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Anomalous mode pulling, instabilities, and chaos in a single-mode, standing-wave 3.39-μm He-Ne laser

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A high-gain, single-mode He-Ne 3.39-μm laser shows extreme mode pulling, as the laser operating frequency varies up to three times more slowly than the corresponding variation of the empty-cavity frequency. When the laser is operated at low pressures, new features appear in the vicinity of, and in conjunction with, the Lamb dip. Dispersive effects from the overlapping of the holes distorts the more nearly linear mode pulling observed for high pressures near line center. When the dispersive effects are strong enough instabilities are observed including self-pulsing, quasiperiodic, and chaotic output. At sufficiently high gain, the dispersion is distorted to give a multivalued region for the laser operating frequency for a single cavity frequency as predicted by Bennett.

I. INTRODUCTION

High-gain lasers have been of particular interest because of the associated anomalous dispersion which enhances many otherwise small corrections to simple laser analyses. When the laser medium is also inhomogeneously broadened, the dispersive effects of the holes burned in the gain profile can lead to single-mode self-pulsing, mode splitting and anomalous tuning of the laser frequency. Several of these effects are studied here using the 3.39-μm transition in neon in a low-Q, Fabry-Perot laser.

II. BACKGROUND

The analysis of single-mode inhomogeneously broadened lasers was provided very early in the development of laser theory by the pioneering contributions of Bennett[1-3] and Lamb.[4] Among the classic predictions were the phenomenon of hole burning by selective saturation of narrow velocity classes of atoms within the Doppler-broadened transition and the appearance of the Lamb dip, a reduced power output when the laser was tuned within a homogeneous linewidth of the resonance line center. General treatments of the dispersive effects of holes burned in the population inversion also lead to an understanding of many forms of mode competition (through "hole repulsion") and other effects variously termed "mode pushing" or "mode pulling." The effects of pressure broadening and interatomic collisions have also been investigated in detail.[5]

One of the often neglected effects predicted by Bennett [see Ref. 3, p. 128] was that as a single-mode laser was tuned through the resonant frequency of the transition (in the vicinity of the Lamb dip), the mode would interact with the dispersive effects of the "mirror-image" hole (burned because of bidirectional propagation of the beam) causing a distortion of the simple mode-pulling effects that would be observed, for example, in a single-mode ring laser. Bennett[6] was able to confirm the existence of this effect by finding that mode pulling could be distorted a few tens of kilohertz out of a few tens of megahertz in a He-Ne 1.15-μm laser. More extreme effects were predicted for higher-gain and lower-Q cavities, but until the present work, no studies have been made to observe the predicted region of multiple frequencies.

Most mode-pulling studies have been made in laser systems where the effects have been, at most, a few percent.[6-8] However, higher-gain systems, such as the 3.51-μm line in xenon in particular, have shown very large anomalous dispersion leading to mode-pulling factors of from 2 to 50.[9-12] These studies were sparked in part by the prediction of special effects in such cases.[13] More recently, renewed interest in the theory of inhomogeneously broadened lasers has also focused on dispersive effects in order to understand the appearance of spontaneous pulsations in dc-excited systems.[14-20] Casperson[15] has developed a simple intuitive picture which helps to demonstrate how dispersive effects may lead to the development of sidebands to the laser operating frequency manifesting themselves as pulsations in the intensity of the laser.

These oscillations have been observed in high-gain gas laser systems, including standing-wave xenon lasers at 3.51 μm,[11,12,14,21-23] unidirectional ring lasers at 3.51 μm,[24] and in standing-wave helium-neon lasers at 3.39 μm.[25] The theories, however, have concentrated almost entirely on predictions for unidirectional ring lasers for simplicity. Reported here is an experiment using a Fabry-Perot helium-neon laser which displays both phenomena, self-pulsing instabilities and anomalous (and multivalued) mode pulling.

III. EXPERIMENTAL SETUP

The experiments reported here used a laser tube and cavity specially designed for high-gain and single-mode operation. A schematic is shown in Fig. 1. This laser has been used previously in the study of single-mode instabilities in xenon lasers.[11,12,21-23] The 909-MHz free spectral range helps to ensure only single longitudinal mode operation, while the 1-mm-diam capillary tube enhances the population inversion (presumably by improving depletion of the lower level through wall collisions). The relatively low-Q cavity (using one 50% reflecting mirror) is necessary for observations of both the single-mode instabilities and the unusually high degree of mode pulling. Considering mirror and diffractive losses as well as absorption in the quartz windows, we calculate a cavity lifetime of 1 nsec, or equivalently, a decay rate for the electric field of 5×10^{8} sec^{-1}.

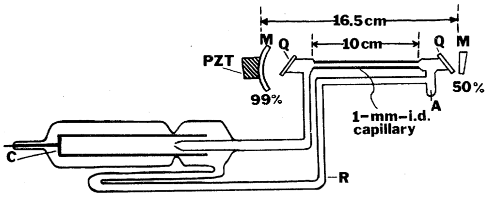

FIG. 1. Schematic diagram of high-gain, single-mode laser. *A*, anode; *C*, cold cathode; *R*, nonexcited return path; *Q*, quartz windows at Brewster's angle; *M*, mirrors (20-cm focal length and wedge); PZT, piezoelectric mirror drive.

The laser was filled with a mixture of single-isotope neon gas (99.9% ^{20}Ne) and research-grade helium at pressures of 150 and 500 mTorr, respectively. The natural linewidth of the neon 3.39-μm transition caused by the spontaneous decay of the upper and lower levels is 20 ± 2 MHz.[26–30] The pressure broadening coefficients have been measured to be 12.4 ± 4.9 MHz/Torr of neon and 25.6 ± 0.6 MHz/Torr of helium.[31] These numbers are in rather considerable dispute from a survey of the literature,[26–35] however, our results indicate that the numbers quoted by Ohi are much more consistent with the data than the generally higher numbers cited elsewhere. The generally accepted value for the Doppler broadening of the transition in a dc discharge is 290 ± 10 MHz.[26,27,35,36]

From our operating conditions we calculate a homogeneously broadened linewidth of 35 ± 3 MHz giving an inhomogeneous to homogeneous linewidth ratio of 9:1 and a polarization decay rate of 10^{8} sec^{-1}. These operating conditions are the same as those used earlier in the discovery of single-mode instabilities in the operation of a 3.39-μm laser using a natural isotopic mixture of neon (91% ^{20}Ne, 9% ^{22}Ne).[25] Note that the cavity linewidth is greater than the polarization decay rate, a condition necessary for the observation of single-mode instabilities.[15–20]

The laser cavity could be tuned through variations in the voltage applied to the piezoelectric crystal on which the spherical mirror was mounted. Nonlinearities in the crystal expansion versus voltage were calibrated using an external He-Ne 0.6328-μm laser and the experimental laser cavity as a passive cavity interferometer.

The laser was operated in a cold-cathode mode with dc excitation using a 1–3 kV power supply and a 100-kΩ series ballast resistor. The power supplies used were measured to have less than 0.2% ripple for the maximum power output used in these experiments.

In order to measure the laser operating frequency, a single-mode reference laser system was designed as shown in Fig. 2. A first single-mode laser was feedback stabilized to the peak of the its output profile after amplification by an external gain cell. Modulating the piezoelectric crystal voltage at a 5-KHz rate with an amplitude that resulted in an optimal frequency modulation of ±5 MHz, an error signal was generated to keep the laser locked to within 0.5 MHz of the peak. As this modulated laser was unsuitable for service as the desired reference laser, a second single-mode laser was added and heterodyned with the first. The resulting modulated beat note generated an error signal at a second lock-in amplifier after detection through an rf tuned filter. The second laser could thus be locked to a frequency offset from the first, within 0.5 MHz of the center of the tuned filter. The tuned filter (actually an rf spectrum analyzer used in a tuned filter mode) could be adjusted to set this stabilized, unmodulated reference laser to any chosen value within the central 100 MHz of its tuning range. A portion of the output of the stabilized (but unmodulated) reference laser was heterodyned with the laser under study with the resulting beat note providing the desired information on the laser operating frequency (or frequencies). The same detector was used to observe the rf power spectrum of the pulsations of the laser when it went unstable.

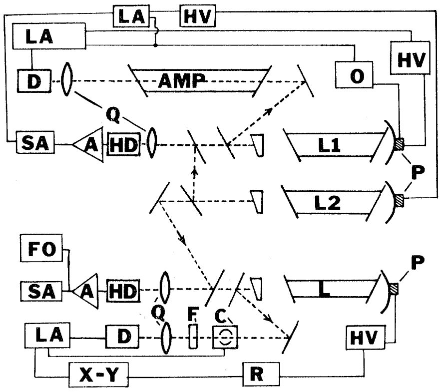

FIG. 2. Schematic of optical bench setup for detection of single-mode laser output and providing of a reference laser frequency: reference lasers *L* 1, *L* 2; laser under study, *L*; external optical amplifier, AMP, quartz lens, *Q*; InAs detector, *D*; reverse-biased InAs detector, HD; piezoelectric mirror drives, *P*; high-voltage optical amplifier, HV; ramp generator, *R*; oscillator, *O*; lock-in amplifier, LA; spectrum analyzer, SA; fast oscilloscope, FO; integral preamplifier, *A*; ir line filter, *F*; plotter, *X-Y*; chopper, *C*.

IV. EXPERIMENTAL RESULTS

Using a programmable high-voltage power supply, a scan of power output discharge current was obtained for fixed cavity length. The laser was allowed to come to thermal equilibrium at an operating current of about 8 mA before the scan was run so that the sudden heating by the increased current would not cause thermal drifts in the cavity length. The results are shown in Fig. 3. The

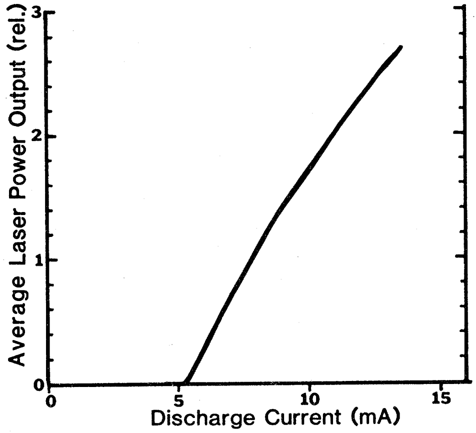

FIG. 3. Average laser power output versus discharge current using a programmable high-voltage power supply.

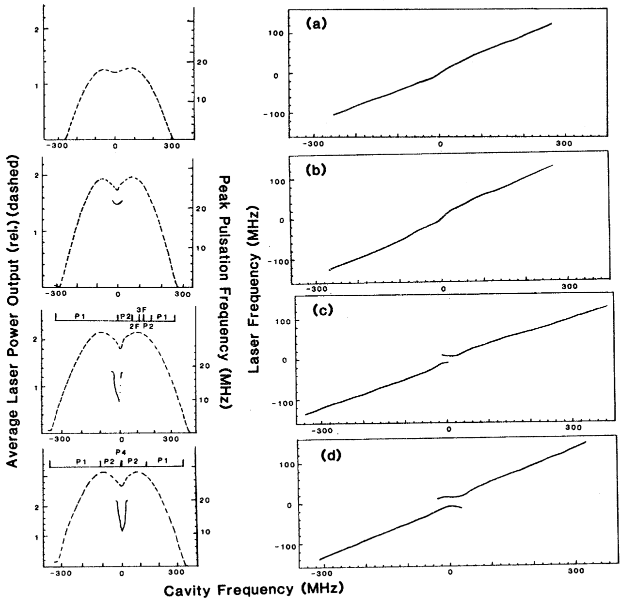

FIG. 4. Average laser power output (dashed line) versus cavity detuning with frequencies of principal peaks in the rf power spectrum of the observed single-mode instabilities as indicated (left). Laser operating frequency or frequencies as determined from heterodyned beat notes with the reference laser versus cavity frequency (right). Discharge currents are (a) 8.0, (b) 10.8, (c) 11.8, and (d) 16.5 mA. Expanded scale regions in (c) and (d) show limits of different dynamical behavior such as simple periodic pulsations ($P1$) as in Fig. 5(a), the appearance of a subharmonic or "period-2" oscillation ($P2$) as in Figs. 5(b) and 5(c), the appearance of an addition subharmonic or "period-4" ($P4$), simultaneous operation of two relatively incommensurate frequencies ($2F$) as in Figs. 5(d) and 5(f), and the relatively anomalous three-frequency region [5(e)]. Expanded scale for Fig. 4(c), -41 MHz to $+15$ MHz and for Fig. 4(d), -31 MHz to $+31$ MHz.

threshold discharge current is 5.2±0.1 mA. The initially linear behavior of the curve in the 5.5–8-mA range is to be expected for a mixed broadening case and the rolloff at higher currents can be explained as evidence of discharge excitation of the lower lasing level.[2] However, on the assumption that the linear relationship between gain and power output continues at high currents, we can use the power output to scale the threshold conditions of the laser even in the nonlinear region of Fig. 3.

Figure 4(a) shows a laser power output scan versus cavity detuning. Using the measurements of the actual laser operating frequency the full width at half maximum (FWHM) of the Lamb dip is found to be 52±6 MHz. This is in reasonable agreement with the predicted homogeneously broadened linewidth of 35±3 MHz and theories of high intensity layers.[37–39]

In addition, the value of the pulsation frequency (21 MHz) at the instability threshold (laser threshold parameter ~1.7) should provide a sensitive prediction of the homogeneous linewidth (cf. Refs. 16–20). While formulas are not provided for the particular case of He-Ne, estimates of the homogeneous linewidth [using, for example, Eq. (5.1) of Ref. 19 and similar results in Refs. 16–20] yield values in the range of 21–50 MHz. The agreement is quite good but more general theory and more precise spectroscopic data on the 3.39 transition are required before precise comparisons can be made.

Figures 4(a)–4(d) show combinations of laser power output versus cavity length, peaks in the intensity power spectrum (cf. Fig. 5) versus cavity length, and laser operating frequency versus cavity detuning. A single-mode instability is observed at the center of the Lamb dip as seen previously[25] using natural isotopic neon.

Figure 5 shows sample rf power spectra of the laser intensity from the regions of instabilities shown in Fig. 4. With small changes in detuning as shown on the expanded scales of Figs. (4c), and 4(d); the pulsation character changes from high-Q pulsations to two-frequency operation, or gives evidence of a "subharmonic bifurcation" (period doubling) with other cases [such as the transition from 5(b) to 5(c)] showing broadening of the peaks and the addition of a broadband spectral component characteristic of deterministic chaos. Details of the transition of single-mode laser systems from periodic to chaotic instabilities are the subject of recent theoretical[17,40] and experimental[12,22–25] reports. (See Ref. 41 for general discussions of dynamical instabilities and the various routes to chaotic behavior in diverse nonlinear systems.) An unusual feature is the three-frequency spectrum shown in Fig. 5(e). Three-frequency operation is of considerable interest because of theoretical disputes over its possibility which seem to be only recently resolved.[42]

Two indirect methods were used to estimate the laser threshold conditions at line-center operation for different currents. The threshold parameter is defined to be the ratio of the unsaturated gain at the lasing threshold. From the power output versus current in Fig. 3 an initially linear dependence can be noted and a clear threshold value is evident. Assuming that nonlinear effects may be due to excitation of the lower atomic level[2] and that the power output scales linearly with threshold parameter the values

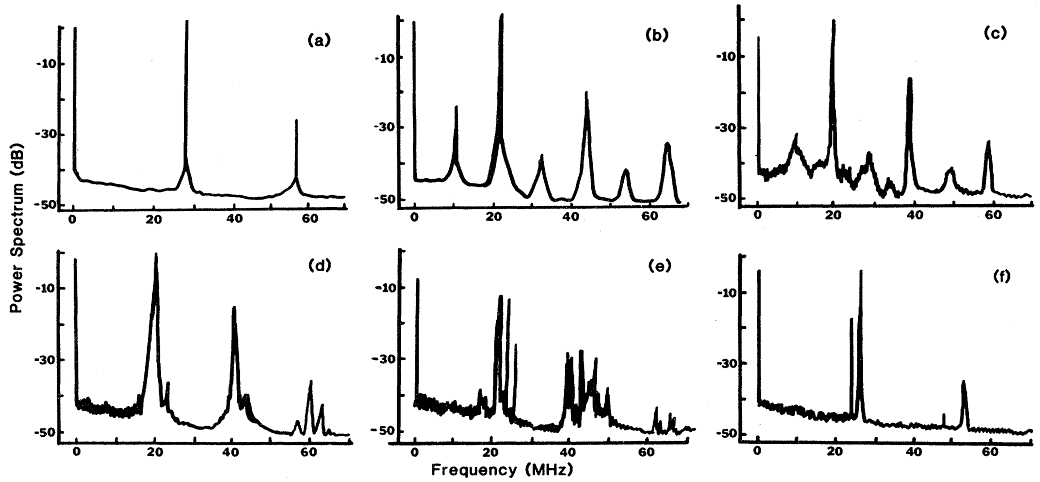

FIG. 5. Sample rf power spectra from the regions of instability indicated in Fig. 4. Samples show typical high-Q period doubling with a broadband chaotic background (b), period doubling without chaos (c), and the appearance of two distinct frequencies from an initial near degeneracy in the sequence (d-f) from quasiperiodic (d) and multiply periodic (e) behavior to stable two-frequency operation (d).

in the first column of Table I could be estimated. However, for conditions of mixed homogeneous and inhomogeneous broadening no analytic scaling law is available, nor does one exist for operation in the presence of instabilities. The line-center threshold parameter values tabulated in the second column were calculated by taking the measured tuning range of the laser operation and the assumption of an approximately Gaussian gain profile with a 292-MHz FWHM ($[(\Delta\nu_D)^2+(\Delta\nu_h)^2]^{1/2}$). As the laser was stable at the extremes of its tuning range this calculation is unaffected by instabilities and thus may be the more reliable estimate of the unsaturated small signal gain. In any case, it is likely that the true threshold conditions were bounded by the values calculated from these two methods. The discrepancies for higher currents cannot readily be explained quantitatively without a more detailed theory of laser operation well above threshold for the unstable single-mode laser. However, the larger threshold parameter values based on the power output are consistent with the results of calculations by Casperson which show increased average power for unstable pulsations in comparison with the cw average power predicted by the steady-state solutions.

A distortion of the nearly linear mode pulling at low gain appears as the laser is operated further above threshold. First a kink develops [Fig. 4(a)] and then an actual break occurs [Fig. 4(c)]. The region of the break is not to be interpreted as the overlapping or hysteresis of single-frequency operation. Rather, observation of the beat frequency between the laser under study and the single-frequency laser shows the emergence of two equally strong beat frequencies.

TABLE I. Calculated laser threshold parameter values.

Discharge current (mA)	Threshold parameter values from power output	Threshold parameter values from detuning range
8.0	1.49±0.05	1.57±0.04
10.8	1.90±0.05	1.73±0.04
11.8	2.1 ±0.1	1.83±0.02
16.5	2.6 ±0.2	1.94±0.02

Bennett's[3] original prediction of high-gain operation was for a multivalued equation of state which would manifest itself as hysteresis in the detuning plots. The more recent theoretical work has demonstrated that the steady-state single-frequency solution is unstable[14–20] so our observations must be taken as measurements of the dynamical state of the self-pulsing laser and not of the predicted steady-state solution. We observe in Fig. 4 that instabilities have already emerged for these threshold conditions and that the apparent multivalued-frequency region requires even higher threshold values with coincidently stronger and more complex instabilities.

V. DISCUSSION

Bennett has provided an approximate formula for the anomalous mode pulling expected in a standing-wave laser:[3]

$$\nu_{\rm cav}=\nu_l+\frac{c}{(F/L)}\Delta\varphi_g(\nu_l)+\Delta\varphi_{\rm hole}(\nu_l)\,, \tag{1}$$

where $\nu_{\rm cav}$ is the cavity frequency, ν_l is the laser oscillation frequency, F is the fractional loss per pass, L is the length of the gain medium, $\Delta\varphi_g$ is the phase shift for the unsaturated gain profile, and $\Delta\varphi_{\rm hole}$ is the phase shift for the hole burned in the gain profile. Using values suitable or typical for our experimental setup, the curves shown in Fig. 6 were generated using Bennett's approximate formula. The formula is limited in applicability to near the center of the Doppler-broadened profile and does not take into account power broadening of the hole or variation in

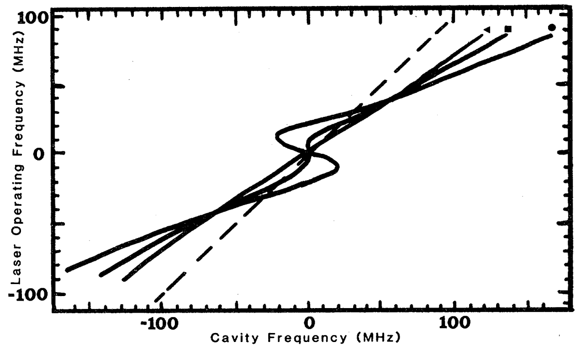

FIG. 6. Plot of Bennett's mode [Eq. (1)] for steady-state laser frequency versus cavity frequency using He-Ne parameters of $\Delta\nu_D=300$ MHz, $\Delta\nu_c=150$ MHz, $\Delta\nu_h=25$ MHz, $F=0.3$, $L=0.1$, and threshold values of $r=1.0$ (▲), 1.67 (■), and 3.0 (●).

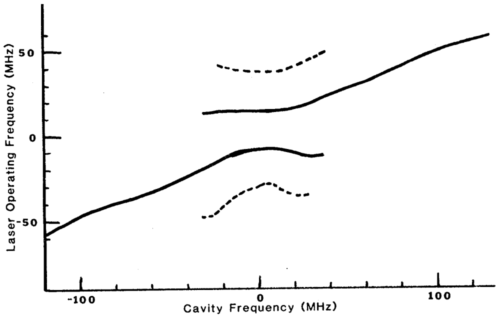

FIG. 7. Expanded scale plot of Fig. 4(d) data including frequencies of sidebands to the primary laser frequencies.

the hole depth with detuning.

The qualitative agreement between the theoretical curves and the experimental ones shown in Fig. 4 indicates that even in the presence of the instabilities, the laser operates close to the steady-state prediction for the optical frequency. Figure 7 shows an expanded scale version of the data in Fig. 4(c) including the weak sidebands that are generated by the instability. A reasonable agreement with the theoretical curve for the steady-state solution is obtained.

The changes in the shape of the Lamb dip in the power output versus detuning curve are distinctive with the onset of strong self-pulsing. The dip appears to deepen and narrow as the pulsing becomes stronger. The dip widens as the laser is raised even further above threshold.

Simple low-gain interpretations of the Lamb dip cannot easily be extended to this case. Not only is there strong mode pulling and the associated distortions predicted by Bennett, but the unstable operation of the laser means that there are significantly strong optical sidebands to the main laser frequency. Often there are two equally strong optical frequencies. No simple mapping is then possible between the cavity frequency and operating frequency as the latter no longer exists uniquely. Similarly, any simple hole-burning picture breaks down as well. A full dynamical model for the standing-wave case is likely to be required to explain these observations in detail.

Study of the detuning curves indicates that the coexisting frequencies are within 20 MHz of each other. The switchback in the steady-state curves from the Bennett theory shows a separation of the curves that is more nearly equal to the homogeneous linewidth. The combination of a narrow Lamb dip under stable operating conditions and the close spacing of the coexisting frequencies may suggest that either or both of the pressure-broadening effects or the natural linewidth have been overestimated previously. Detailed study of standing-wave laser operating conditions in the presence of single-mode instabilities will be required before these discrepancies can be resolved. Preliminary studies have indicated qualitative agreement with the features of an enhancement of the instability at the Lamb dip, a dip in the pulsing frequency at the Lamb dip, and a narrow range of detunings that support the instability.[43,44]

VI. SUMMARY

Studies of the single-mode laser operation in a high-gain medium using a low-*Q*, Fabry-Perot cavity indicate both spontaneous pulsations and anomalies in the mode pulling. Features include a narrow range of pulsations at the Lamb dip, and an increase in the depth (and narrowing of the width) of the dip in the power output versus detuning curves. A link is thus established between the dispersive effects of hole burning predicted for steady-state laser operation and the more recently predicted dispersion-induced instabilities for hole burning in inhomogeneously broadened lasers.

Note added in proof. It has been brought to our attention that the high intensity gas laser theory papers of Refs. 37—39 predict population pulsations in "steady state" which may be the precursors of a Lamb dip enhanced laser instability. We also note with interest a new paper on mode pushing in a single mode laser.[45]

VII. ACKNOWLEDGMENTS

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