Electrostatic-plasma-wave energy flux

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The direct loss of fusion plasma energy via electrostatic plasma waves propagating across field lines has previously been shown to be significant. We investigate the theoretical foundations of this phenomenon from a strictly transport perspective. The critical role assumed by the displacement current in sustaining cross-field transport of electrostatic energy is shown. An accompanying analysis of the composition of electrostatic-wave-energy flux reveals a far larger involvement of the field degrees of freedom than that associated with wave-energy density. A phenomenological Fourier-type law describing wave-energy-current degradation by general wave-particle interactions is shown to be the appropriate description of wave-energy conduction in the nonthermal regime.

I. INTRODUCTION

The phenomenon of thermal conduction by lattice waves in semiconductors has long been appreciated as significant within the solid-state physics community. As the field of fusion plasma physics is a comparatively young one, a similar recognition of the importance of plasma-wave-energy transport has not been forthcoming. With regard to energy confinement, this is an uncomfortable state of affairs. Present magnetic confinement approaches heavily emphasize the particle degrees of freedom over the field degrees of freedom. While the external magnetic fields may prove satisfactory in suppressing the cross-field particle component of energy transport, the consequences of unhindered transport of waves across the field have received only limited interest within the fusion community.

The problem was first posed in a thermal equilibrium estimate by Rosenbluth and Liu.[1] They found that energy losses induced by cross-field transport of electrostatic waves can be comparable in magnitude to particle thermal diffusive losses. Subsequently, Molvig, Bers, and Tekula found that runaway-induced electron-plasma-wave energy transport was greater than neoclassical ion thermal conduction and of the order of observed transport in intermediate density tokamaks.[2] Recently, an estimate has been made of cross-field wave-energy losses by ion-cyclotron modes driven unstable by the Drummond-Rosenbluth ion-cyclotron instability.[3] This estimate was distinguished by a departure from the previous use[1,2] of collision-induced wave-emission mechanisms in favor of an induced emission version. A correspondingly large enhancement over the thermal level was demonstrated.

This succession of estimates underscores the timely need for an investigation of electrostatic-wave-energy flux from first principles. The study undertaken here will address the following fundamental questions:

(1) What is the underlying physical basis for cross-field electrostatic-wave-energy transport in the strong-field limit?

(2) How is the integrity of electrostatic-wave-energy transport influenced by a varying degree of plasma anisotropy?

(3) Do the usual divisions of wave-energy *density* into particle and field components directly carry over into wave-energy *flux*?

(4) Are the usual ambiguities in locally defining energy density and flux circumvented by demanding an equivalence between group velocity and velocity of energy transport?

(5) How is wave thermal conduction manifested in the nonthermal, collisionless regime?

The collective intent of these questions is to understand the nature of electrostatic plasma waves from a *transport* perspective.

In Sec. II we begin by deriving an energy continuity equation for an electrostatic system with direct use of Joule's law. The electrostatic limit employed is distinguished from previous conventions by the necessary requirement that the induced magnetic induction $\vec{B}$ also vanishes as the speed of light c approaches infinity. This allows for a tractable treatment of "electrostatic" waves even in anisotropic media. In Sec. III the role of the electrostatic flux vector in electrostatic-wave-energy transport is investigated. The key term responsible for unhindered cross-field transport of non-cyclotron-type wave energy is identified, and the constitution of electrostatic-wave-energy transport in an anisotropic plasma is examined. A discussion of the usual ambiguities associated with defining local-energy flux within the context of an observable wave then follows. In Sec. IV, we investigate the role of weak wave dissipation mechanisms in degrading a wave-energy current by formulating a Fourier-type phenomenological law that is valid in a nonthermal, weakly turbulent setting.

II. A VLASOV DESCRIPTION OF ELECTROSTATIC ENERGY FLUX

The rate at which particle kinetic energy W_p is dissipated into field energy is embodied in Joule's law

$dW_p/dt = -\vec{J}\cdot\vec{E}$, where $\vec{J}$ and $\vec{E}$ are the induced particle current and electric field, respectively. The usual derivation of Poynting's theorem involves use of the Maxwell-Vlasov system of equations:

$$\vec{\nabla}\times\vec{B} = \frac{1}{c}\frac{\partial\vec{D}}{\partial t}\,,\quad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}\,, \tag{1}$$

$$\frac{\partial f_\sigma}{\partial t}(\vec{x},\vec{v};t) + \vec{v}\cdot\frac{\partial f_\sigma}{\partial\vec{x}}(\vec{x},\vec{v};t) + \frac{e_\sigma}{m_\sigma}\left[\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right]\cdot\frac{\partial f_\sigma}{\partial\vec{v}}(\vec{x},\vec{v};t) = 0 \tag{2}$$

in reformulating Joule's law to give

$$\frac{\partial}{\partial t}\left[\frac{|\vec{E}|^2 + |\vec{B}|^2}{8\pi} + \sum_\sigma \int f_\sigma(\vec{x},\vec{v};t)\frac{m_\sigma v^2}{2}d\vec{v}\right] + \vec{\nabla}\cdot\left[\frac{c}{4\pi}\vec{E}\times\vec{B} + \sum_\sigma \int f_\sigma(\vec{x},\vec{v};t)\vec{v}\frac{m_\sigma v^2}{2}d\vec{v}\right] = 0\,. \tag{3}$$

Here $\vec{D}$ is the electric displacement and $f_\sigma(\vec{x},\vec{v};t)$ is the Vlasov distribution function for particles of species σ. In Eq. (3), the total-energy flux includes kinetic energy flux and the Poynting flux $(c/4\pi)\vec{E}\times\vec{B}$. In most treatments on plasma-energy transport that invoke the electrostatic approximation, $\vec{B}$ is simply set equal to zero and the particle heat current only considered.[4] In an anisotropic or magnetized plasma, $\vec{\nabla}\times\vec{B}c\neq\vec{0}$ and the Poynting vector is finite. Only when c is taken to be infinite can $\vec{B}$ be set equal to zero. This defines the electrostatic limit for our purposes but not the limiting form of the Poynting flux. This impasse is resolved by formulating an alternative to Poynting's theorem. Consider Joule's law again but with $\vec{E}$ rewritten in terms of potentials as follows:

$$\vec{E} = -\vec{\nabla}\varphi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\,, \tag{4}$$

where $\vec{A}$ is the vector potential from which $\vec{B}$ is derivable: $\vec{B} = \vec{\nabla}\times\vec{A}$. Insertion of Eq. (4) into Joule's law together with use of Eqs. (1) and (2) yields the following alternative to Eq. (3):

$$\frac{\partial}{\partial t}\left[\frac{|\vec{E}|^2}{8\pi} + \frac{|\vec{B}|^2}{8\pi} + \sum_\sigma \int f_\sigma(\vec{x},\vec{v};t)\frac{m_\sigma v^2}{2}d\vec{v}\right] + \vec{\nabla}\cdot\left[\frac{\partial\vec{D}}{\partial t}\frac{\varphi}{4\pi} - \frac{\vec{B}}{4\pi}\times\frac{\partial\vec{A}}{\partial t} + \sum_\sigma \int f_\sigma(\vec{x},\vec{v},t)\frac{m_\sigma v^2}{2}\vec{v}\,d\vec{v}\right] = 0\,, \tag{5}$$

where the identity $(1/4\pi)\partial\vec{D}/\partial t = \vec{j} + (1/4\pi)\partial\vec{E}/\partial t$ is used. In the electrostatic limit, i.e., $c\to\infty$, and $\vec{B}\to 0$, we obtain a local-energy conservation law for an electrostatic system:[5–7]

$$\frac{\partial}{\partial t}\left[\frac{|\vec{E}|^2}{8\pi} + \sum_\sigma \int f_\sigma(\vec{x},\vec{v};t)\frac{m_\sigma v^2}{2}d\vec{v}\right] \vec{\nabla}\cdot\left[\frac{\partial\vec{D}}{\partial t}\frac{\varphi}{4\pi} + \sum_\sigma \int f_\sigma(\vec{x},\vec{v};t)\frac{m_\sigma v^2}{2}\vec{v}\,d\vec{v}\right] = 0\,, \tag{6}$$

where the term $(\varphi/4\pi)(\partial\vec{D}/\partial t) \equiv \vec{S}_{es}$ is identified as electrostatic energy flux. Other local expressions for electrostatic energy flux are readily obtainable,[8] but the ambiguities inherent to a local consideration of transport can be overcome with an appropriate use of a spatial average. For this reason, we now analyze $\vec{S}_{es}$ within the framework of electrostatic-*wave*-energy transport.

III. ELECTROSTATIC-WAVE-ENERGY TRANSPORT

The role of $\vec{S}_{es}$ in electrostatic-wave-energy transport will now be indicated. To this end, we call upon previous work on electromagnetic-wave-energy transport.[9]

Upon expressing all field variables in terms of complex Fourier amplitudes, Eq. (1) may be reduced to the following relation:

$$\overleftrightarrow{Y}(\vec{k},\omega)\cdot\vec{E}(\vec{k},\omega) \equiv [\overleftrightarrow{\epsilon}(\vec{k},\omega) - (kc/\omega)^2(\overleftrightarrow{I} - \vec{k}\vec{k}/k^2)]\cdot\vec{E}(\vec{k},\omega) = 0\,, \tag{7}$$

where $\overleftrightarrow{\epsilon}$ is the dielectric tensor, and $\vec{k}$ and ω are, respectively, the wave vector and frequency. Equation (7) admits nontrivial solutions only if the determinant of $\overleftrightarrow{Y}$ vanishes, i.e., $\det(\overleftrightarrow{Y}) \equiv Y(\vec{k},\omega) = 0$. This condition determines the wave-vector frequency dispersion relation for electromagnetic waves. Multiplying Eq. (7) on the left by $\vec{E}^*(\vec{k},\omega)$ and allowing small perturbations in $\vec{k}$ and ω yields to lowest order an expression for group velocity $\vec{v}_g(\vec{k})$:[9]

$$\frac{d\omega}{d\vec{k}} \equiv \vec{v}_g(\vec{k}) = \frac{(\vec{E}^*_{\vec{k}}\times\vec{B}_{\vec{k}} + \vec{E}_{\vec{k}}\times\vec{B}^*_{\vec{k}})c - \omega\vec{E}^*_{\vec{k}}\cdot(\partial\overleftrightarrow{\epsilon}_H/\partial\vec{k})\cdot\vec{E}_{\vec{k}}}{|\vec{B}_{\vec{k}}|^2 + \omega\vec{E}^*_{\vec{k}}\cdot(\partial\overleftrightarrow{\epsilon}_H/\partial\omega)\cdot\vec{E}_{\vec{k}}}\,, \tag{8}$$

where $\overleftrightarrow{\epsilon}_H$ denotes the loss-free or Hermitian part of $\overleftrightarrow{\epsilon}$. The numerator of Eq. (8) represents the sum of the spatially averaged Poynting flux and nonelectromagnetic energy flux, while the denominator is associated with wave-energy density. In the electrostatic limit, Eq. (8) reduces to

$$\vec{v}_g(\vec{k}) = \frac{-(\omega/k)|E_{\vec{k}}|^2\overleftrightarrow{\epsilon}_H\cdot\hat{k} - \omega|E_{\vec{k}}|^2\hat{k}\cdot(\partial\overleftrightarrow{\epsilon}_H/\partial\vec{k})\cdot\hat{k}}{\omega|E_{\vec{k}}|^2\partial\epsilon_H/\partial\omega}\,, \tag{9}$$

where $\epsilon_H(\vec{k},\omega) \equiv \hat{k}\cdot\vec{\epsilon}_H\cdot\hat{k}$ is the real part of the dielectric response function, $\hat{k}$ is the unit vector $\vec{k}/k$, and Eq. (1) has been used to demonstrate the equivalence of the electrostatic limit of the Poynting flux and $\vec{S}_{es}$ on a spatial average. The second term in the numerator of Eq. (9) may be rewritten as follows:

$$-\omega |E_{\vec{k}}|^2\hat{k}\cdot(\partial\vec{\epsilon}_H/\partial\vec{k})\cdot\hat{k} = -\omega|E_{\vec{k}}|^2\partial\epsilon_H/\partial\vec{k} + 2\omega|E_{\vec{k}}|^2\hat{k}\cdot\vec{\epsilon}_H\cdot\partial\hat{k}/\partial\vec{k} \,. \tag{10}$$

From the electrostatic mode condition $\epsilon_H=0$, the last term in Eq. (10) can be shown to cancel exactly with the first term of the numerator in Eq. (9), leaving the electrostatic analog of Eq. (8):

$$\vec{v}_g(\vec{k}) = \frac{-(|E_{\vec{k}}|^2/8\pi)\partial\epsilon_H\omega/\partial\vec{k}}{(|E_{\vec{k}}|^2/8\pi)\partial\epsilon_H\omega/\partial\omega} \equiv \frac{\vec{S}_W(\vec{k})}{\xi(\vec{k})} \,. \tag{11}$$

Although Eq. (11) follows trivially from the definition of exact differential, its physical basis is embodied in Eq. (10). Electrostatic-wave-energy flux is the sum of spatially averaged electrostatic energy flux and wave kinetic energy flux. This wave kinetic energy flux is generally undistinguished physically except as the amount of kinetic energy flux required for the wave to carry energy at the group velocity. Only in an isotropic media can it be identified as nonresonant kinetic energy flux.[10] The electrostatic component of the wave-energy flux consists of two parts: $\vec{j}_{\vec{k}}\varphi_{-\vec{k}}+\varphi_{-\vec{k}}\partial\vec{E}_{\vec{k}}/\partial t$, where the last term is strictly field dependent. For noncyclotron-type modes, this term associated with the displacement current *solely* gives rise to energy transport across the field in the strong-field limit.

We now verify that indeed the induced charge current and kinetic energy flux have no cross-field component in such a limit. This involves use of a Fourier-Bessel expansion procedure,[11]

$$\vec{j}_{\vec{k}}(t) = \sum_{\sigma,l}\int e_\sigma J_l^2(k_\perp v_\perp/\omega_{c\sigma}) f^{(1)}_{\sigma,\vec{k},l}(v_\perp,v_\parallel;t)\vec{v}_{\sigma,l}(\vec{k})d\vec{v} \,, \tag{12}$$

$$\vec{j}^{\,E}_{\vec{k}}(t) = \sum_{\sigma,l}\int \tfrac{1}{2}m_\sigma v^2 J_l^2(k_\perp v_\perp/\omega_{c\sigma}) f^{(2)}_{\sigma,\vec{k},l}(v_\perp,v_\parallel;t)\vec{v}_{\sigma,l}(\vec{k})d\vec{v} \,, \tag{13}$$

where

$$\vec{v}_{\sigma,l}(\vec{k}) = \hat{e}_1\left[\frac{k_\parallel v_\parallel + l\omega_{c\sigma}}{k}\right] + \hat{e}_2\left[\frac{k_\perp v_\parallel}{k} - \frac{k_\parallel l\omega_{c\sigma}}{kk_\perp}\right] + \hat{e}_3\frac{v_\perp}{i}\frac{J_l'(k_\perp v_\perp/\omega_{c\sigma})}{J_l(k_\perp v_\perp/\omega_{c\sigma})} \tag{14}$$

satisfies the symmetry relation $\vec{v}_{\sigma,l}(-\vec{k}) = \vec{v}^{\,*}_{\sigma,-l}(\vec{k})$, and $f^{(1)}_{\sigma,\vec{k},l}$ and $f^{(2)}_{\sigma,\vec{k},l}$ are given in the time asymptotic limit by the following:

$$f^{(1)}_{\sigma,\vec{k},l}(v_\perp,v_\parallel;t) = \frac{e_\sigma}{m_\sigma}\varphi_{\vec{k}}(t)\frac{\left[\dfrac{l\omega_{c\sigma}}{v_\perp}\dfrac{\partial f^{(0)}_\sigma}{\partial v_\perp} + k_\parallel\dfrac{\partial f^{(0)}_\sigma}{\partial v_\parallel}\right]}{(k_\parallel v_\parallel + l\omega_{c\sigma}\omega_{\vec{k}})} \,, \tag{15}$$

$$f^{(2)}_{\sigma,\vec{k},l}(v_\perp,v_\parallel;t) = \frac{e_\sigma^2}{2m_\sigma^2}\frac{|\varphi_{\vec{k}}(t)|^2}{\gamma_{\vec{k}}}\left[k_\parallel\frac{\partial}{\partial v_\parallel} + \frac{l\omega_{c\sigma}}{v_\perp}\right]J_l^2(k_\perp v_\perp/\omega_{c\sigma})\frac{\left[\dfrac{l\omega_{c\sigma}}{v_\perp}\dfrac{\partial f^{(0)}_\sigma}{\partial v_\perp} + k_\parallel\dfrac{\partial f^{(0)}_\sigma}{\partial v_\parallel}\right]}{(k_\parallel v_\parallel + l\omega_{c\sigma} - \omega_{\vec{k}})} \,, \tag{16}$$

respectively. Here $\omega_{c\sigma}=e_\sigma B_0/m_\sigma c$ is the cyclotron frequency for particle species σ, J_l is the ordinary Bessel function of order l, $\omega_{\vec{k}}=\Omega_{\vec{k}}+i\gamma_{\vec{k}}$ is the complex wave frequency, $f^{(i)}_{\sigma,\vec{k},l}(\vec{v},t)$ refers to an expansion of $f_\sigma(\vec{x},\vec{v};t)$ in powers of $\varphi^i_{\vec{k}}(t)$, and $\{\hat{e}_\alpha\}$ are orthonormal vectors defined relative to $\vec{k}$ and the external field $B_o\hat{z}$:

$$\hat{e}_1=\vec{k}/k, \quad \hat{e}_2=\frac{\vec{k}\times(\hat{z}\times\vec{k})}{kk_\perp}, \quad \hat{e}_3=\frac{\vec{k}\times\hat{z}}{k_\perp} \,. \tag{17}$$

The directional dependence of Eqs. (12) and (13) is governed by the behavior of $\vec{v}_{\sigma,l}(\vec{k})$. In the limit as $k_\perp v_\perp/\omega_{c\sigma}\to 0$, the harmonic number summation $\sum_l$ collapses to an $l=0$ evaluation. In this case $\vec{v}_{\sigma,l}(\vec{k})$ reduces to a component only in the $\vec{z}$ direction, i.e., $\vec{v}_{\sigma,l}(\vec{k})=v_\parallel\hat{z}$. Thus the energy-flux terms involving a dependence on the particle degrees of freedom have a suppressible cross-field component when $l=0$. In contrast, the displacement current contribution does not share this feature and is fundamentally responsible for the unhindered cross-field transport of energy by noncyclotron electrostatic waves. For cyclotron modes, all terms will contribute to cross-field transport, regardless of the magnetic field strength.

It is customary in the literature to label a mode as purely electrostatic when the magnetic field component is absent. An anisotropic media ($\vec{D}\neq 0$) cannot support such a category of waves, however, when c is kept finite. Consequently, exact evaluation of the Poynting flux term involves complete knowledge of the dispersion tensor $\overleftrightarrow{Y}$.

The foregoing formulation of an electrostatic flux vector represents an attractive alternative to an exact analysis when the wave magnetic field component is small compared to the electric field component. As an illustration, we examine the composition of electrostatic-wave-energy flux for plasma modes in general.

For the case of an electron plasma wave in a strong magnetic field characterized by the dispersion relation $\Omega_{\vec{k}}=\omega_{pe}k_{\parallel}/k$, explicit evaluation of

$$\vec{S}_{es}^{(0)}\equiv\sum_{\vec{q}}[\vec{j}_{\vec{q}}\varphi_{-\vec{q}}+(1/4\pi)(\partial\vec{E}_{\vec{q}}/\partial t)\varphi_{-\vec{q}}]$$

by use of Eqs. (12), (14), (15), and (17) yields

$$\vec{S}_{es}^{(0)}=\sum_{\vec{q}}\vec{v}_g(\vec{q})\xi_{\vec{q}}\ ,$$

where $\xi_{\vec{q}}$ is the wave-energy density $|E_{\vec{q}}|^2/4\pi$. This implies that all of the wave-energy flux for this particular mode comes from $\vec{S}_{es}$ and *none* from the kinetic energy flux component of Eq. (10). A more general argument can be advanced by dot multiplying Eq. (10) with $\hat{k}$ and using Eq. (11) to yield

$$\hat{k}\cdot\vec{v}_g(\vec{k})\xi_{\vec{k}}=-\hat{k}\cdot[\hat{k}\cdot(\Omega_{\vec{k}}\partial\overleftrightarrow{\epsilon}_H/\partial\vec{k})\cdot\hat{k}\,|E_{\vec{k}}|^2/8\pi]\ , \quad (18)$$

where the quantity in square brackets corresponds to wave kinetic energy flux. For modes characterized by $\vec{v}_g(\vec{k})\cdot\hat{k}\neq 0$, this component is nonvanishing. Since $\vec{v}_g(\vec{k})\cdot\hat{k}=0$ for exclusively an electron plasma wave, other modes will generally exhibit a mixture of electrostatic and kinetic energy flux in an anisotropic media. Note that while the wave-energy density for an electron plasma wave is split evenly between the electrostatic and kinetic energy, the *transport* of this energy is due entirely to the electrostatic component. Other modes in general have only a very small fraction of electrostatic field energy involved in the total wave energy. However, the relative role assumed by the field component becomes considerably enhanced as this wave energy is transported in a strongly magnetized plasma. It becomes quite evident that the integrity of wave-energy transport and wave-energy density are very distinct. From the perspective of the confinement challenge, these transport features of electrostatic waves clearly comprise the more relevant description of wave properties.

For an isotropic, electrostatic plasma, the electric displacement $\vec{D}(\vec{k},\omega)$ vanishes leaving kinetic energy flux as the sole surviving energy transport constituent. This agrees with previous claims that electrostatic-wave-energy flux is composed entirely of wave-induced kinetic energy flux.[7]

In Sec. II, we derived an expression for local-energy flux. Although the form obtained was convenient for taking the electrostatic limit, an ambiguity arose in the sense that the curl of any vector field could be added without altering the form of the local-energy conservation equation. Only through the use of an appropriate spatial average could energy flow be uniquely described. Note how the identification of group velocity and velocity of energy transport in Eqs. (8) and (11) automatically incorporated a spatial average and resolved the uniqueness question. For this reason alone, waves which transport energy at the group velocity are the sole physical entities of interest. Occasionally, articles appear in the literature which purport to show how wave group velocity and velocity of energy transport may differ in particular physical situations.[12] In such cases, the suitability of the spatial average taken relative to the geometry assumed warrants more careful scrutiny.

IV. PLASMA-WAVE-ENERGY CONDUCTION

Until now, the undiminished transport of electrostatic wave energy has exclusively occupied our attention. We have come to understand the character of electrostatic-wave-energy transport and the crucial role of displacement current in allowing for unhindered cross-field energy transport. However, the significance of wave transport can only be realized once the waves have interacted with the particles. Then, the entire process of convective wave-energy losses will be completed. This section deals with the latter step of this process from the standpoint of wave-energy conduction. As damping mechanisms degrade the wave-energy current, an accompanying spatial nonuniformity in wave-energy density will arise. This inspires a consideration of a Fourier-type phenomenological description of wave-energy transport in a nonthermal, collisionless environment.

Consider the steady-state wave-kinetic equation,

$$\vec{v}_g(\vec{k})\cdot\partial\xi_{\vec{k}}/\partial\vec{x}=\sum_i 2\gamma_{\vec{k}}^{(i)}\xi_{\vec{k}}\ , \quad (19)$$

where $\gamma_{\vec{k}}^{(i)}$ refers to a particular type of wave-particle interaction, e.g., linear or nonlinear Landau damping. If Eq. (19) is inserted into Eq. (11), the following alternative description for wave-energy current $\vec{S}_W$ is obtained:

$$\vec{S}_W(\vec{k})\equiv-\overleftrightarrow{\kappa}_W\cdot\vec{\nabla}\xi_{\vec{k}}=-\left[\frac{\vec{v}_g(\vec{k})\vec{v}_g(\vec{k})}{\sum_i 2\gamma_{\vec{k}}^{(i)}}\right]\cdot\vec{\nabla}\xi_{\vec{k}}\ , \quad (20)$$

where the plasma-wave-energy conduction tensor $\overleftrightarrow{\kappa}$ is appropriately defined. Equation (20) supersedes the previous thermal equilibrium version by not invoking temperature and an effective particle-particle collisional mean free path to model the wave mean free path.[1]

Because quasiparticles are not conserved in number, a fundamental distinction between quasiparticle and particle thermal conduction naturally arises. In the latter case, an open circuit constraint ensures no net momentum of the particle system. Specifically, convective effects are excluded. However, no such constraint can be imposed on quasiparticles since their number is not fixed. Therefore, in the absence of wave scattering mechanisms, wave energy "conductivity" is infinite since a finite energy current still persists. In the presence of a damping mechanism, knowledge of $\overleftrightarrow{\kappa}_W$ in Eq. (20) allows one to determine the effectiveness of a given wave-particle interaction in degrading a convective wave-energy flow. This alternate representation of $\vec{S}_W$ in terms of $\gamma_{\vec{k}}^{(i)}$ quantitatively reveals the explicit role of damping in affecting wave-energy *flux*.

V. DISCUSSION

Whether the intentional introduction of wave energy or an unintentional loss of wave energy is considered, a thorough description of cross-field electrostatic-wave-energy flux is needed. From an energy-density standpoint, the various properties of electrostatic waves are well understood. The same does not hold true within the context of energy transport. Simply multiplying the energy density by the wave group velocity is the usual avenue to a "thorough" treatment of electrostatic-wave-energy transport. Of primary concern to us has been the directional features of group velocity for an anisotropic plasma in the electrostatic approximation. In this approximation, a usual neglect of the Poynting flux implicitly renders the kinetic-energy flux as the dominant constituent of cross-field wave-energy flux. If such were indeed the case, an increasing magnetic field strength would reduce cross-field wave-energy convection since the particle degrees of freedom assume an ostensibly dominant role. The consequences of this for either the lower-hybrid wave heating schemes or cross-field leakage of wave energy are apparent. We have determined that the displacement current assumes a crucial role in supporting cross-field electrostatic energy transport. As this term is strictly field dependent, the imposition of an external magnetic field will have no effect in suppressing it. Since the displacement current has also been found to be a significant feature of cross-field wave-energy transport, previous concern about electrostatic wave-energy losses is well founded.[1–3]

The main concerns with confinement generally focus around assorted transport phenomena. Yet, the understanding of the critical roles assumed by the field degrees within a transport framework has been incomplete. Already, we have noted how a transport representation reveals a startling preponderance of field energy transport over field energy density in waves. Furthermore, we observed how the concept of wave-energy conductivity provided direct measure of the role of wave damping in the degradation of wave-energy *current.*

At present the problem of energy transport in tokamaks is an unsolved problem that is treated with largely empirical scaling laws by experimentalists. Among the currently favored theoretical proposals to illuminate the transport problem are unstable drift modes and tearing modes that lead to anomalous transport by particles. Rough calculations indicate that such anomalous transport could explain the experimental observations to date. However, one cannot legislate that future measurements will conform to these theories. Indeed the past history is that practically every new machine has produced surprises. The purpose of this paper has been to emphasize that the transport by waves instead of particles is usually of the same order of magnitude and can be much larger in a sufficiently strong magnetic field.

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