Routes to chaotic emission in a cw He-Ne laser

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Chaotic emission is observed from a 3.39-μm He-Ne laser. We find different sequences leading from coherent to chaotic states depending on the laser frequency. In particular, all three theoretically known sequences—"period doubling," "Ruelle-Takens," and "intermittency"—are observed.

We have recently identified for the first time chaotic states in a laser which represents an autonomous system.[1] Chaos in a nonautonomous system has also been reported recently.[2] In our recent work the laser frequency was a relatively uncertain parameter and the nature of the initial oscillatory state developing into a chaotic one was not clear.

We have therefore controlled the laser frequency by heterodyning with a frequency-stabilized auxiliary laser, which has allowed us to observe for the first time different "routes" the system takes from a coherent to a chaotic state. Routes of all three known types—the period-doubling route,[3] the three-bifurcation route,[4] and the intermittency route[5]—are observed for the first time in a laser. The initial oscillation is found to be a secondary combination tone produced by the simultaneous oscillation of three longitudinal modes.

The laser used to observe chaotic emission is the same as described in Ref. 1. To the measurement system was added a CH_4-Lamb-dip-stabilized He-Ne laser[6] with which the output of the "test" laser was heterodyned on the fast detector used for the measurements (Fig. 1). Between the ^{20}Ne test laser and the ^{22}Ne CH_4-stabilized laser an isotope shift of 70 MHz was found.

The test laser would oscillate in general in five longitudinal modes. In the case of perfect mirror alignment the laser can operate single mode (by mode competition) in a range of $\sim$20 MHz around the gain line center. Within this frequency range we have studied the sequences of laser emission frequencies leading to chaos which appear when one resonator mirror is progressively tilted.[1]

In a range of 6-MHz width, centered at $+5$ MHz from the gain line center, we observe the period-doubling approach to chaos[3] shown in Fig. 2: Tilting one resonator mirror away from the perfect alignment condition, the sequence starts with an oscillation at 6.8 MHz at a certain tilting angle [2(a)]. By increasing the angle, the amplitude of the oscillation increases up to a point where the oscillation frequency suddenly halves [2(b)]. By increasing the angle further, two more period doublings (frequency halvings) can be observed [2(c) and 2(d)]. Here we are able to resolve one period-doubling step more than in Ref. 1 due to better acoustical isolation. Further frequency doublings cannot be observed due to noise sources in the system, and the laser then goes into the chaotic state [2(e)].

In the range -1 to -3 MHz from the gain line center we find a Ruelle-Takens sequence[4] (Fig. 3). Starting at a laser frequency difference of -2 MHz and tilting the resonator mirror, this sequence starts with the appearance of a 3.2-MHz oscillation [3(a)] (notably different from the starting oscillation of the period-doubling sequence [2(a)], but it is not impossible that this starting frequency [3(a)] is $\frac{1}{2}$ of the starting frequency [2(a)] since the oscillation frequencies vary slightly with the resonator length). This oscillation increases in amplitude with increasing tilting angle until a second frequency of 1.4 MHz, incommensurate with 3.2 MHz appears [3(b)]. This state corresponds to motion in phase space on a two-torus, after which according to Ref. 4 chaos should set in. This occurs indeed, when the tilting angle is further increased [3(c)].

Within the range of -3 to -10 MHz from the gain line center a sequence is observed in which chaos sets in after the initial oscillation, without appearance of any further frequencies.

Since the spectra in the chaotic states are reminiscent of $1/f$ noise[7] and some relation appears to exist between "intermittency"[5] and low-frequency divergence of the chaotic spectra[8] the above sequence was examined for occurrence of intermittency.

The intermittency route to chaos[5] is characterized by the occurrence of short drastic deviations from a seemingly stationary oscillation (the "laminar" phase).[9] These deviations have durations comparable to the oscillation period and occur statistically. They occur more frequently, the closer the control parameter is to the chaotic re-

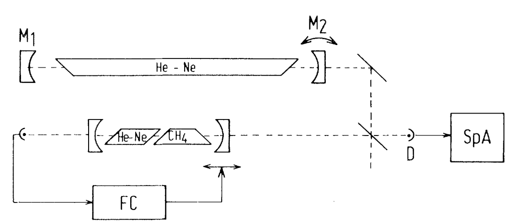

FIG. 1. Measurement setup consists of a 2.5-m-long He-Ne laser with M_1, gold mirror 98% reflectivity, $R=2.5$ m; M_2, uncoated SiO_2 planoconcave mirror $R=2.5$ m. Mirrors "gimbal" mounted (very small change of resonator length with tilting angle). Discharge tube length 1.6 m, inner diameter 2.9 mm, filling: 2.4 Torr, 1:9=[^{20}Ne]:[^{3}He], discharge current 14 mA, laser output power 20 mW. CH_4-stabilized He-Ne laser as described in Ref. 6. Frequency control on CH_4 Lamb dip. Observation of oscillations in and frequency measurement of test laser by fast photodiode D and 1-GHz radio-frequency spectrum-analyzer (SpA). For measurement M_2 is tilted.

gion. Within the chaotic region the duration of the laminar phase has gone to zero.

Intermittency does not produce pronounced spectral features. The time dependence of the laser output was therefore observed simultaneously with the rf spectra. The result is shown in Fig. 4.

After single-mode oscillation for the perfectly aligned mirror, the 3.2-MHz oscillation appears [Fig. 4(a)]. Increase of the tilting angle produces short excursions from the regular oscillation [4(b)]. These become more frequent as the tilting angle is further increased [4(c)]. The phases of regular oscillations become shorter and finally disap-

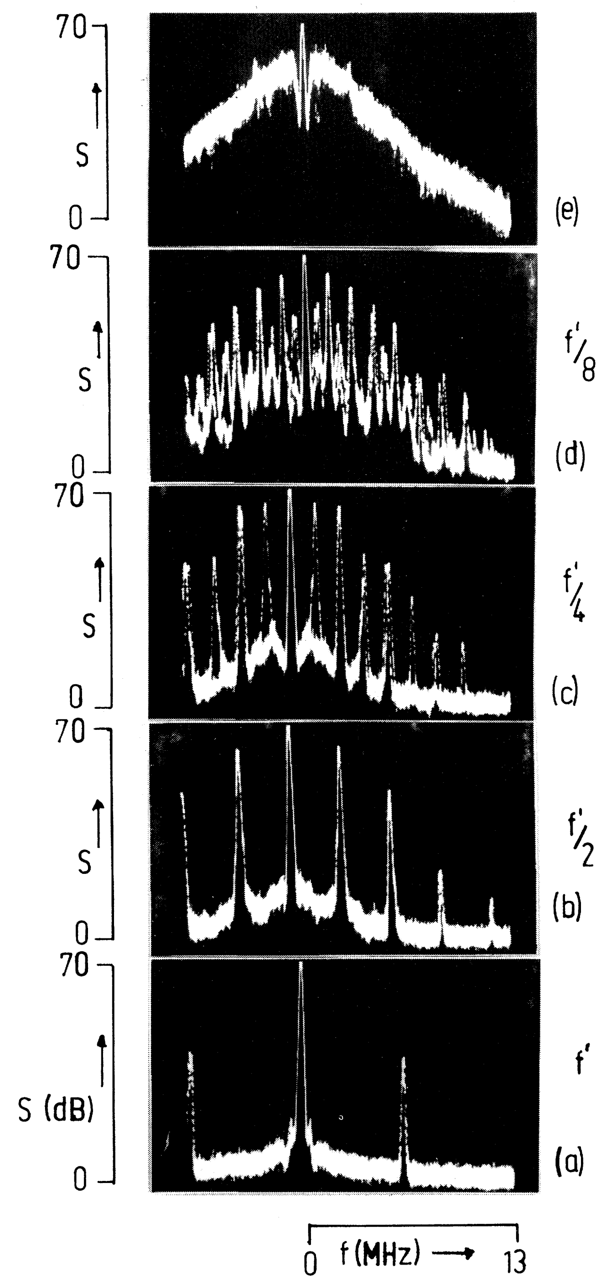

FIG. 2. Starting from single-mode oscillation, tilting of one resonator mirror leads to an oscillation f' [(a)], period doubling [$f'/2$, (b)], period doubling [$f'/4$ (c)], period doubling [$f'/8$, (d)], and chaos [(e)]. Laser frequency offset from the gain line center + 5 MHz. Spectrum is not given correctly in all measurements below 1 MHz due to lower cutoff frequency of detector amplifier. (S, relative intensity of oscillation; f, oscillation frequency.)

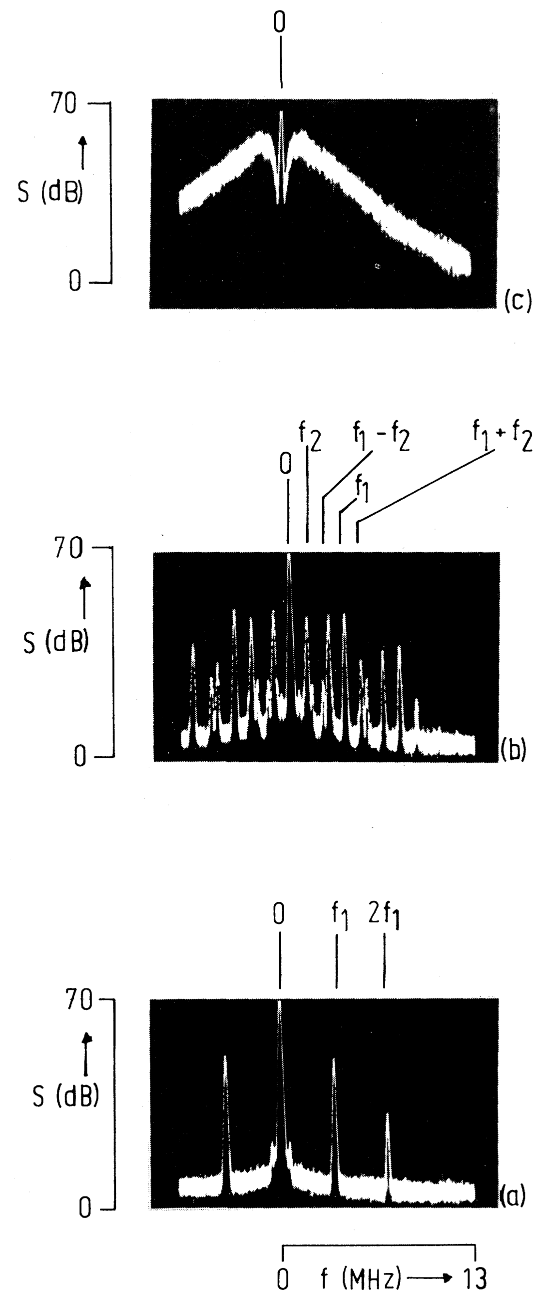

FIG. 3. Starting from single-mode oscillation, tilting of one resonator mirror leads to an oscillation f_1 [(a)] followed by a two-periodic state f_1, f_2 [(b)] followed by chaos [(c)]. Laser frequency offset -2 MHz.

pear completely [1(c)—1(e)]. The rf spectra corresponding to cases 1(a) and 1(e) are also shown in Fig. 4, indicating that 1(e) corresponds to a fully chaotic case. The short excursions have a duration of 100—300 ns. It would appear difficult to imagine disturbances of a technical kind with high enough Fourier frequencies to produce this behavior. We can therefore identify the sequence in the range −3 to −10 MHz from the gain line center with the intermittency route[5] to chaos.

In the frequency range +8 to +11 MHz from the gain line center we find another "Ruelle-Takens"[4] sequence, starting with the oscillation from 2(a) [Figs. 5(a)—5(c)].

In the range −1 to +2 MHz the sequence starts with the 3.2-MHz oscillation of Fig. 3(a) but then switches to the 6.8 MHz of Fig. 2(a) and continues the route to chaos via period doublings as in the range of +2 to +8 MHz (Fig. 2). This change between two sequences is probably caused by the small frequency change accompanying the resonator mirror tilting. Figure 6 sums up in which frequency ranges which sequences are observed. It may be significant to note that above the line center all sequences start with an oscillation of 6.5—7.5 MHz, while below the line center they start with an oscillation of 3—3.5 MHz.

The nature of the oscillations starting the sequences was found to be of the "secondary combination tone" type.[10] Whenever the mirror is tilted, two additional longitudinal modes separated from the initial single mode by roughly the laser resonator free-spectral range of 60 MHz come above threshold and increase in intensity with the tilting angle. The initial oscillation frequency is found in all cases to be exactly equal to the difference of the beat frequencies of the two additional modes with the main mode and the intensity of the initial oscillation increases with increasing intensity of the additional modes. Figure 7 shows the rf spectrum of the laser output from 0 to 100 MHz for a mirror tilting corresponding to Fig. 2(a). The two mode beats at ∼60 MHz are seen, and it is evident that the

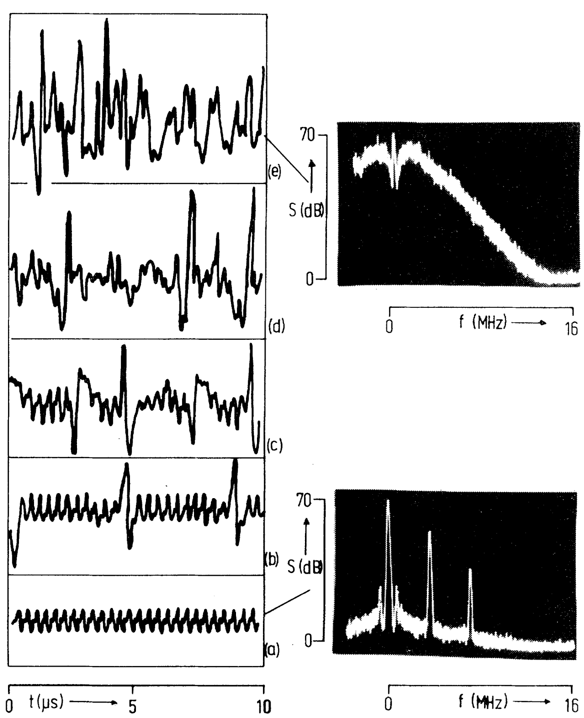

FIG. 4. Time dependence of laser output as the control parameter (mirror tilting angle) is varied from stable oscillatory state (a) to chaotic state (e). Spectra corresponding to (a) and (e) are also shown. (Observed −9 MHz from the gain line center.)

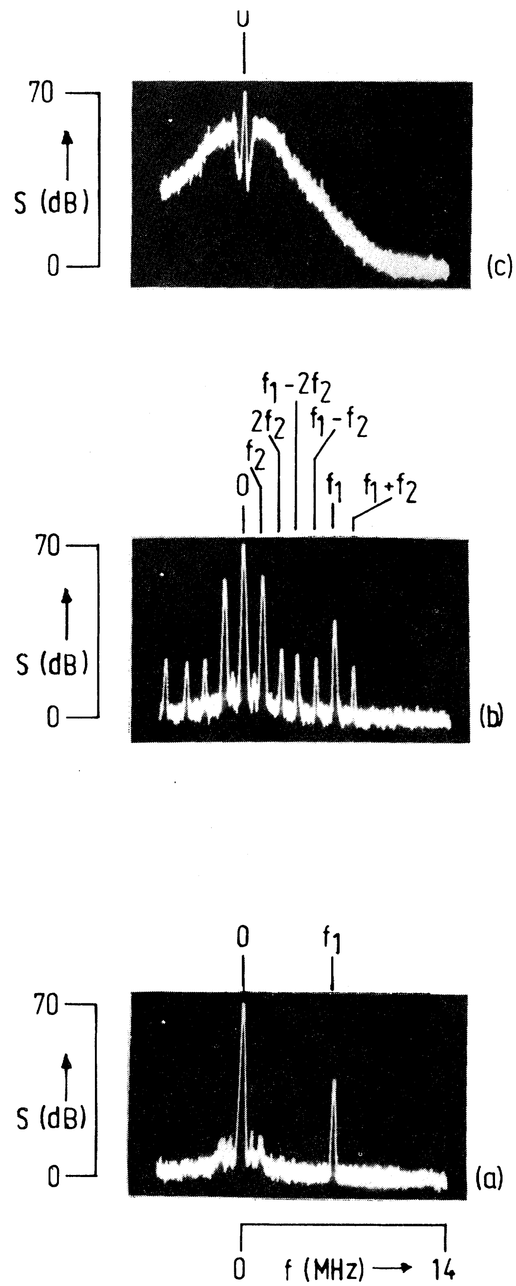

FIG. 5. Ruelle-Takens sequence observed at +10 MHz from the gain line center.

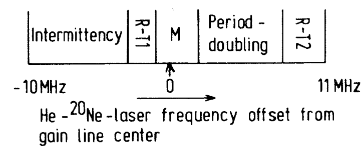

FIG. 6. Different routes to chaos appearing at different laser frequency settings when all other laser parameters are held constant. Intermittency, see Fig. 4. RT 1, Ruelle-Takens sequence, Fig. 3; *M*, transitional sequence described in text; period doubling, Feigenbaum sequence, Fig. 2; RT 2, Ruelle-Takens sequence, Fig. 5.

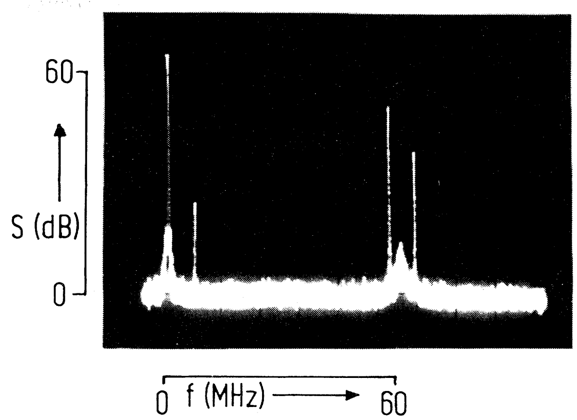

FIG. 7. RF spectrum of the laser output from 0 to 100 MHz showing two different mode beats at approximately the resonator free-spectral range of 60 MHz. The difference of the beat frequencies is probably due to the laser medium dispersion. Low-frequency oscillation is equal in frequency to the difference of the mode beats. [Mirror tilting corresponds to Fig. 2(a)]. Incoherent signals are probably due to modes below threshold.

low-frequency oscillation corresponds in frequency to the difference of the two mode beats.

It is not currently understood by which mechanism the intensity of the additional two modes is increased by tilting the mirror; however, it is clear that the phenomena observed are related to the three-mode operation of the laser and the phase locking of the three modes (which can very clearly be observed in this laser).

Although we cannot compare our observations with calculations at present, it appears safe to identify the observed incoherent emission with chaotic states of the laser since the routes correspond to known patterns. It has also been argued recently[11] that chaos should be a common phenomenon in multimode lasers and a numerical calculation has shown "violent fluctuations" of the mode intensities for a multimode laser.[12]

We summarize that we have observed different routes to chaos in a laser for the first time. The chaotic emission is associated with three-mode operation somewhere in the vicinity of the phase-locking condition.

Although the physical details producing the observed sequences are not understood at present, it is worth noting that the three routes regarded to date as universal[9] are all observed.

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