$$M_{ij}(\vec{r},\vec{r}\,',\tau)=\int_0^{\infty}\frac{d\omega}{\pi}\sin(\omega\tau)\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\times[-\mathrm{Re}\mathcal{G}^{EH}_{ij}(\vec{r},\vec{r}\,',\omega)]\ . \tag{3.6}$$

As a consequence of (3.6) the averaged Poynting vector vanishes:

$$\langle \hat{S}_i(\vec{r})\rangle = \frac{c}{4\pi}\epsilon_{ijk}M_{jk}(\vec{r},\vec{r},\tau=0)$$
$$= \frac{c}{4\pi}\epsilon_{ijk}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]$$
$$\times[-i\,\mathrm{Re}\mathscr{G}_{ij}^{EH}(\vec{r},\vec{r},\omega)]\ . \quad (3.7)$$

For $\tau=0$ the positive frequency $(+\omega)$ contributions cancel with the negative frequency $(-\omega)$ contributions in the spectrum of (3.3) (detailed balance).

For lossless systems the calculation of (3.4)—(3.6) must also include the fundamental requirement of causality (Kramers-Kronig relations for the generalized susceptibilities). This requirement can be achieved if we assume dissipation and take the limit $\epsilon''\to 0$ (and/or $\mu''\to 0$) in the suited expressions. This is nothing else but the insertion and extraction of the famous dust particle.

IV. FLUCTUATION-DISSIPATION THEOREM OF THE SECOND KIND

Here the EM fields are interpreted as stochastic variables which are driven by the Langevin forces $\vec{P}$ and $\vec{M}$. These forces describe a stochastic Gaussian process, i.e., the knowledge of the quadratic force correlation functions is enough to describe the process completely. Generalizing the formulas of Landau and Lifschitz[21] we can write

$$[\vec{P}(\vec{r},\omega)]_{av}\equiv[\vec{M}(\vec{r},\omega)]_{av}\equiv 0\ , \quad (4.1)$$

$$[P_i(\vec{r},\omega)P_j(\vec{r}\,',\omega')]_{av}=\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\epsilon_{ij}''(\vec{r},\vec{r}\,',\omega)$$
$$\times\delta(\omega+\omega')\ , \quad (4.2)$$

$$[M_i(\vec{r},\omega)M_j(\vec{r}\,',\omega')]_{av}=\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\mu_{ij}''(\vec{r},\vec{r}\,',\omega)$$
$$\times\delta(\omega+\omega')\ , \quad (4.3)$$

$$[P_i(\vec{r},\omega)M_j(\vec{r}\,',\omega')]_{av}\equiv 0\ . \quad (4.4)$$

[For the symmetries of (4.2) and (4.3) see the remarks following Eq. (2.16).]

We form the expression

$$E_i(\vec{r},t)E_j(\vec{r}\,',t')=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}e^{-i\omega t}e^{-i\omega' t'}$$
$$\times E_i(\vec{r},\omega)E_j(\vec{r}\,',\omega') \quad (4.5)$$

and insert in the right-hand side the particular solutions (2.7) of the classical Maxwell equations. We average with respect to the stochastic process (4.1)—(4.4) and use (2.15). We obtain

$$[E_i(\vec{r}_1,t)E_j(\vec{r}_2,t')]_{av}\equiv[E_{ij}(\vec{r}_1,\vec{r}_2,\tau)]_{av}$$
$$=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}\frac{1}{4\pi}\int_V d^3r\int_V d^3r'\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]$$
$$\times[\mathscr{G}_{ik}^{EE}(\vec{r}_1,\vec{r},\omega)\epsilon_{kl}''(\vec{r},\vec{r}\,',\omega)\mathscr{G}_{jl}^{EE*}(\vec{r}_2,\vec{r}\,',\omega)$$
$$+\mathscr{G}_{ik}^{EH}(\vec{r}_1,\vec{r},\omega)\mu_{kl}''(\vec{r},\vec{r}\,',\omega)\mathscr{G}_{jl}^{EH*}(\vec{r}_2,\vec{r}\,',\omega')]\ . \quad (4.6)$$

The same procedure takes us to the formulas

$$[H_i(\vec{r}_1,t)H_j(\vec{r}_2,t')]_{av}\equiv[H_{ij}(\vec{r}_1,\vec{r}_2,\tau)]_{av}$$
$$=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}\frac{1}{4\pi}\int_V d^3r\int_V d^3r'\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]$$
$$\times[\mathscr{G}_{ik}^{HE}(\vec{r}_1,\vec{r},\omega)\epsilon_{kl}''(\vec{r},\vec{r}\,',\omega)\mathscr{G}_{jl}^{HE*}(\vec{r}_2,\vec{r}\,',\omega)$$
$$+\mathscr{G}_{ik}^{HH}(\vec{r}_1,\vec{r},\omega)\mu_{kl}''(\vec{r},\vec{r}\,',\omega)\mathscr{G}_{jl}^{HH*}(\vec{r}_2,\vec{r}\,',\omega)]\ , \quad (4.7)$$

$$[E_i(\vec{r}_1,t)H_j(\vec{r}_2,t')]_{av}\equiv[M_{ij}(\vec{r}_1,\vec{r}_2,\tau)]_{av}$$
$$=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}\frac{1}{4\pi}\int_V d^3r\int_V d^3r'\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]$$
$$\times[\mathscr{G}_{ik}^{EE}(\vec{r}_1,\vec{r},\omega)\epsilon_{kl}''(\vec{r},\vec{r}\,',\omega)\mathscr{G}_{jl}^{HE*}(\vec{r}_2,\vec{r}\,',\omega)$$
$$+\mathscr{G}_{ik}^{EH}(\vec{r}_1,\vec{r},\omega)\mu_{kl}''(\vec{r},\vec{r}\,',\omega)\mathscr{G}_{jl}^{HH*}(\vec{r}_2,\vec{r}\,',\omega)]\ . \quad (4.8)$$

The left-hand sides of (4.6)—(4.8) have to be real and the frequency integration range can be restricted to the positive part, i.e.,

$$[A_i(\vec{r}_1,t)B_j(\vec{r}_2,t')]_{\rm av}=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}f(\vec{r}_1,\vec{r}_2,\omega)=\int_0^{\infty}\frac{d\omega}{\pi}{\rm Re}[e^{-i\omega\tau}f(\vec{r}_1,\vec{r}_2,\omega)] \ . \tag{4.9}$$

The relation (2.15) guarantees the condition $f(\vec{r}_1,\vec{r}_2,\omega)=-f^*(\vec{r}_1,\vec{r}_2,-\omega)$ in (4.6)—(4.8).

The averaged Poynting vector can be written in accordance to (3.7):

$$[S_i(\vec{r})]_{\rm av}=\frac{c}{4\pi}\epsilon_{ijk}[M_{jk}(\vec{r},\vec{r},\tau=0)]_{\rm av}$$
$$=\frac{c}{4\pi}\epsilon_{ijk}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{4\pi}\int_V d^3r'\int_V d^3r''\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right][\mathcal{G}^{EE}_{jl}(\vec{r},\vec{r}',\omega)\epsilon''_{lm}(\vec{r}',\vec{r}'',\omega)\mathcal{G}^{HE*}_{km}(\vec{r},\vec{r}'',\omega)$$
$$+\mathcal{G}^{EH}_{jl}(\vec{r},\vec{r}',\omega)\mu''_{lm}(\vec{r}',\vec{r}'',\omega)\mathcal{G}^{HH*}_{km}(\vec{r},\vec{r}'',\omega)] \ . \tag{4.10}$$

In contrast to (3.4)—(3.6) the τ dependence of (4.9) cannot be described by a pure cosine or sine behavior. Only the imaginary and real parts of the Green functions have a well-defined parity with respect to the frequency and not the products of Green functions appearing in (4.6)—(4.8).

The application of (4.1)—(4.4) has to be taken with care. In lossless systems or in systems with unbounded vacuum regions, i.e., the distance between vacuum regions and dissipative regions becomes infinite, the limit of vanishing dissipation in the vacuum regions must be taken *after* the integration over the vacuum regions has been performed in (4.6)—(4.8).[27] If the vacuum regions are bounded (e.g., by perfectly conducting walls) and if dissipation is possible in the system the imaginary parts of the permeabilities ϵ and μ may be set equal to zero in these regions from the beginning. To discuss this problem in more detail we start from (2.23). We split up the volume integrations in (4.6)—(4.8) in two parts ($\sum_i V_{1i}=V_1$, $V=V_1+V_2$):

$$\int_V d^3r\int_V d^3r'\{\overleftrightarrow{\mathcal{G}}:\overleftrightarrow{\epsilon}'':\overleftrightarrow{\mathcal{G}}^{*T}+\overleftrightarrow{\mathcal{G}}:\overleftrightarrow{\mu}'':\overleftrightarrow{\mathcal{G}}^{*T}\}=I_1+I_2 \ , \tag{4.11}$$

where

$$I_1=\int_{V_1}d^3r\int_{V_1}d^3r'\{\overleftrightarrow{\mathcal{G}}(\vec{r}_1,\vec{r}):\overleftrightarrow{\epsilon}^{(1)''}(\vec{r},\vec{r}'):\overleftrightarrow{\mathcal{G}}^{*T}(\vec{r}_2,\vec{r}')+\overleftrightarrow{\mathcal{G}}(\vec{r}_1,\vec{r}):\overleftrightarrow{\mu}^{(1)''}(\vec{r},\vec{r}'):\overleftrightarrow{\mathcal{G}}^{*T}(\vec{r}_2,\vec{r}')\} \tag{4.12}$$

and

$$I_2=\lim_{\epsilon^{(2)}\to 1,\ \mu^{(2)}\to 1}\int_{V_2}d^3r\{\epsilon^{(2)''}\overleftrightarrow{\mathcal{G}}(\vec{r}_1,\vec{r}):\overleftrightarrow{\mathcal{G}}^{*T}(\vec{r}_2,\vec{r})+\mu^{(2)''}\overleftrightarrow{\mathcal{G}}(\vec{r}_1,\vec{r}):\overleftrightarrow{\mathcal{G}}^{*T}(\vec{r}_2,\vec{r})\} \ . \tag{4.13}$$

(For simplicity we omitted the irrelevant arguments and notations.) In the Green functions in (4.11) for which all $\vec{r}$ and $\vec{r}'$ are elements of V_1 [first part of (4.11)] $\epsilon^{(2)''}$ and $\mu^{(2)''}$ may be set equal to zero.

In (4.11) we can distinguish four different cases.

(i) Homogeneous systems with dissipation[16–18]:

$$I_1\neq 0,\quad I_2=0,\quad V=V_1 \ .$$

(ii) Homogeneous systems without dissipation[12,16–18]:

$$I_1=0,\quad I_2\neq 0,\quad V=V_2 \ .$$

(iii) Inhomogeneous bounded systems[27]

$$I_1\neq 0,\quad I_2=0,\quad V=V_1+V_2 \ .$$

(iv) Inhomogeneous unbounded systems[20,27]:

$$I_1\neq 0,\quad I_2\neq 0,\quad V=V_1+V_2 \ .$$

The application of the FDT of the second kind has the advantage that the contributions to the correlations can be distinguished with respect to their origin from different dissipative regions of the system. We have mentioned above that this interpretation must be handled with care. Once more we want to emphasize that in (4.6)—(4.8) the Green functions signify the classical particular solutions of the inhomogeneous Maxwell equations. For the discussion of nonmagnetic systems we simply may replace $\overleftrightarrow{\mathcal{G}}^{HH}$ and $\overleftrightarrow{\mathcal{G}}^{EH}$ by zero in the formulas (4.6)—(4.8) and (4.10). In Appendix B this procedure is justified extensively.

V. FDT'S IN GLOBAL THERMODYNAMIC EQUILIBRIUM

In the global thermodynamic equilibrium (i.e., at every point of the system the same temperature T is defined) we expect that both theorems lead to identical results. By comparison of (3.1)—(3.3) with (4.6)—(4.8) we find the conditions (the factor $\hbar\coth\hbar\omega/2k_BT$ can be written in front of the volume integrations):

$${\rm Im}\mathcal{G}^{EE}_{ij}(\vec{r}_1,\vec{r}_2,\omega)=\frac{1}{4\pi}\int_V d^3r'\int_V d^3r''[\mathcal{G}^{EE}_{ik}(\vec{r}_1,\vec{r}',\omega)\epsilon''_{kl}(\vec{r}',\vec{r}'',\omega)\mathcal{G}^{EE*}_{jl}(\vec{r}_2,\vec{r}'',\omega)$$
$$+\mathcal{G}^{EH}_{ik}(\vec{r}_1,\vec{r}',\omega)\mu''_{kl}(\vec{r}',\vec{r}'',\omega)\mathcal{G}^{EH*}_{jl}(\vec{r}_2,\vec{r}'',\omega)] \ , \tag{5.1}$$

$$\mathrm{Im}\mathcal{G}_{ij}^{HH}(\vec{r}_1,\vec{r}_2,\omega)=\frac{1}{4\pi}\int_V d^3r'\int_V d^3r''[\mathcal{G}_{ik}^{HE}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''_{kl}(\vec{r}\,',\vec{r}\,'',\omega)\mathcal{G}_{jl}^{HE*}(\vec{r}_2,\vec{r}\,'',\omega)$$
$$+\mathcal{G}_{ik}^{HH}(\vec{r}_1,\vec{r}\,',\omega)\mu''_{kl}(\vec{r}\,',\vec{r}\,'',\omega)\mathcal{G}_{jl}^{HH*}(\vec{r}_2,\vec{r}\,'',\omega)]\ , \tag{5.2}$$

$$-i\,\mathrm{Re}\mathcal{G}_{ij}^{EH}(\vec{r}_1,\vec{r}_2,\omega)=\frac{1}{4\pi}\int_V d^3r'\int_V d^3r''[\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''_{kl}(\vec{r}\,',\vec{r}\,'',\omega)\mathcal{G}_{jl}^{HE*}(\vec{r}_2,\vec{r}\,'',\omega)$$
$$+\mathcal{G}_{ik}^{EH}(\vec{r}_1,\vec{r}\,',\omega)\mu''_{kl}(\vec{r}\,',\vec{r}\,'',\omega)\mathcal{G}_{jl}^{HH*}(\vec{r}_2,\vec{r}\,'',\omega)]\ . \tag{5.3}$$

The relations (5.1)—(5.3) can easily be proven.

(i) We replace ϵ''_{ik} and μ''_{ik} by the inverse Green functions according to (2.20)—(2.22).

(ii) We use the completeness relations among the Green functions, which are obtained by inserting (2.9) and (2.10) in (2.7) and (2.8).

(iii) We use the symmetry relations (2.11)—(2.15) and we note that the inverse Green functions (2.18) and (2.19) are purely imaginary.

This proof is explicitly performed in Appendix A. (For nonmagnetic systems see Appendix B.)

We split up the right-hand sides of (5.1)—(5.3) in the two parts (4.12) and (4.13). In general, both parts will be complex. The expressions (5.1) and (5.2) are purely real while (5.3) is purely imaginary. Therefore, we obtain the following relations (for simplicity we restrict to nonmagnetic media):

$$\mathrm{Im}\left[\frac{1}{4\pi}\int_{V_1}d^3r'\int_{V_1}d^3r''\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''_{kl}(\vec{r}\,',\vec{r}\,'',\omega)\mathcal{G}_{jl}^{EE*}(\vec{r}_2,\vec{r}\,'',\omega)\right]$$
$$=-\mathrm{Im}\left[\frac{1}{4\pi}\lim_{\epsilon''\to 0}\int_{V_2}d^3r'\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''(\omega)\mathcal{G}_{jk}^{EE*}(\vec{r}_2,\vec{r}\,',\omega)\right]\ , \tag{5.4}$$

$$\mathrm{Im}\left[\frac{1}{4\pi}\int_{V_1}d^3r'\int_{V_1}d^3r''\mathcal{G}_{ik}^{HE}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''_{kl}(\vec{r}\,',\vec{r}\,'',\omega)\mathcal{G}_{jl}^{HE*}(\vec{r}_2,\vec{r}\,'',\omega)\right]$$
$$=-\mathrm{Im}\left[\frac{1}{4\pi}\lim_{\epsilon''\to 0}\int_{V_2}d^3r'\mathcal{G}_{ik}^{HE}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''(\omega)\mathcal{G}_{jk}^{HE*}(\vec{r}_2,\vec{r}\,',\omega)\right]\ , \tag{5.5}$$

$$\mathrm{Re}\left[\frac{1}{4\pi}\int_{V_1}d^3r'\int_{V_1}d^3r''\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''_{kl}(\vec{r}\,',\vec{r}\,'',\omega)\mathcal{G}_{jl}^{HE*}(\vec{r}_2,\vec{r}\,'',\omega)\right]$$
$$=-\mathrm{Re}\left[\frac{1}{4\pi}\lim_{\epsilon''\to 0}\int_{V_2}d^3r'\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''(\omega)\mathcal{G}_{jk}^{HE*}(\vec{r}_2,\vec{r}\,',\omega)\right]\ . \tag{5.6}$$

Since no confusion is possible we have omitted in (5.4)—(5.6) the upper indices of the dielectric function. The limit $\epsilon''\to 0$ in (5.4)—(5.6) corresponds to the limiting process performed in (4.13).

Only in case (iv) of Sec. IV the relations (5.4)—(5.6) are nontrivially fulfilled; nontrivially in the sense that the left- and right-hand sides of (5.4)—(5.6) are different from zero.

VI. LOCAL INTERPRETATION OF THE FLUCTUATION-DISSIPATION THEOREMS

A. FDT of the first kind

The commutator-anticommutator relation is caused by the special form of the equilibrium density operator $\hat{\rho}_0\sim\exp(-\hat{H}_0/k_BT)$ where

$$\hat{H}_0=\hat{H}_M+\hat{H}_R+\hat{H}_{\mathrm{int}}\ . \tag{6.1}$$

The condensed matter part, which includes the short-wave (nonretarded) components of the EM field, is represented by $\hat{H}_M$. The free parts of the EM fields are given by $\hat{H}_R$:

$$\hat{H}_R=\left\{\int_{V_1}+\int_{V_2}\right\}d^3r(\hat{\vec{E}}^2+\hat{\vec{B}}^2)\ . \tag{6.2}$$

The interaction Hamiltonian $\hat{H}_{\mathrm{int}}$ describes the interaction of the long-wave ($\lambda\gg\alpha$) EM field with the condensed matter. On the macroscopic level this interaction is expressed by the susceptibilities $\vec{\epsilon}$ and $\vec{\mu}$.

Up to now in our considerations the integration in (6.2) was performed with respect to the total volume V

($V=V_1+V_2$). If we omit the integration over V_2 in (6.2), we obtain a density operator $\hat{\rho}'_0$. The independent internal variables are now the $\vec{E}(\vec{r})$ and $\vec{H}(\vec{r})$ fields with $\vec{r}\in V_1$. Consequently, the perturbation Hamiltonian $\hat{H}_1$ refers only to V_1 and via the linear-response theory and the FDT we only can calculate the EM field fluctuations inside the condensed matter, e.g.,

$$E_{ij}(\vec{r},\vec{r}\,',\omega)=\hbar\coth\left[\frac{\hbar\omega}{2k_BT_0}\right]\mathrm{Im}\mathcal{G}^{EE}_{ij}(\vec{r},\vec{r}\,',\omega)\,, \tag{6.3}$$

where $\vec{r},\vec{r}\,'\in V_1$. The temperature T_0 is only defined in V_1.

With the aid of (5.1) we may write (6.3) in an alternative form (nonmagnetic media provided):

$$E_{ij}(\vec{r}_1,\vec{r}_2,\omega)=\hbar\coth\left[\frac{\hbar\omega}{2k_BT_0}\right]\frac{1}{4\pi}\left[\int_{V_1}d^3r'\int_{V_1}d^3r''\mathcal{G}^{EE}_{ik}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''_{kl}(\vec{r}\,',\vec{r}\,'',\omega)\mathcal{G}^{EE*}_{jl}(\vec{r}_2,\vec{r}\,'',\omega)\right.$$
$$\left.+\lim_{\epsilon''\to 0}\int_{V_2}d^3r'\mathcal{G}^{EE}_{ik}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''(\omega)\mathcal{G}^{EE*}_{jk}(\vec{r}_2,\vec{r}\,',\omega)\right]\,, \tag{6.4}$$

where $\vec{r}_1,\vec{r}_2\in V_1$. Starting from the stochastic interpretation of the Maxwell equations we can write, on the other hand,

$$[E_{ij}(\vec{r}_1,\vec{r}_2,\omega)]_{\mathrm{av}}=\int_V d^3r'\int_V d^3r''\mathcal{G}^{EE}_{ik}(\vec{r}_1,\vec{r}\,',\omega)\mathcal{G}^{EE*}_{jl}(\vec{r}_2,\vec{r}\,'',\omega)[P_{ij}(\vec{r}\,',\vec{r}\,'',\omega)]_{\mathrm{av}} \tag{6.5}$$

with

$$[P_i(\vec{r}\,',\omega)P_j(\vec{r}\,'',\omega')]_{\mathrm{av}}=[P_{ij}(\vec{r}\,',\vec{r}\,'',\omega)]_{\mathrm{av}}\delta(\omega+\omega')\,. \tag{6.6}$$

We want to find out the explicit structure of (6.6) which reproduces (6.4). If the second term in (6.4) vanishes the (constant) temperature need not be defined in V_2. The comparison of (6.4) with (6.5) yields

$$[P_{ij}(\vec{r},\vec{r}\,',\omega)]_{\mathrm{av}}=\begin{cases}\hbar\coth\left[\dfrac{\hbar\omega}{2k_BT_0}\right]\epsilon''_{ij}(\vec{r},\vec{r}\,',\omega) & \text{for } \vec{r},\vec{r}\,'\in V_1\\[2ex] 2\dfrac{\eta(\omega)}{\omega}\epsilon''(\omega) & \text{for } \vec{r},\vec{r}\,'\in V_2\,,\end{cases} \tag{6.7}$$

where at this stage of our considerations $\eta(\omega)$ denotes an arbitrary and $\vec{r}$-independent function of ω. Alternately, we can start from (6.7) and calculate (6.5) for $\vec{r}_1\notin V_1$ and/or $\vec{r}_2\notin V_1$. This procedure leads us back to (6.4) with $\vec{r}_1\notin V_1$ and/or $\vec{r}_2\notin V_1$. The expression (6.4) again can be written in the form (6.3) even for $\vec{r}_1\notin V_1$ and/or $\vec{r}_2\notin V_1$.

Therefore, in inhomogeneous bounded systems it is completely arbitrary whether the Hamiltonian (6.1) contains the second part in (6.2) or not. The character of a bounded system and the stationarity of the process automatically guarantees that the condensed matter is surrounded by its thermal equilibrium radiation.

We used the two different interpretations of the Maxwell equations: In (6.3) $\mathcal{G}_{ij}$ is considered as an averaged field commutator which is initially defined only for $\vec{r},\vec{r}\,'\in V_1$. The extension of the validity of (6.4) to the whole system was made possible by the identity (5.1) which in turn represents a relation between classical Green functions. We can conclude that in bounded systems the two FDT's are identical and that it suffices to define the temperature only in the condensed matter part of the system.

Let us now assume that the second term in (6.4) does not vanish. In this case the validity of (6.3) in the whole system can be achieved only if we define in V_2 specific properties. It is obvious that the specification

$$\eta(\omega)=\frac{\hbar\omega}{2}\coth\left[\frac{\hbar\omega}{2k_BT_0}\right] \tag{6.8}$$

leads to the global validity of (6.3). Therefore, the assumption (6.8) is equivalent to the consideration of the second integral in (6.2) and corresponds to the prescription in Sec. IV of how to apply the FDT of the second kind in the case of global thermal equilibrium.

B. FDT of the second kind

We insert (6.7) in (6.5):

$$[E_{ij}(\vec{r},\vec{r}_2,\omega)]_{\mathrm{av}}=\frac{\hbar\omega}{2}\coth\left[\frac{\hbar\omega}{2k_BT_0}\right]\frac{1}{4\pi}\int_{V_1}d^3r'\int_{V_1}d^3r''\mathcal{G}^{EE}_{ik}(\vec{r}_1,\vec{r}\,',\omega)\frac{2}{\omega}\epsilon''_{kl}(\vec{r}\,',\vec{r}\,'',\omega)\mathcal{G}^{EE*}_{jl}(\vec{r}_2,\vec{r}\,'',\omega)$$
$$+\eta(\omega)\frac{1}{4\pi}\lim_{\epsilon''\to 0}\int_{V_2}d^3r'\mathcal{G}^{EE}_{ik}(\vec{r}_1,\vec{r}\,',\omega)\frac{2}{\omega}\epsilon''(\omega)\mathcal{G}^{EE*}_{jk}(\vec{r}_2,\vec{r}\,',\omega)\,. \tag{6.9}$$

From now on we give up the demand that (6.9) should reproduce (6.3) in the total system, i.e., the function $\eta(\omega)$ no longer has the global equilibrium form (6.8).

From a quantum electrodynamical point of view it is evident that the polarization fluctuation (6.7) should nevertheless represent the long-wave zero-point fluctuations in the vacuum. Therefore, we set

$$\eta(\omega)=\eta_0(\omega)=\frac{\hbar\omega}{2}\quad\text{for } \vec{r},\vec{r}\,'\in V_2\,. \tag{6.10}$$

Starting from (6.9) and using (6.10) we can no longer reproduce (6.3) in open systems. It can easily be seen however that the second term in (6.9) vanishes if the distance of the points $\vec{r}_1$ and $\vec{r}_2$ ($\in V_1$) to the boundary layer is sufficiently large, i.e., if the points $\vec{r}_1$ and $\vec{r}_2$ are deep in the interior of the dissipative part of the system. In this case the Green functions in the second term of (6.9) will vanish. If, on the other side, the points $\vec{r}_1$ and $\vec{r}_2$ are lying in a surface layer or even in V_2, both parts of (6.9) will contribute to the correlations.

It follows that (6.7) with (6.10) is no longer equivalent to the local FDT of the first kind of (6.3) ("local" with respect to $\hat{\rho}'_0$). Furthermore, it is obvious that the local FDT of the first kind cannot be valid for open systems because it implies a thermal equilibrium between matter and radiation up to the sharp surface of the condensed matter. This assumption cannot be correct if we do not define the global equilibrium from the beginning. For example, we can calculate the averaged Poynting vector (3.7) with respect to the density operator $\hat{\rho}'_0$. It is clear that this vector will also vanish at the surface of the condensed matter. On the other hand, this open material system must radiate. But due to the continuity of the Poynting vector this is impossible.

The polarization fluctuations (6.7) with the function (6.10) are equivalent to a local temperature distribution in (4.2) and (4.3):

$$T(\vec{r})=\begin{cases} T_0 & \text{for } \vec{r}\in V_1 \\ 0 & \text{for } \vec{r}\in V_2 \,. \end{cases} \tag{6.11}$$

The expression (6.11) suggests a simple interpretation of the second FDT as a neoclassical macroscopic theory. It is assumed that—independent of the state of the long-wave ($\lambda \gg \alpha$) EM field—the distribution over the states of the condensed matter system (including the short-wave parts of the EM field) is the equilibrium distribution with respect to T_0 [in (6.1) $\hat{H}_R$ and $\hat{H}_{\rm int}$ are omitted, semiclassical theory].

Taking into account the zero-point contributions (neoclassical theory[22–25]) this equilibrium distribution of the condensed matter system leads to the polarization fluctuations (6.7) with (6.10) and these in turn lead via the classical macroscopic Maxwell equations to the correlations (4.6)–(4.8) with the local temperature distribution (6.11). We have seen that in bounded systems and far inside the dissipative regions of open systems the thermal equilibrium of the matter leads to the thermal equilibrium radiation (global thermal equilibrium). In surface and vacuum regions of open systems the EM radiation field must differ from its thermal equilibrium properties.

Crucial for the applicability of the local FDT of the second kind in open systems is the validity of the assumption that the distribution over the material states is the equilibrium distribution even near the boundary layers. The short-wave additive parts of the EM field mainly represent the binding energy (apart from the nonadditive van der Waals forces) of the condensed matter system and determine the heat contact. Therefore, the long-wave components of the nonequilibrium radiation in the surface regions cannot essentially disturb the equilibrium distribution of the matter. (One could correct this approximation by introducing a material-dependent effective temperature $T_0^{\rm eff}$ which must be smaller than T_0.)

We just stated that the local interpretation of the second FDT does not take into account the back-reaction of the free radiation on the matter. On the other hand, we have seen that in bounded systems this back-reaction is included in the same formalism (equivalence of the first and second FDT). This fictitious contradiction is caused by the two different interpretations of the Maxwell equations. In bounded systems the Green functions may be interpreted as field commutators averaged with respect to $\hat{\rho}_0$. The density operator $\hat{\rho}_0$ describes the global thermal equilibrium and consequently the back-reaction is included (detailed balance).

This interpretation implies that a microscopic calculation of $\overleftrightarrow{\epsilon}$ and $\overleftrightarrow{\mu}$ principally must include all field components [see (2.16) and (2.17)] whereas the local interpretation of the FDT of the second kind implies that only the short-wave components contribute to $\overleftrightarrow{\epsilon}$ and $\overleftrightarrow{\mu}$. Therefore, the local interpretation of the second FDT in open systems is based on the premises that $\overleftrightarrow{\epsilon}$ and $\overleftrightarrow{\mu}$ are only determined by the short-wave components of the EM field.[3]

VII. STATIONARY RADIATIVE HEAT TRANSFER

We refer to nonmagnetic media. We insert (6.11) in (4.10) and transform (4.10) to the positive frequency part [see (4.9)]. The relation (5.6) allows us to write the Poynting vector in the form

$$[S_i(\vec{r})]_{\rm av}=\frac{c}{4\pi}\epsilon_{ijk}\int_0^\infty \frac{d\omega}{\pi}\hbar\omega\left[\exp\left[\frac{\hbar\omega}{k_B T_0}\right]-1\right]^{-1}\left[-\mathrm{Re}\left[\frac{1}{4\pi}\lim_{\epsilon''\to 0}\int_{V_2} d^3r'\,\mathcal{G}^{EE}_{jl}(\vec{r},\vec{r}\,',\omega)\frac{2\epsilon''}{\omega}\mathcal{G}^{HE*}_{kl}(\vec{r},\vec{r}\,',\omega)\right]\right]. \tag{7.1}$$

Owing to (5.6) the zero-point contributions canceled out.

The Poynting vector (7.1) makes sense if a finite and connected condensed matter system (V_1,T_0) is embedded in the infinite vacuum or if the radiation of an infinitely extended dissipative region (V_1,T_0) which lies completely in the half-space $z<0$ is considered. In a strict manner only the latter case should be treated by (7.1). Here, the dissipative matter acts as a heat reservoir. Nevertheless, we may assume stationarity for periods in which the radiated energy can be neglected compared to the internal energy of the dissipative matter. In this sense also the first case can be treated by (7.1).

The Poynting vector can no longer be written in the form (7.1) if the system consists of different connected dissipative regions V_{1i} $(i=1,2,\ldots,N)$ with different temperatures T_i. In this case we start from (4.10) and use the temperature distribution:

$$T(\vec{r})=\begin{cases} T_i & \text{for } \vec{r}\in V_{1i} \\ 0 & \text{for } \vec{r}\in V_2 \, . \end{cases} \tag{7.2}$$

In any case the relation (5.6) guarantees that there are no zero-point contributions in the Poynting vector.

Let us now consider two infinitely extended dielectric half-spaces with arbitrary macroscopic surface structures which are in the regions $z<0$ (V_{11},T_1) and $z>0$ (V_{12},T_2), respectively. For this system the right-hand side of (5.6) vanishes and we obtain

$$[S_i(\vec{r})]_{\rm av}=\frac{c}{4\pi}\epsilon_{ijk}\int_0^\infty \frac{d\omega}{\pi}\hbar\omega\left\{\left[\exp\left[\frac{\hbar\omega}{k_BT_1}\right]-1\right]^{-1}-\left[\exp\left[\frac{\hbar\omega}{k_BT_2}\right]-1\right]^{-1}\right\}$$
$$\times\mathrm{Re}\left[\frac{1}{4\pi}\int_{V_{11}}d^3r'\int_{V_{11}}d^3r''\,\mathcal{G}^{EE}_{jl}(\vec{r},\vec{r}',\omega)\frac{2}{\omega}\epsilon''_{lm}(\vec{r}',\vec{r}'',\omega)\mathcal{G}^{HE*}_{km}(\vec{r},\vec{r}'',\omega)\right] . \tag{7.3}$$

The local interpretation of the second FDT can be extended to systems in which externally imposed temperature gradients are present. This extension is possible since the Hamiltonian $\hat{H}_M$ of the condensed matter is purely additive.

In each case the temperature function must be given by a solution of a macroscopic transport equation in which the transport coefficients are determined solely by the short-wave components of the EM field. Furthermore, we have to demand that the length scale on which the temperature changes must be much larger than the coherence length ξ of all possible field correlations inside the matter [ξ can be calculated with the aid of the first FDT; replace T_0 by $T(\vec{r})$ in (6.3); $\vec{r},\vec{r}'\in V_1$]:

$$\frac{T}{|\vec{\nabla}T|}\gg\xi \, . \tag{7.4}$$

In spatial dispersive media also the "coherence length" of the permeability ϵ must fulfill the inequality (7.4).

VIII. KIRCHHOFF'S RADIATION LAW

We assume that the temperature depends only on z and that the electric permeability has the simple form

$$\epsilon_{ik}(\vec{r},\vec{r}',\omega)=\epsilon(z,\omega)\delta_{ij}\delta(\vec{r}-\vec{r}') \, . \tag{8.1}$$

The considered system is invariant with respect to translations in the x-y plane and therefore all Green functions can be expanded in terms of two-dimensional plane waves:

$$\mathcal{G}(\vec{r},\vec{r}',\omega)=\int\frac{d^2q}{(2\pi)^2}e^{i\vec{q}\cdot(\vec{\rho}-\vec{\rho}')}\mathcal{G}(z,z',\vec{q},\omega) \, , \tag{8.2}$$

where $\vec{\rho}=(x,y)$.

Owing to the symmetry the heat can only be transferred along the z direction [see (4.10) and (5.6)]:

$$[S_z(z)]_{\rm av}=\int_0^\infty d\omega\frac{c}{4\pi^2}\epsilon_{zjk}\frac{1}{4\pi}\int_{-\infty}^{\infty}dz'\hbar\omega\left[\exp\left[\frac{\hbar\omega}{k_BT(z')}\right]-1\right]^{-1}$$
$$\times\mathrm{Re}\int_{-\infty}^{\infty}\frac{d^2q}{(2\pi)^2}\mathcal{G}^{EE}_{jl}(z,z',\vec{q},\omega)\frac{2}{\omega}\epsilon''(z',\omega)\mathcal{G}^{HE*}_{kl}(z,z',\vec{q},\omega) \, . \tag{8.3}$$

As a further consequence of the symmetry it suffices to solve the Maxwell equations for partial waves of the form

$$\mathcal{G}(z,z',q,\omega)e^{iq(y-y')} \, . \tag{8.4}$$

[In Ref. 19 the differential equations for the partial waves (8.4) of the Green functions are given.] Here, we will discuss the thermal radiation of a dielectric half-space, i.e.,

$$\epsilon(z,\omega)=\begin{cases}\epsilon(\omega) & \text{for } z<0 \\ 1 & \text{for } z>0 \, ,\end{cases} \tag{8.5}$$

and

$$T(z)=\begin{cases}T_0 & \text{for } z<0 \\ 0 & \text{for } z>0 \, .\end{cases} \tag{8.6}$$

In the considered system the Poynting vector (8.3) does not contain any evanescent contributions (i.e., contributions for which $q^2>\omega^2/c^2$) and we may introduce the substitution $q=(\omega/c)\sin\theta$. We obtain

$$[S_z(z)]_{\rm av}=c\int_0^\infty d\omega\,E_\omega(z) \, . \tag{8.7}$$

In (8.7) we defined the quantity:

$$E_\omega(z)=\int_{(2\pi)} d\Omega\,\cos\theta\frac{\omega^2}{\pi^2c^3}\hbar\omega\left[\exp\left[\frac{\hbar\omega}{k_BT_0}\right]-1\right]^{-1}\times\frac{1}{4\pi}\frac{1}{2}\mathrm{Re}\left[\int_{-\infty}^{0}dz'\frac{c}{\omega}\epsilon''(\omega)\epsilon_{zjk}\mathcal{G}_{jl}^{EE}[z,z',(\omega/c)\sin\theta,\omega]\mathcal{G}_{kl}^{HE*}[z,z',(\omega/c)\sin\theta,\omega]\right]. \tag{8.8}$$

We emphasize that the solid angle integration in (8.8) only extends over the half-space $z>0$.

Finally, the solutions of the Maxwell equations lead to the result

$$E_\omega(z)=\int_{(2\pi)} d\Omega\,\cos\theta B_\omega(\theta)\,, \tag{8.9}$$

where

$$B_\omega(\theta)=\frac{\omega^2}{\pi^2c^3}\hbar\omega\left[\exp\left[\frac{\hbar\omega}{k_BT_0}\right]-1\right]^{-1}\times\frac{1}{4\pi}\frac{1}{2}\{[1-R_{||}(\omega,\theta)]+[1-R_\perp(\omega,\theta)]\}\,. \tag{8.10}$$

In (8.10), $R_{||}$ and $R_\perp$ represent the reflection coefficients for waves which are polarized parallel and perpendicular to the plane of incidence, respectively (incident angle θ):

$$R_{||}(\omega,\theta)=\left|\frac{[\epsilon(\omega)-\sin^2\theta]^{1/2}-\epsilon(\omega)\cos\theta}{[\epsilon(\omega)-\sin^2\theta]^{1/2}+\epsilon(\omega)\cos\theta}\right|^2, \tag{8.11}$$

$$R_\perp(\omega,\theta)=\left|\frac{[\epsilon(\omega)-\sin^2\theta]^{1/2}-\cos\theta}{[\epsilon(\omega)-\sin^2\theta]^{1/2}+\cos\theta}\right|^2. \tag{8.12}$$

We have written our result in the form (8.9) to make contact with the quantities which are usually used in the phenomenological theory of radiation.[31,32] $B_\omega(\vec{r},\theta)$ and $E_\omega(\vec{r})$ are denoted as "radiance" and "radiant emittance," respectively.

The expression (8.10) represents Kirchhoff's radiation law: The spectral emissivity divided by the absorption coefficient $A=\frac{1}{2}[(1-R_{||})+(1-R_\perp)]$ yields the spectral emissivity of the black-body radiation. Usually, the law (8.10) is derived for bodies which are in thermal equilibrium with the radiation. In global equilibrium the integration in (8.9) formally extends to the total solid angle 4π. This extension is due to the second part in (4.11) and due to the condition (6.8) for global thermal equilibrium. Consequently, (8.9) vanishes and (8.10) represents the principle of detailed balance. In principle, we have to distinguish between the meanings of the reflection coefficients for local equilibrium on the one hand and global equilibrium on the other hand (we refer to the remarks at the end of Sec. VI).

The problem of the validity of Kirchhoff's radiation law for freely radiating bodies has been discussed in literature.[28-30] Based on considerations completely different from ours, Weinstein[28] stated that Kirchhoff's law is valid for freely radiating bodies as long as the distribution over the material states is the equilibrium distribution. This statement is in complete agreement with our assumptions about the validity of the local FDT of the second kind.

For a black body ($R_{||}\equiv R_\perp\equiv 0$) Eq. (8.7) leads to the famous black-body radiation law:

$$[S_z]_{\mathrm{av}}=\frac{\pi^2}{60c^2\hbar^3}(k_BT_0)^4\,. \tag{8.13}$$

Starting from (7.3) we can calculate the radiative heat transfer between two black bodies. Let us assume two dielectric half-spaces separated by a vacuum gap. If the distance of the half-spaces is very large compared to all relevant wavelength we may use the Green functions for the dielectric half-space in (7.3) [see (8.8) and (8.10); $R_{||}\equiv R_\perp\equiv 0$]. We find the well-known result

$$[S_z]_{\mathrm{av}}=\frac{\pi^2}{60c^2\hbar^3}k_B^4(T_1^4-T_2^4)\,. \tag{8.14}$$

IX. SUMMARY

In this paper we discussed and compared the two methods for the macroscopic description of EM fluctuations. In bounded systems the application of the first FDT is correct in the sense of quantum electrodynamics. The Green functions are interpreted as commutators of the EM field operators which are averaged with respect to the global equilibrium distribution (matter and radiation). We proved that in bounded systems the stochastic electrodynamic theory which we called the second FDT is equivalent to the first FDT and, consequently, the second FDT is nothing else than the corresponding Langevin theory which can always be constructed to the usual linear response result.[33] This equivalence is also valid for infinitely extended systems with open vacuum regions if global thermodynamic equilibrium is postulated from the beginning.

We saw that the first FDT cannot be applied locally to open systems (local in the sense that thermodynamic equilibrium is postulated only in the material part of the system). This is due to the fact that equilibrium in the sense of the first FDT includes the long-wave radiation: The Hamiltonian in the corresponding density operator, on which the proof of the first FDT is based, must include all internal variables of the system. The assumption of thermal equilibrium between matter and radiation up to the sharp surface of the condensed matter leads to a vanishing Poynting vector although an open system should radiate. This contradiction can only be avoided by postulating global thermodynamic equilibrium from the beginning.

Furthermore, we proposed a local interpretation of the second FDT in open systems which represents an extension and new interpretation compared with the above stated equivalence with the first FDT. This new interpretation at least must include the following assumptions:

(i) A separation of the short-wave and long-wave radiation is possible.

(ii) The back-reaction of the long-wave radiation onto the matter can be neglected.

(iii) The form of the polarization fluctuations is unaltered although it is assumed that this form is based on an ensemble average with respect to the condensed matter part alone.

(iv) Only the short-wave components of the EM field are relevant in the microscopic calculation of the susceptibilities ϵ and μ.

In this local interpretation it can formally be assumed that $T=0$ in the vacuum part and $T=T_0$ in the condensed matter part of the system (i.e., the notion temperature is only related to the condensed matter). Nevertheless, we showed that in bounded systems this assumption leads via the Maxwell equations to the equilibrium radiation temperature T_0 in the whole system. This result is a consequence of the c-number properties of the field propagators: The Green functions cannot be distinguished with respect to the density operator on which they are based. In open systems, in which there is no equilibrium between matter and radiation, no true equilibrium temperature of the radiation exists.[34]

APPENDIX A: PROOF OF (5.1)

We insert the relations (2.20) and (2.21) in (5.1) and note the symmetries (2.11) and (2.15):

$$\begin{aligned}
&\mathcal{G}_{ji}^{EE*}(\vec{r}_2,\vec{r}_1,\omega)-\mathcal{G}_{ij}^{EE}(\vec{r}_1,\vec{r}_2,\omega)\\
&\quad=\int_V d^3r'\int_V d^3r''[\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}',\omega)\mathcal{G}_{jl}^{EE}(\vec{r}_2,\vec{r}'',-\omega)(\{[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r}'',\vec{r}',\omega)]^{-1}\}_{lk}-\{[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r}',\vec{r}'',\omega)]^{-1*}\}_{kl})\\
&\qquad\qquad+\mathcal{G}_{ik}^{EH}(\vec{r}_1,\vec{r}',\omega)\mathcal{G}_{jl}^{EH}(\vec{r}_2,\vec{r}'',-\omega)(\{[\overleftrightarrow{\mathcal{G}}^{HH}(\vec{r}'',\vec{r}',\omega)]^{-1}\}_{lk}-\{[\overleftrightarrow{\mathcal{G}}^{HH}(\vec{r}',\vec{r}'',\omega)]^{-1*}\}_{kl})]\ . \qquad (A1)
\end{aligned}$$

The linear equations (2.7)—(2.10) lead to the completeness relations:

$$\int_V d^3r'(\mathcal{G}_{ik}^{\{\substack{HE\\EE}\}}(\vec{r},\vec{r}',\omega)\{[\overleftrightarrow{\mathcal{G}}^{\{\substack{EH\\EE}\}}(\vec{r}',\vec{r}'',\omega)]^{-1}\}_{kl}+\mathcal{G}_{ik}^{\{\substack{HH\\EH}\}}(\vec{r},\vec{r}',\omega)\{[\overleftrightarrow{\mathcal{G}}^{\{\substack{HH\\HE}\}}(\vec{r}',\vec{r}'',\omega)]^{-1}\}_{kl})=\delta_{il}\delta(\vec{r}-\vec{r}'')\ , \qquad (A2)$$

$$\int_V d^3r'(\mathcal{G}_{ik}^{\{\substack{HE\\EE}\}}(\vec{r},\vec{r}',\omega)\{[\overleftrightarrow{\mathcal{G}}^{\{\substack{EE\\EH}\}}(\vec{r}',\vec{r}'',\omega)]^{-1}\}_{kl}+\mathcal{G}_{ik}^{\{\substack{HH\\EH}\}}(\vec{r},\vec{r}',\omega)\{[\overleftrightarrow{\mathcal{G}}^{\{\substack{HE\\HH}\}}(\vec{r}',\vec{r}'',\omega)]^{-1}\}_{kl})=0\ . \qquad (A3)$$

From (A3) we obtain the formulas:

$$\int_V d^3r'\mathcal{G}_{ik}^{EH}(\vec{r}_1,\vec{r}',\omega)\{[\overleftrightarrow{\mathcal{G}}^{HH}(\vec{r}',\vec{r}'',\omega)]^{-1}\}_{kl}=-\int_V d^3r'\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}',\omega)\{[\overleftrightarrow{\mathcal{G}}^{EH}(\vec{r}',\vec{r}'',\omega)]^{-1}\}_{kl}\ , \qquad (A4)$$

$$\int_V d^3r''\mathcal{G}_{jl}^{EH*}(\vec{r}_2,\vec{r}'',\omega)\{[\overleftrightarrow{\mathcal{G}}^{HH}(\vec{r}',\vec{r}'',\omega)]^{-1*}\}_{kl}=-\int_V d^3r''\mathcal{G}_{jl}^{EE*}(\vec{r}_2,\vec{r}'',\omega)\{[\overleftrightarrow{\mathcal{G}}^{EH}(\vec{r}'',\vec{r}',\omega)]^{-1*}\}_{lk}\ . \qquad (A5)$$

We put (A4) and (A5) in the second part of (A1) and use (2.13):

$$\begin{aligned}
&\mathcal{G}_{ji}^{EE*}(\vec{r}_2,\vec{r}_1,\omega)-\mathcal{G}_{ij}^{EE}(\vec{r}_1,\vec{r}_2,\omega)\\
&\quad=\int_V d^3r'\int_V d^3r''(\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}',\omega)\mathcal{G}_{jl}^{EE*}(\vec{r}_2,\vec{r}'',\omega)\{[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r}',\vec{r}'',\omega)]^{-1}\}_{kl}\\
&\qquad\qquad-\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}',\omega)\mathcal{G}_{jl}^{EE*}(\vec{r}_2,\vec{r}'',\omega)\{[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r}'',\vec{r}',\omega)]^{-1*}\}_{lk}\\
&\qquad\qquad-\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}',\omega)\{[\overleftrightarrow{\mathcal{G}}^{EH}(\vec{r}',\vec{r}'',\omega)]^{-1}\}_{kl}\mathcal{G}_{jl}^{EH*}(\vec{r}_2,\vec{r}'',\omega)\\
&\qquad\qquad+\mathcal{G}_{ik}^{EH}(\vec{r}_1,\vec{r}',\omega)\mathcal{G}_{jl}^{EE*}(\vec{r}_2,\vec{r}'',\omega)\{[\overleftrightarrow{\mathcal{G}}^{EH}(\vec{r}'',\vec{r}',\omega)]^{-1*}\}_{lk})\\
&\quad=\int_V d^3r'\int_V d^3r''[\mathcal{G}_{jl}^{EE*}(\vec{r}_2,\vec{r}'',\omega)(\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}',\omega)\{[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r}',\vec{r}'',\omega)]^{-1}\}_{kl}\\
&\qquad\qquad\qquad+\mathcal{G}_{ik}^{EH}(\vec{r}_1,\vec{r}',\omega)\{[\overleftrightarrow{\mathcal{G}}^{EH}(\vec{r}'',\vec{r}',\omega)]^{-1*}\}_{lk})\\
&\qquad\qquad-\mathcal{G}_{ik}^{EE}(\vec{r}_1,\vec{r}',\omega)(\mathcal{G}_{jl}^{EE*}(\vec{r}_2,\vec{r}'',\omega)\{[\overleftrightarrow{\mathcal{G}}^{EE}(\vec{r}'',\vec{r}',\omega)]^{-1*}\}_{lk}\\
&\qquad\qquad\qquad+\{[\overleftrightarrow{\mathcal{G}}^{EH}(\vec{r}',\vec{r}'',\omega)]^{-1}\}_{kl}\mathcal{G}_{jl}^{EH*}(\vec{r}_2,\vec{r}'',\omega))]\ . \qquad (A6)
\end{aligned}$$

In the bold parentheses of the second expression for (A6) we collected the first and fourth and the second and third terms of (A6), respectively.

If we now note that $(\overleftrightarrow{\mathcal{G}}^{EH})^{-1}$ and $(\overleftrightarrow{\mathcal{G}}^{HE})^{-1}$ are purely imaginary and are connected by (2.14), we can use (A2) and reproduce the left-hand side of (A1). The proofs of (5.2) and (5.3) are carried out in a completely analogous manner.

Implicitly, in this proof the correct succession of operations was used (see Sec. IV). If we split up (A1) according to (4.13) we are not allowed to omit the second part for open systems. In this case the validity of (A2) and (A3) demands the integration over the total volume whereas in bounded systems the integration over V_1 suffices.

APPENDIX B: NONMAGNETIC SYSTEMS

1. FDT of the first kind

In (2.6) we assume that $M\equiv 0$ and $\mu=1$. The perturbation Hamiltonian is given by $H_1=-\int_V d^3r\,\hat{\vec{E}}\cdot\vec{P}$. The independent internal variable is only the $\vec{E}$ field. In $\hat{H}_0$ the free components of the $\hat{\vec{B}}$ field are omitted.

The Maxwell equation (2.1) connects the electric field operator and its mean value with the magnetic field operator and its mean value, respectively. We insert (2.1) in (2.2) and obtain the equations which are interpreted as the linear-response equations in the sense of the QED perturbation theory:

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E})=\frac{\omega^2}{c^2}(\overleftrightarrow{\epsilon}\cdot\vec{E}+4\pi\vec{P})\ , \quad \text{(B1)}$$

$$\vec{\nabla}\cdot(\overleftrightarrow{\epsilon}\cdot\vec{E}+4\pi\vec{P})=0\ . \quad \text{(B2)}$$

Equations (B1) and (B2) define the Green function:

$$E_i(\vec{r},\omega)=\int_V d^3r'\mathscr{G}_{ik}(\vec{r},\vec{r}\,',\omega)P_k(\vec{r}\,',\omega)\ , \quad \text{(B3)}$$

$$P_k(\vec{r}\,',\omega)=\int_V d^3r''\{[\overleftrightarrow{\mathscr{G}}(\vec{r}\,',\vec{r}\,'',\omega)]^{-1}\}_{kj}E_j(\vec{r}\,'',\omega)\ . \quad \text{(B4)}$$

The comparison of (B4) with (B1) yields

$$\{[\overleftrightarrow{\mathscr{G}}(\vec{r},\vec{r}\,',\omega)]^{-1}\}_{ik}=-\frac{1}{4\pi}\epsilon_{ik}(\vec{r},\vec{r}\,',\omega)+\frac{c^2}{\omega^2}\left[\frac{\partial^2}{\partial x_i\partial x_k}-\delta_{ik}\Delta\right]\delta(\vec{r}-\vec{r}\,')\ . \quad \text{(B5)}$$

The symmetry relation (2.11) can be derived from (B5) (here, the symmetry relation for $\overleftrightarrow{\epsilon}$ must be presumed) or, alternatively, from the identification

$$\mathscr{G}_{ik}(\vec{r},\vec{r}\,',t-t')=\frac{i}{\hbar}\Theta(t-t')\langle[E_i(\vec{r},t),E_j(\vec{r}\,',t')]\rangle\ . \quad \text{(B6)}$$

Furthermore, we find the completeness condition

$$\int_V d^3r'\mathscr{G}_{ik}(\vec{r},\vec{r}\,',\omega)\{[\overleftrightarrow{\mathscr{G}}(\vec{r}\,',\vec{r}\,'',\omega)]^{-1}\}_{kj}=\delta_{ij}\delta(\vec{r}-\vec{r}\,'') \quad \text{(B7)}$$

and the relation

$$\frac{1}{2i}(\{[\overleftrightarrow{\mathscr{G}}(\vec{r},\vec{r}\,',\omega)]^{-1*}\}_{ik}-\{[\overleftrightarrow{\mathscr{G}}(\vec{r}\,',\vec{r},\omega)]^{-1}\}_{ki})=\frac{1}{4\pi}\epsilon''_{ik}(\vec{r},\vec{r}\,',\omega)\ . \quad \text{(B8)}$$

Since (B5) differs from (2.16) by a purely real term, (B8) and (2.20) have identical forms.

The FDT connects (B6) with the correlation function:

$$E_{ij}(\vec{r},\vec{r}\,',\tau=t-t')=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}E_{ij}(\vec{r},\vec{r}\,',\omega) \quad \text{(B9)}$$

where

$$E_{ij}(\vec{r},\vec{r}\,',\omega)=\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\mathrm{Im}\mathscr{G}_{ij}(\vec{r},\vec{r}\,',\omega)\ . \quad \text{(B10)}$$

In order to obtain all other correlation functions we write (B9) in the form

$$E_{ij}(\vec{r},\vec{r}\,',\tau)=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}e^{-i\omega t}e^{-i\omega' t'}\times[E_{ij}(\vec{r},\vec{r}\,',\omega)\delta(\omega+\omega')]\ . \quad \text{(B11)}$$

It is now clear, that (Wiener-Khinchin theorem for stationary random processes)

$$\langle\tfrac{1}{2}\{E_i(\vec{r},\omega),E_j(\vec{r}\,',\omega')\}\rangle=E_{ij}(\vec{r},\vec{r}\,',\omega)\delta(\omega+\omega')\ . \quad \text{(B12)}$$

If we now use the operator relation (2.1), i.e., $\hat{H}_i(\vec{r},\omega)=(c/i\omega)\epsilon_{ikl}(\partial/\partial x_k)E_l$, we immediately can write down the spectra of all other correlation functions:

$$H_{ij}(\vec{r},\vec{r}\,',\omega)=\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\frac{c^2}{\omega^2}\epsilon_{ikl}\epsilon_{jmp}\times\frac{\partial^2}{\partial x_k\partial x'_m}\mathrm{Im}\mathscr{G}_{lp}(\vec{r},\vec{r}\,',\omega)\ , \quad \text{(B13)}$$

$$M_{ij}(\vec{r},\vec{r}\,',\omega)=\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\frac{ic}{\omega}\epsilon_{jkl}\frac{\partial}{\partial x'_k}\times\mathrm{Im}\mathscr{G}_{il}(\vec{r},\vec{r}\,',\omega)\ . \quad \text{(B14)}$$

Naturally, the results (B10), (B13), and (B14) must coincide with the results of Sec. III. This can easily be shown: The determining equation for $\mathscr{G}_{ij}$ [insert (B3) in (B1)] coincides with the determining equation for $\mathscr{G}^{EE}_{ij}$ [insert (2.7) and (2.8) in (2.1) and (2.2), respectively]:

$$\left[\frac{\partial^2}{\partial x_i\partial x_k}-\delta_{ik}\Delta\right]\mathscr{G}_{km}(\vec{r},\vec{r}\,',\omega)-\frac{\omega^2}{c^2}\int_V d^3r''\epsilon_{ik}(\vec{r},\vec{r}\,'',\omega)\mathscr{G}_{km}(\vec{r}\,'',\vec{r}\,',\omega)=4\pi\frac{\omega^2}{c^2}\delta_{im}\delta(\vec{r}-\vec{r}\,')\ . \quad \text{(B15)}$$

Furthermore, the "full" Maxwell equations ($\vec{M}\neq\vec{0}$, $\mu=1$) lead to the relations

$$\mathscr{G}^{HE}_{ik}(\vec{r},\vec{r}\,',\omega)=-\frac{ic}{\omega}\epsilon_{ipl}\frac{\partial}{\partial x_p}\mathscr{G}^{EE}_{lk}(\vec{r},\vec{r}\,',\omega)\ , \quad \text{(B16)}$$

$$\mathscr{G}^{EH}_{ik}(\vec{r},\vec{r}\,',\omega)=-\mathscr{G}^{HE}_{ki}(\vec{r}\,',\vec{r},\omega)\ , \quad \text{(B17)}$$

$$\mathscr{G}^{HH}_{ik}(\vec{r},\vec{r}\,',\omega)=-4\pi\delta_{ik}\delta(\vec{r}-\vec{r}\,')+\frac{c^2}{\omega^2}\epsilon_{ijl}\epsilon_{kpm}\frac{\partial^2}{\partial x_j\partial x'_p}\mathscr{G}^{EE}_{lm}(\vec{r},\vec{r}\,',\omega)\ . \quad \text{(B18)}$$

2. FDT of the second kind

The Maxwell equations (2.1)–(2.6) with $\vec{M}\equiv 0$ and $\mu=1$ are interpreted as classical stochastic differential equations. The fields are driven by the stochastic force $\vec{P}$ which describes a Gaussian stochastic process:

$$[P_i(\vec{r},\omega)]_{av}=0\;, \tag{B19}$$

$$[P_i(\vec{r},\omega)P_j(\vec{r}\,',\omega')]_{av}=\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\frac{1}{4\pi}\times\epsilon''_{ij}(\vec{r},\vec{r}\,',\omega)\delta(\omega+\omega')\;. \tag{B20}$$

The particular solutions of the classical Maxwell equations (B1) and (B2) directly lead to the spectrum [see (B9)]:

$$[E_{ij}(\vec{r}_1,\vec{r}_2,\omega)]_{av}=\frac{1}{4\pi}\int_V d^3r'\int_V d^3r''\hbar\coth\left[\frac{\hbar\omega}{2k_BT}\right]\mathcal{G}_{ik}(\vec{r}_1,\vec{r}\,',\omega)\epsilon''_{kl}(\vec{r}\,',\vec{r}\,'',\omega)\mathcal{G}^*_{jl}(\vec{r}_2,\vec{r}\,'',\omega)\;. \tag{B21}$$

All other spectra are again obtained by using the Wiener-Khinchin theorem:

$$[E_i(\vec{r},\omega)E_j(\vec{r}\,',\omega')]_{av}=[E_{ij}(\vec{r},\vec{r}\,',\omega)]_{av}\delta(\omega+\omega') \tag{B22}$$

and Eq. (2.1) with $\mu=1$.

If we replace in these results the spatial derivatives of $\mathcal{G}_{ij}$ ($\mathcal{G}_{ij}=\mathcal{G}^{EE}_{ij}$) by $\mathcal{G}^{HE}_{ij}$ according to (B16), we are led back to the expressions (4.6)–(4.8) in which the Green functions $\overleftrightarrow{\mathcal{G}}^{EH}$ and $\overleftrightarrow{\mathcal{G}}^{HH}$ formally have been set equal to zero. The identity of (B21) with (B10) can again be proven if global thermal equilibrium is presumed: We replace ϵ''_{kl} in (B21) by (B8), use the symmetry relations of the inverse Green function (B9), and the completeness relation (B7).

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[34]Formally, a local anisotropic and frequency-dependent radiation temperature can be defined. The definition is based on the comparison of the true radiant emittance [see (6.8)] with the radiant emittance of the blackbody radiation. See Ref. 10, Sec. 63.