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New values for some ^{4}He I $1snl$ energy levels, ionization energies, and Lamb shifts

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Recent experimental determinations of energy separations within the $1snl$ term system ($n=2-6$) have been used to reevaluate 35 levels. Most of the levels have estimated errors less than 0.001 cm^{-1} relative to the $2\,^3P$ levels. Addition of accurate theoretical term values (ionization energies) available for several $1snl$ levels to the corresponding experimental level values gives generally consistent values for the principal ionization energy (E_I). The theoretical energies are further confirmed by the agreement of the weighted average of seven of these E_I values with a value obtained by fitting Ritz formulas to three accurately determined $1snl$ series; the suggested new E_I is 198 310.7745(40) cm^{-1} on an energy scale fixed by the value 171 135.0000 cm^{-1} for $2\,^1P$. Lamb shifts are derived for the 2, 3, $4\,^3S_1$, $2\,^1S_0$, $2\,^3P_1$, and 2^1P_1 levels as differences between experimental term values obtained with the new E_I and corresponding calculated term values not including Lamb shifts. The experimental and calculated values for the $1s^2\,^1S_0$ ground level relative to the present $1snl$ excited-level system are 0.00 ± 0.15 and 0.073 ± 0.009 cm^{-1}, respectively, so that a ~20-fold increase in the experimental accuracy would be required to test the calculated ground-level Lamb shift.

I. INTRODUCTION

Recent investigations of the ^{4}He I spectrum have included accurate wave-number determinations for a number of optical transitions between $1snl$ levels[1–4] ($n=2-6$). The energies of some of these levels have been calculated with sufficient accuracy to allow meaningful comparisons of particular experimental level separations with theoretical values. Such comparisons are important tests of nonrelativistic term values calculated from presumably the most accurate wave functions (involving correlations) ever obtained[5–10] as well as calculated relativistic, Lamb-shift, and mass-polarization contributions.[5–9,11,12] One purpose of this paper is to extend such comparisons for particular separations to a more general comparison of the predicted values obtained for a single important quantity, the principal ionization energy relative to the excited $1snl$ level system. This comparison not only implicitly includes all optical-region term separations that have been both calculated and experimentally determined with sufficient accuracy but also contributes to an improved value for the ionization energy. A reevaluation of the energy levels required by this approach has been extended to include 35 $1snl$ levels, and a new value for the ionization energy is supported by Ritz-formula fits of three reevaluated $1snl$ series. The values of the Lamb shifts for several terms are predicted as differences between experimental and uncorrected theoretical term values.

II. EXPERIMENTAL LEVELS AND THE IONIZATION ENERGY

The reevaluated levels are given in Table I. The $1s^2\,^1S_0$ ground level is included to show the experimental uncertainty of ±0.15 cm^{-1} in the connection between it and the $1snl$ excited-term system.[13,14] The previously adopted experimental value for the $2\,^1P_1$ level,[14,15] 171 135.00 cm^{-1} (the basic $1s^2\,^1S_0-1s2p\,^1P_1$ resonance-line wave number) has been arbitrarily extended to five decimal places to fix the excited-term system. The estimated uncertainty given with any level, however, pertains to the separation of that level from the $2\,^3P$ term, the fine-structure intervals of which are given to six decimal places.[16] This choice for the reference term allows the most accurately known He I separation in the optical region,[3,4] $2\,^3P-3\,^3D$, to be exhibited.

Other experimental term separations used for the determination of the levels were $2\,^1P-3\,^1D$ and $2\,^1P-3\,^3D$ (Ref. 4), $2\,^3S-n\,^3D$ (Refs. 1 and 3), $3\,^1D-3\,^1P$ (Ref. 17), $3\,^1D-n\,^1F$ (Ref. 18), $3\,^1S-3\,^1P$ (Ref. 19), $4\,^3S-4\,^3P$ (Refs. 19 and 20), and $n\,^3D-n\,^1D$ for $n=3$ (Ref. 4) and $n=5$ and 6 (Ref. 21). The $2\,^1S-3\,^1P$, $2\,^1S-4\,^1P$, and $2\,^1P-4\,^1D$ separations are from older data.[14,15,22] The $n\,^3D$ fine-structure intervals were taken from level-crossing measurements,[23,24] as were the $4\,^3P$ intervals.[25]

The error discussions in most of these references do not specify quantitative confidence levels for the errors. The discussions for the important term-separation measurements, however, usually give quantitative information or qualitative descriptions implying confidence levels of about $\geq 90\%$. The errors for the levels in Table I were obtained by using or combining (in quadrature) the errors of the pertinent separations as given in the references, except in cases where an averaging procedure seemed appropriate.[19] All of the triplet levels in Table I, and also the $n\,^1D$ levels except $4\,^1D$, have estimated errors ≤ 0.001 cm^{-1} with respect to $2\,^3P$. The errors of the 2,3 ^1S, 3,4 ^1P, and $4\,^1D$ levels are 0.002 to 0.004 cm^{-1}. No levels with errors greater than 0.004 cm^{-1} were included except the $n\,^1F$ levels. The full accuracies of the best determinations of particular level separations, including the fine-structure intervals, are in most cases not displayed in Table I.

TABLE I. Some ^{4}He I $1snl$ energy levels ($n=1-6$). The value of n precedes the term symbol. The estimated error in the last decimal place (given in parentheses) refers to the separation of a level from the $2\,^3P$ levels.

Term	J	Level (cm^{-1})	Term	J	Level (cm^{-1})
$1\,^1S$	0	0.00±0.15[a]	$4\,^3D$	3	191 444.5843(5)
				2	191 444.5855(5)
$2\,^3S$	1	159 856.0776(5)		1	191 444.6040(5)
$2\,^1S$	0	166 277.542(3)	$4\,^1D$	2	191 446.559(2)
$2\,^3P$	2	169 086.869 782	$4\,^1F$	3	191 451.995(5)
	1	169 086.946 208	$4\,^1P$	1	191 492.812(4)
	0	169 087.934 120			
$2\,^1P$	1	171 135.000 00(11)	$5\,^3S$	1	193 347.0947(5)
			$5\,^3D$	3	193 917.2546(5)
$3\,^3S$	1	(183 236.890 6)[b]		2	193 917.2552(5)
$3\,^1S$	0	184 864.932(2)		1	193 917.2647(5)
$3\,^3D$	3	186 101.649 50(3)	$5\,^1D$	2	193 918.3931(5)
	2	186 101.652 04(3)	$5\,^1F$	3	193 921.194(8)
	1	186 101.696 22(3)			
$3\,^1D$	2	186 105.069 84(9)	$6\,^3S$	1	194 936.2234(10)
$3\,^1P$	1	186 209.467(2)	$6\,^3D$	3	195 260.1753(9)
				2	195 260.1757(9)
$4\,^3S$	1	190 298.2165(5)		1	195 260.1812(9)
$4\,^3P$	2	191 217.1440(7)	$6\,^1D$	2	195 260.8744(9)
	1	191 217.1530(7)	$6\,^1F$	3	195 262.502(9)
	0	191 217.2633(7)			
			Limit		198 310.7745(40)

[a] This is the experimental value. An apparently more accurate position of 0.073(9) cm^{-1} is obtained for the ground level relative to the other levels in this table by using a calculated value for the ionization energy (see text).

[b] Level value calculated from a series formula (see text). The error is estimated to be less than ±0.001 cm^{-1}.

Four members of each of the series $n\,^3S$, $n\,^3D$, and $n\,^1D$ are given in Table I with sufficient accuracy to make these series candidates for yielding an improved value for the ionization energy relative to the $1snl$ excited-term system. The absolute term value $T(nSLJ)$ associated with a $1snl$ SLJ level is the binding energy of the nl electron for that level, taken as a positive quantity. According to this definition,

$$E_I(\text{He II } 1s\,^2S_{1/2})-E_{nl}=T=R(^4\text{He})/(n^*)^2\;,$$

where E_I represents the principal ionization energy, E_{nl} the energy of the $1snl$ level, $R(^4\text{He})$ is the Rydberg constant[26] for ^{4}He, and n^* is the effective principal quantum number for the level. We assume that the n^* values for the above three series vary according to an extended Ritz formula

$$n-n^*=a+b(n-\delta_0)^{-2}+c(n-\delta_0)^{-4}\;.$$

Here δ_0 is fixed at the approximate value of the series quantum defect[27] ($\delta_0 \approx a$), and a, b, and c are series constants. These three constants and the ionization energy (E_I) can be adjusted to fit exactly a four-member series.

The results of such fits are given in Table II. The error given with each E_I value represents the maximum variation that still allows a fit of the levels within the errors by a least-squares adjustment of the three constants.[28] These E_I errors thus correspond to true error estimates only if a three-constant Ritz formula is capable of representing more accurate and extensive data for the series than those used here. The results in Table II are not inconsistent with such an assumption. It would of course be desirable to test the formulas against high-accuracy measurements of additional members of these series and also to have such measurements for other series.

The value for the ionization energy indicated in Table II, near 198 310.775 cm^{-1}, agrees with but should be more accurate than the previous value[29,30,15,18] of 198 310.76(2) cm^{-1}. A part of the increase is no doubt due to the redetermination of the singlet-triplet connection, which raised the triplet levels by about 0.0065 cm^{-1} relative to the singlet system.[4] With adjustments for this change, the existing data for other He I series yield limits (values for the ionization energy) consistent with the values in Table II but with much larger errors. A (two-constant) series formula fit of the 4,5,6 ^1F levels in Table I, for example, gives an E_I of 198 310.78(7) cm^{-1}, the indicated error having the meaning explained above.

III. THEORETICAL TERM VALUES AND THE IONIZATION ENERGY

The data assembled in Table III pertain to the ionization energy and are also of fundamental interest. Nonrelativistic energies have been calculated for a number of ^{4}He I $1snl$ levels[5-10] with estimated uncertainties less than 0.005 cm^{-1}. Calculated term values are given in Table III for all such levels having experimental values in Table I. The total term value T is given by the sum

$$T=T_{\text{nr}}+\Delta_r-\epsilon_M+\Delta_L=T_r+\Delta_L\;,$$

where T_{nr} is the nonrelativistic term value and Δ_r, $-\epsilon_M$, and Δ_L are the relativistic, mass-polarization, and Lamb-shift contributions, respectively. Calculated term values lacking only the Lamb shift will be denoted by T_r.

The values of T_{nr}, Δ_r, and $-\epsilon_M$ given for the nS and

TABLE II. Constants and ionization energy (E_I) obtained by fitting an extended Ritz formula to three He I series (see text). Four members of each series were fitted, as indicated by the n values preceding the term symbols. The $n\,^3D_{cg}$ series formula represents the ^3D baricenter positions. The value of the ^{4}He Rydberg constant was taken as 109 722.2731 cm^{-1}.

Series	δ_0	a	b	c	E_I (cm^{-1})
$2,4-6\,^3S_1$	0.3	0.296 657 660	0.038 153 534	0.008 126 37	198 310.7740(35)
$3-6\,^3D_{cg}$	0	0.002 892 870	−0.006 517 22	0.001 056 5	0.7753(35)
$3-6\,^1D_2$	0	0.002 115 610	−0.003 223 10	0.000 446 7	0.7751(65)

TABLE III. Values of the ionization energy for the He I 1snl term system. The calculated term value T (binding energy) for a particular level is equal to the sum $T_r+\Delta_L=T_{\mathrm{nr}}+\Delta_r-\epsilon_M+\Delta_L$, where T_{nr} is the nonrelativistic term value, and Δ_r, $-\epsilon_M$, and Δ_L are the relativistic, mass-polarization, and Lamb-shift contributions, respectively. The values of all calculated quantities have been adjusted to the current values for pertinent atomic constants (see text). Values for the principal ionization energy (E_I) were obtained by adding values of $T_r+\Delta_L$ to values of the corresponding levels in Table I. All values in cm^{-1}.

Term	T_{nr}	Δ_r	$-\epsilon_M$	T_r	Δ_L	E_I
$2\,^3S_1$	38 453.131 385[a]	1.922 236[a]	−0.223 923[a]	38 454.829 70(10)[a]	−0.135[b]	198 310.772
$2\,^1S_0$	32 033.20 78[c]	0.399 4[c]	−0.286 0[c]	32 033.321 2(20)[c]	−0.106(16)[d]	0.757(16)
$2\,^3P_1$	29 222.155 36[e]	−0.314 65[f]	1.942 89[e]	29 223.783 6(2)[f]	0.038[b]	0.768
$2\,^1P_1$	27 176.689 8[e]	0.467 57[f]	−1.385 41[e]	27 175.772 0(10)[f]	(0.000)[g]	0.772
$3\,^3S_1$	15 073.439 8[h]	0.526 9[h]	−0.057 1[h]	15 073.909 6(20)[h]		
$3\,^1D_2$	12 205.668 0(20)[i]	0.032 2[j]	0.007 2[k]	12 205.707 4(30)	(0.000)[l]	0.777(4)
$4\,^3S_1$	8 012.377 4[h]	0.213 7[h]	−0.022 37[h]	8 012.568 7(20)[h]		
$4\,^1D_2$	6 864.194 7(50)[i]	0.019 2[j]	0.004 6[j]	6 864.218 5(50)	(0.000)[l]	0.777(5)
$4\,^1F_3$	6 858.771 11(20)[i]	0.009 0[j]	0.000 3[j]	6 858.780 4(4)	(0.000)[l]	0.775(5)
$5\,^1F_3$	4 389.538 3(40)[i]	0.006 4[j]	0.000 2[j]	4 389.544 9(40)	(0.000)[l]	0.739(9)

[a] Pekeris (Refs. 5 and 6). These values were obtained from a wave function resulting from the solution of a determinant of order 1078 (Table VI of Ref. 6).

[b] Ermolaev (Ref. 34).

[c] Pekeris (Ref. 6).

[d] Suh and Zaidi (Ref. 33).

[e] Schiff *et al.* (Ref. 7).

[f] Values from Schiff *et al.* (Ref. 7), corrected to include the singlet-triplet mixing contributions (Ref. 9).

[g] Assumed value; see Refs. 4 and 34.

[h] Accad *et al.* (Ref. 8).

[i] Sims *et al.* (Ref. 10). See text for explanation of estimated error.

[j] Cok and Lundeen (Ref. 11).

[k] Drake (Ref. 12).

[l] Value assumed.

nP levels in Table III were calculated by Pekeris and his co-workers.[5–9] The T_{nr} values for the $n\,^1D$ and $n\,^1F$ levels are from Sims *et al.*,[10] with Δ_r and $-\epsilon_M$ contributions from Cok and Lundeen[11] or Drake (ϵ_M for $3\,^1D_2$).[12] All calculated energies quoted here have been converted to a value of 109 722.273 09(11) cm^{-1} for $R(^4\mathrm{He})$, and the Δ_r and ϵ_M values calculated by Pekeris and co-workers[5–8] have been adjusted to the values $\alpha^2R(^4\mathrm{He})=5.842\,861$ cm^{-1} and $m_e/M_\alpha=1.370\,9337\times10^{-4}$, respectively.[26,31]

Pekeris's calculations[6] of $T_r(2\,^3S)$ extended to the solution of a determinant of order 1078, at which point the results had converged to agreement with the extrapolated value to within less than 10^{-5} cm^{-1}. The error of 1×10^{-4} cm^{-1} given here arises mainly from a doubling of the one standard-deviation error of 4×10^{-5} cm^{-1} due to the Rydberg constant. A recent adjustment of Pekeris's $T_{\mathrm{nr}}(2\,^3S)$ value by 0.004 19 cm^{-1} seems unwarranted and is not adopted here.[32] The errors given for the $T_r(2\,^1S)$, $T_r(2\,^3P_1)$, and $T_r(2\,^1P)$ levels are based on the discussion in Refs. 6 and 7. The $T_r(3\,^3S)$ and $T_r(4\,^3S)$ values from Accad *et al.*,[8] originally given by them to three decimal places, are listed with errors of ±0.0020 cm^{-1} based on their statement that the results "... are estimated to differ by not more than 1, or occasionally 2, in the last digit quoted from the extrapolated value, unless otherwise stated."[8] The T_{nr} values from Sims *et al.*[10] correspond to their "estimated exact" energies, and the errors have been taken equal to the difference between these values and their final calculated values. The estimated errors are included with the T_{nr} values for these $n\,^1D$ and $n\,^1F$ levels, since the other contributions to the T_r sums were calculated by different authors. The T_r errors for the $3\,^1D$ and $4\,^1D$ levels take into account estimated errors in Δ_r and ϵ_M.

Lamb shifts have been calculated for three of the levels in Table III.[32–34] Ermolaev[34] did not estimate the errors of his calculated Δ_L values for $2\,^3S$ and $2\,^3P_1$, but the resulting total term values $T=T_r+\Delta_L$ give a calculated term separation $T(2\,^3S_1)-T(2\,^3P_1)$ only 0.0046 cm^{-1} greater than the experimental separation from Table I. It thus seems at least marginally preferable to use Ermolaev's Δ_L value for $2\,^3S_1$ instead of the value −0.129(11) cm^{-1} calculated by Suh and Zaidi.[33] Hata and Grant[32] obtained a total Lamb shift of −0.1820 cm^{-1} for the $2\,^3S-2\,^3P$ transition but do not give the calculated individual shifts for $2\,^3S$ and $2\,^3P$. A value of zero has been assumed for the $2\,^1P$ Lamb shift, which has been shown[4] to have an absolute value probably less than 0.01 cm^{-1}. Zero Lamb shifts were also assumed for the $n\,^1D$ and $n\,^1F$ levels.

The values of $T=T_r+\Delta_L$ for eight levels in Table III have been added to the corresponding level values from Table I to obtain values for the principal ionization energy (E_I in Table III). The estimated errors given with five of the E_I values were obtained by combining the contributing errors in quadrature. The error of the zero Lamb shift assumed for $3\,^1D$ was taken as less than 0.003 cm^{-1}, and the $4\,^1D$ and $n\,^1F$ Lamb shifts were assumed to contribute negligibly to E_I errors of 0.005 to 0.009 cm^{-1}. No errors are given with the E_I values from the $2\,^3S_1$, $2\,^3P_1$, and

$2\,{}^1P_1$ terms because of a lack of uncertainty estimates for the Lamb shifts.

The agreement of the E_I values in Table III is on the whole gratifying, only the value from the $5\,{}^1F$ term having an unexpectedly large deviation. It seems almost certain that either the calculated $5\,{}^1F$ energy[10] is in error by much more than the uncertainty assumed in Table III or (less probable in the author's opinion) the experimental level error is several times larger than originally estimated.[18] It is interesting to note that the uncertainties of the different E_I values in Table III are essentially due to different contributors according to several cases: the error is wholly or mainly from Δ_L for the $2S$ and $2P$ E_I values, from $T_{\rm nr}$ for the $4\,{}^1D$ E_I value, and from the experimental level value for the $4\,{}^1F$ E_I value. A determination of the $4\,{}^1F$ level value within an error of ± 0.0002 cm^{-1} or so would give the E_I value with an estimated error smaller by an order of magnitude than the smallest E_I error in Table III.

In order to obtain an average of all of the E_I values in Table III except that from $5\,{}^1F$, the somewhat arbitrary errors of ± 0.007 cm^{-1} were assumed for the Δ_L values from Ref. 34 ($2\,{}^3S$ and $2\,{}^3P_1$) and ± 0.010 cm^{-1} for the zero Δ_L value assumed for $2\,{}^1P_1$. The usual procedure (weights according to the inverse squares of the errors) of averaging the seven E_I values then gives a weighted mean of 198 310.7745 cm^{-1}, with a value of 0.0022 cm^{-1} for the error of the mean.

On the basis of the results in Tables III and II, it seems reasonable to adopt a value of 198 310.7745(40) cm^{-1} for the ^{4}He I ionization energy relative to the levels in Table I. The confidence level of the error is estimated to be in the range above 95% despite the difficulty of estimating the errors of the calculated quantities in Table III. The results in Table II support the adopted E_I value but have not been used directly because of a lack of accurate data for higher series members. The adopted E_I value agrees so closely with the values in Table II that there is at present no need to recalculate these series. The values of higher series members predicted by the formulas should be accurate to ± 0.005 cm^{-1}, and the error of the predicted $3\,{}^3S_1$ level in Table I should be less than ± 0.001 cm^{-1}.

The most accurate term values now available for the levels in Table I should be given by subtracting the level values from the E_I value 198 310.7745(40) cm^{-1}. One such experimental term value ($T_{\rm expt}$) of interest is that for $5\,{}^1F$, since it can be combined with the Δ_r, $-\epsilon_M$, and Δ_L values for $5\,{}^1F$ in Table III to obtain a value for $T_{\rm nr}(5\,{}^1F)$ independent of the calculated value.[10] The result indicates that the calculated $T_{\rm nr}(5\,{}^1F)$ value quoted in Table III is too small by 0.036(9) cm^{-1}.

IV. LAMB SHIFTS FOR SEVERAL EXCITED LEVELS

The difference of the experimental term value and the T_r value for a level constitutes a predicted value for the Lamb shift, $T_{\rm expt}-T_r$. Values of the Lamb shifts so obtained for several levels are given in Table IV, all errors having been combined in quadrature. The uncertainties of the $T_{\rm expt}-T_r$ values are entirely or mainly due to the estimated uncertainty of ± 0.0040 cm^{-1} in the ionization energy.

In considering the $\Delta_L(2\,{}^3S)$ and $\Delta_L(2\,{}^3P)$ values, it is important to note that the difference of these two shifts is determined to within an error much smaller than the error of the individual $T_{\rm expt}-T_r$ values for these terms in Table IV. The difference between the experimental $2\,{}^3S_1-2\,{}^3P_1$ separation (Table I) and the corresponding theoretical separation obtained from the T_r values in Table III is the Lamb-shift contribution to the $2\,{}^3S-2\,{}^3P$ separation,[4] $\Delta_L(2\,{}^3S-2\,{}^3P)=-0.1775(5)$ cm^{-1}. The values of $\Delta_L(2\,{}^3S-2\,{}^3P)$ calculated by Ermolaev[34] and by Hata and Grant,[32] -0.173 and -0.1820 cm^{-1}, respectively, are apparently in error by about $2\frac{1}{2}$% in opposite directions. The $T_{\rm expt}-T_r$ value for $2\,{}^3P_1$ indicates that Ermolaev's calculated Δ_L value,[34] 0.038 cm^{-1}, is too small. The $T_{\rm expt}-T_r$ value for $2\,{}^3S$ falls between the Δ_L values calculated in Refs. 34 and 33, respectively.

The two calculated values[33,34] of $\Delta_L(2\,{}^1S_0)$ in Table IV are in agreement but differ from the $T_{\rm expt}-T_r$ value by about the error estimated in Ref. 33 (15%). Previous results[4,34] indicating a relatively small value for $\Delta_L(2\,{}^1P)$ are supported by the $T_{\rm expt}-T_r$ value in Table IV. The $T_{\rm expt}-T_r$ value for $3\,{}^1D$ is consistent with the value 0.000(3) cm^{-1} assumed for $\Delta_L(3\,{}^1D)$ in Table III.

One might expect an approximate $(n^*)^{-3}$ dependence for the $\Delta_L(n\,{}^3S)$ values, the Lamb shifts of the hydrogen ns levels being proportional to n^{-3} to a good approximation. The ratio of the $T_{\rm expt}-T_r$ values for $3\,{}^3S$ and $4\,{}^3S$ agrees with the ratio predicted by an $(n^*)^{-3}$ dependence to within 7%, a small discrepancy compared to the uncertainties, but the ratio of the $T_{\rm expt}-T_r$ values for $2\,{}^3S$ and $3\,{}^3S$ corresponds to an $(n^*)^{-3.5}$ dependence. A decrease of the E_I value by 0.04 cm^{-1} would about halve the latter discrepancy from an $(n^*)^{-3}$ dependence, but the n^*-power behavior of the resulting three $n\,{}^3S$ $T_{\rm expt}-T_r$ values would be irregular. It seems likely that the upward Lamb shift of the $2\,{}^3S$ level relative to the shifts of the higher $n\,{}^3S$ levels is larger than in the case of the hydrogen ns levels.

TABLE IV. Lamb shifts for some He I terms. The predicted shifts are given as the differences $T_{\rm expt}-T_r$, where $T_{\rm expt}$ is the experimental term value, $T_{\rm expt}=198\,310.7745(40)-E_{nl}$, and T_r is the corresponding calculated term value not including the Lamb shift (T_r values from Table III). Theoretically calculated Lamb shifts Δ_L are listed for three terms. Units are cm^{-1}.

Term	$T_{\rm expt}$	$T_{\rm expt}-T_r$	Δ_L
$2\,{}^3S_1$	38 454.6969(40)	$-0.1328(40)$	-0.135[a] $-0.129(11)$[b]
$2\,{}^1S_0$	32 033.2325(50)	$-0.0887(54)$	-0.103[a] $-0.106(16)$[b]
$2\,{}^3P_1$	29 223.8283(40)	0.0447(40)	0.038[a]
$2\,{}^1P_1$	27 175.7745(40)	0.0025(41)	
$3\,{}^3S_1$	15 073.8839(40)	$-0.0257(45)$	
$3\,{}^1D_2$	12 205.7047(40)	$-0.0027(50)$	
$4\,{}^3S_1$	8 012.5580(40)	$-0.0107(45)$	

[a] Ermolaev (Ref. 34).

[b] Suh and Zaidi (Ref. 33).

V. NEED FOR MORE ACCURATE MEASUREMENTS: IONIZATION ENERGY, POSITION AND LAMB SHIFT OF THE GROUND LEVEL

A more accurate determination of the ionization energy relative to the excited 1*snl* levels would be well worthwhile. It seems likely that Doppler-free measurements of two-photon transitions to high series members in a low-pressure, relatively field-free source would give the limit with an order of magnitude greater accuracy than that quoted above. Experimental term values obtained with an E_I value of this accuracy would, for example, give more accurate values for all the Lamb shifts in Table IV; the errors of the resulting $T_{\text{expt}}-T_r$ values for $2\,^3S$ and $2\,^3P$ should be less than 0.001 cm^{-1}. Experimental term values for levels having very small Lamb shifts test the calculated term values T_r to within the experimental uncertainty. The determination of the ionization energy to within ± 0.0005 cm^{-1} or less would give considerable incentive for additional theoretical and experimental research on the 1*snl* term system.

It seems appropriate to recall, in conclusion, the present situation regarding the ground-term energy. In 1959 Pekeris[5] calculated a value of T_r for the ^{4}He $1s^2\,^1S_0$ ground level that converts to 198 312.0361(5) cm^{-1} with the atomic constants quoted above.[35] Subtraction of this calculated term value from the experimental ionization energy gives a value for the Lamb shift $\Delta_L(1s^2\,^1S_0) = -1.26 \pm 0.15$ cm^{-1}, the error being the uncertainty of the connection between the ground level and the excited 1*snl* level system. A reduction of this uncertainty by 2 to 3 orders of magnitude would thus be required to obtain a value of $\Delta_L(1s^2\,^1S_0)$ with an accuracy limited by the uncertainty of Pekeris's result. Combining the value of $\Delta_L(1s^2\,^1S_0)$ calculated by Aashamar and Austvik,[36] $-1.335(8)$ cm^{-1}, with Pekeris's T_r value gives a total term value $T(1s^2\,^1S_0) = T_r + \Delta_L = 198\,310.701(8)$ cm^{-1}. The difference between the new E_I value 198 310.7745(40) cm^{-1} and this calculated term value is probably the most accurate available value for the ground level with respect to the excited levels in Table I, the value being $+0.073(9)$ cm^{-1}. A new measurement of the ground-level connection accurate enough to test the calculated $\Delta_L(1s^2\,^1S_0)$ value ($\sim$20-fold increase in accuracy) would be a major step towards meeting the challenge set by Pekeris's calculation of 25 years ago.

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