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Exactly solvable model of a physical system exhibiting multidimensional chaotic behavior

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A generalization to two dimensions of the model given previously for the driven diode resonator is used to explain the main features of the actual system. Measurements of the characteristics of the diode are shown to justify the new assumptions. Thus, we have isolated the physical characteristic of the diode responsible for the multidimensional chaotic behavior of this system.

We have shown previously[1] that a model of a simple nonlinear system, the driven diode resonator, comprised of an oscillator, resistor, inductor, and diode in series, can be solved exactly to yield a one-dimensional map which exhibits the period doubling route to chaos. The model of the diode includes a forward bias voltage, junction capacitance, and a reverse recovery time. The reverse recovery time was chosen to be a simple saturating function of the maximum forward current $|I_m|$ through the diode during a cycle. The map, formed by plotting $|I_m|_{n+1}$ vs $|I_m|_n$ for certain values of the circuit parameters in the chaotic regime, is a smooth unimodal function. Existing[2,3] and new measurements given below on the actual system show that the map has more than one branch and, furthermore, the period-3 window shows hysteresis whereas the model did not.

The purpose of this article is to demonstrate that these effects can be explained easily using our model with a simple modification: the diode recovery time depends not only on $|I_m|_n$, but also on $|I_m|_{n-1}$. We will first show an experimental map, then give experimental evidence for the diode being able to "remember" two cycles, and finally incorporate this into our model and compare the results with experiments.

Figure 1 shows the map obtained just below the period-3 stable orbit. The circuit is the series combination of a 14-mH inductor with an internal resistance of 80 Ω and a 1N1221 Si *p*-*n* junction rectifier driven at 100 kHz by a generator with negligible internal resistance. The current is converted to a voltage using an operational amplifier. Every several cycles the peak current and the next peak current, approximately one period later, are sampled and held, and applied to the *x* and *y* inputs of an oscilloscope. The technique is similar to that of Jeffries and Perez,[4] who show a map that is quite different than ours.[2] The difference may be due to their sampling the current at a different point in the cycle. In all cases the resulting maps show branches indicating a higher dimensionality.

The higher-dimensional character could be caused by a number of effects: the voltage-dependent capacitance, the curved *I*-*V* characteristics, or a more complicated function relating the recovery time to the forward current. Experiments using the transistor analog circuit,[5] which simulates the diode, clearly indicate that the recovery time (which can be varied) is the cause of the effects. With a very short recovery time a nearly one-dimensional map could be obtained, and by lengthening the time, maps similar to that shown in Fig. 1 could be obtained. Further increasing the time would result in maps with many branches.

Figure 2 is a double exposure of the voltage across the diode in response to first one, then two voltage pulses applied to our diode. Pulses 2 μsec long and separated by 10 μsec (to mirror the time scale used in the experiments) were applied to the diode in series with a 1.5-kΩ resistor. As described in texts,[6] the voltage across the diode initially rises to 0.5 V—the forward bias voltage. After the applied voltage pulse returns to zero the diode recovers in two

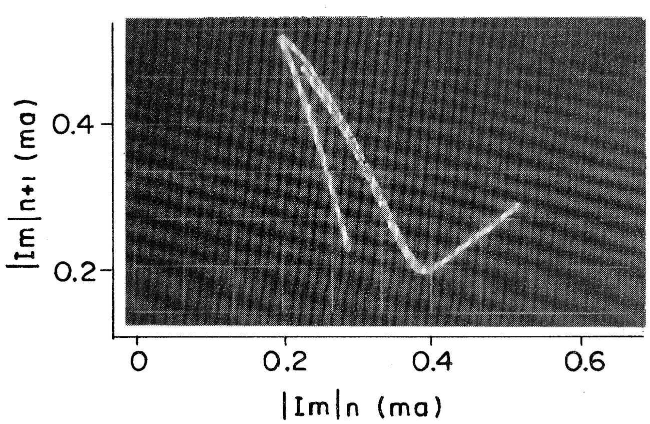

FIG. 1. Measured map, $|I_m|_{n+1}$ vs $|I_m|_n$, of the current peaks just below the period-3 window.

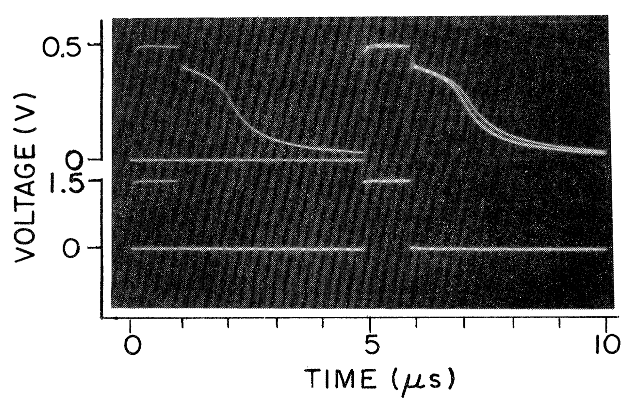

FIG. 2. Double exposure of the response of the diode to one- and two voltage pulses through a 1.5-kΩ resistor. The recovery time following the two-pulse sequence is longer. The applied pulses (lower trace) are 2 μsec long and occur 10 μsec apart.

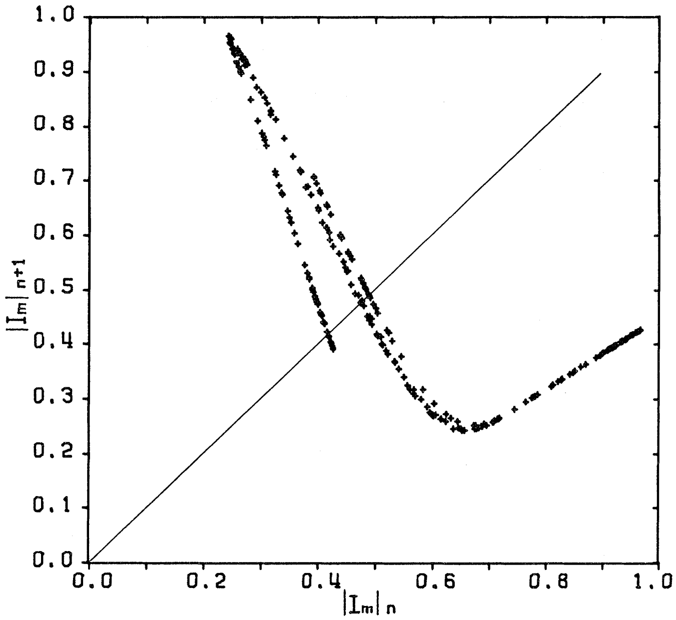

FIG. 3. Calculated map of the current peaks in units of V_f/R. The parameters, defined in Ref. 1 and Eq. (1), are $V_0/V_f = 6.00$, $\omega/\omega_0 = 1.00$, $Q = 15.0$, $\omega_0\tau_m/2\pi = 1.2$, $I_cR/V_f = 2.00$, and $\alpha = 0.4$.

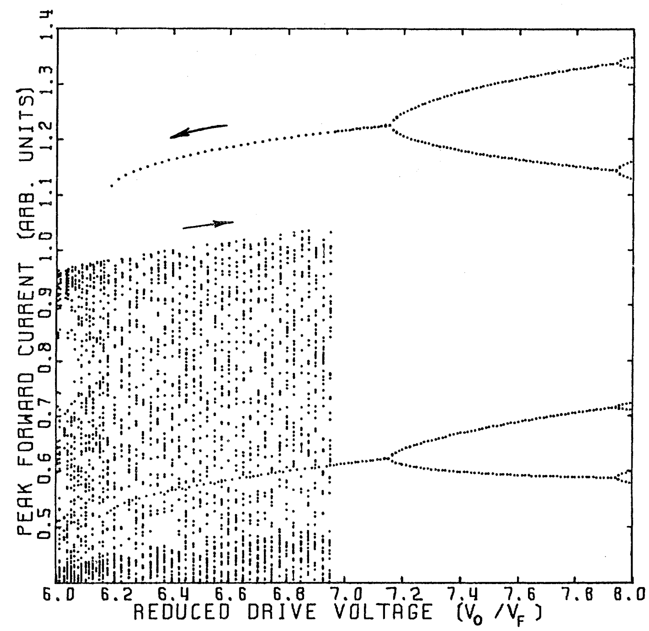

FIG. 4. Portion of the bifurcation diagram in the vicinity of the period-3 window showing the hysteresis. This may be compared with the experimental diagram given in Figs. 2 and 4 of Ref. 4.

steps. The voltage remains nearly constant for the first step, while the injected minority carriers diffuse back across the junction and supply a current through the 1.5 kΩ resistor and pulse generator. The time for this first step, which we call the recovery time, τ_r, increases with the forward current because of the increased number of minority carriers. This recovery time also increases with the circuit resistance unless the time exceeds the recombination time, which in our case is about 100 μsec. The second step is the junction capacity discharging through the source resistance.

Figure 2 shows that if a second pulse is applied before the diode has completely recovered, the recovery time following the second pulse has increased. It is apparent that the diode has a short-term memory that is not totally erased by a second conduction phase.

In our previous work we chose a simple function for the diode recovery time, $\tau_r = f(|I_m|)$ for the nth cycle, where

$$f(|I_m|) = \tau_m[1 - \exp(-|I_m|/I_c)] \ ;$$

$|I_m|$ is the maximum forward current during the previous conducting cycle, and τ_m and I_c are parameters describing the particular diode. Since τ_r also depends on the current during the previous cycle, we choose for simplicity

$$\tau_{r,n} = f(|I_m|_n) + \alpha f(|I_m|_{n-1}) \ , \tag{1}$$

where α is a parameter. It follows from Ref. 1 that

$$|I_m|_{n+1} = F(|I_m|_n, |I_m|_{n-1}) \ , \tag{2}$$

which is a form of a two-dimensional map. Successive iterations of Eq. (2) just below the period-3 window give the map shown in Fig. 3. The parameters are given in the figure caption. The calculated map has all the pertinent features of the experimental map shown in Fig. 1. Only minimal efforts were made adjusting the parameters to obtain this map.

Using the model we find the hysteresis associated with the period-3 window as described by Jeffries and Perez.[4] Figure 4 shows a portion of the calculated bifurcation diagram upon increasing and decreasing the drive voltage in small steps.

The branches of the map and the hysteresis are the two most noteworthy features of multidimensional behavior in the diode resonator. Other features were reported very recently by Brorson, Dewey, and Lindsay,[7] who observe a self-replicating attractor, hysteresis, and bifurcation diagram with frequency-adding sequences similar to that described by Jeffries.[8] We shall show in a future article that our model reproduces these features simply by changing the parameter τ_m. Jeffries[8] and Brorson *et al.*[7] also give models which yield qualitative agreement with the data. Their models are similar to ours in that their large forward differential capacitance gives the diode the combined characteristics of our forward bias voltage and reverse recovery time. Our model has several advantages: it clearly indicates which physical properties are essential for the behavior, it can be solved exactly, and it provides a clear connection with mapping theory. The last feature can be very helpful in gaining a qualitative understanding of the behavior of the system.

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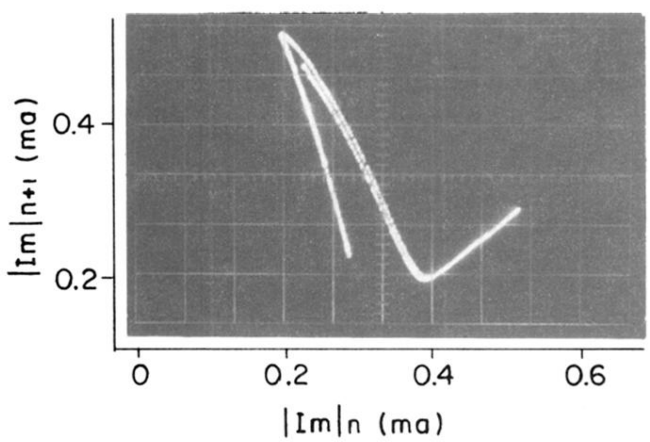
[1]R. W. Rollins and E. R. Hunt, Phys. Rev. Lett. 49, 1295 (1982).
[2]C. Jeffries and J. Perez, Phys. Rev. A 26, 2117 (1982).
[3]H. Ikezi, J. S. deGrassie, and T. H. Jensen, Phys. Rev. A 28, 1207 (1983).
[4]C. Jeffires and J. Perez, Phys. Rev. A 27, 601 (1983).
[5]E. R. Hunt, Phys. Rev. Lett. 49, 1054 (1982).
[6]See, for example, J. Millman and C. Halkias, *Electronic Devices and Circuits* (McGraw-Hill, New York, 1967).
[7]S. D. Brorson, D. Dewey, and P. S. Lindsay, Phys. Rev. A 28, 1201 (1983).
[8]C. Jeffries, reported at the NATO Advanced Study Workshop, June, 1983, Haverford College, Haverford, PA. Workshop reviewed by N. B. Abraham, J. P. Gollub and H. L. Swinney [Physica D (to be published)].



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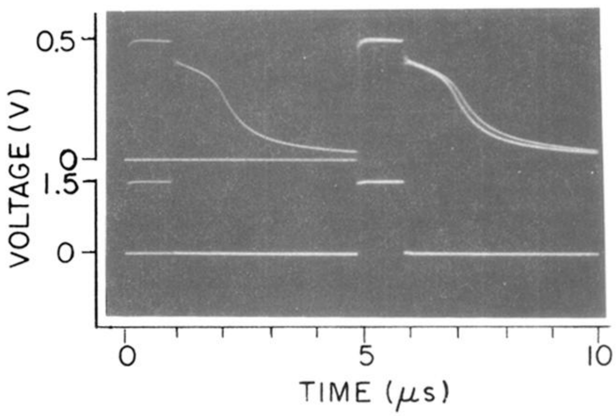

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