# Energy loss and energy straggling of protons and pions in the momentum range 0.7 to 115 GeV/$c$

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The energy-loss distribution for pions and protons, with momenta between 0.7 and 115 GeV/$c$, has been measured in an ion-implanted silicon detector of 300 $\mu$m thickness. The data are well described by an empirical energy-loss distribution.

## I. INTRODUCTION

The detection of charged particles by measuring their energy loss in thin silicon detectors has led to widespread study of the corresponding energy-loss spectrum.[1-3] This spectrum is given, to a first approximation, by the Landau[4] and Vavilov[5] theories, which are based on Rutherford scattering on free electrons.

In order to achieve high spatial resolution and to detect change in charged multiplicity due to decay of short-lived particles,[6,7] very thin ($<300$ $\mu$m) silicon detectors and low-noise electronics are used. Under these conditions, significant deviations from the Landau-Vavilov theory are expected because of the importance of the interactions where the atomic electron binding energy cannot be neglected.[8-10] This deviation has been observed, but not explained[11] for relativistic pions with momenta greater than 6 GeV/$c$.

The purpose of this experiment is to measure the energy-loss spectra of pions and protons over a large momentum range in a very thin silicon detector (300 $\mu$m), using low-noise ($\sigma \approx 4$ keV) electronics.

## II. ENERGY STRAGGLING

The statistical nature of the ionization process during the passage of a fast charged particle through matter results in large fluctuations of the energy loss in absorbers which are thin compared with the particle range. The calculation of these fluctuations was first performed by Landau.[4] Subsequently Vavilov[5] refined the Landau solution to the problem by introducing the maximum allowable energy transfer in the Rutherford macroscopic cross section $\omega'(\epsilon)$:

$$\omega'(\epsilon)=\begin{cases}(\xi/\epsilon^2)(1-\beta^2\epsilon/\epsilon_m), & 0<\epsilon<\epsilon_m\\ 0, & \epsilon>\epsilon_m\end{cases} \tag{1}$$

where $\epsilon_m$ (Ref. 12) is the maximum amount of energy that can be transferred to an atomic electron in a single collision with the incident particle of mass $m$ and velocity $\beta c$, and where $\epsilon$ is the actual energy transferred in the collision (Landau neglected the term $\beta^2\epsilon/\epsilon_m$).

The quantity $\xi$ is given, in keV, by

$$\begin{aligned}\xi&=(2\pi z^2 e^4/m_e c^2\beta^2)NZx\rho/A\\ &=153.4(z^2/\beta^2)(Z/A)x\rho\ ,\end{aligned} \tag{2}$$

where $N$ is the Avogadro number, $m_e$ and $e$ are the electron mass and charge, respectively, $z$ is the charge of the incident particle, $Z$, $A$, and $\rho$ are the atomic number, atomic weight, and density (g/cm$^3$) of the material, and where $x$ is the distance in cm traversed through the material.

However, the effects of atomic binding of the electrons have been disregarded in both the Landau and Vavilov theories. The theories can be improved by using a modified cross section to take into account the electron binding energy.[13] The modified energy-loss distributions can be expressed as the convolution of a Gaussian function with a Landau or Vavilov distribution, respectively.[8,10,14] Thus

$$f(\Delta,x)=(1/\sigma\sqrt{2\pi})\int_{-\infty}^{+\infty} f_{L,V}(\Delta',x)\times\exp[-(\Delta-\Delta')^2/2\sigma^2]d\Delta'\ , \tag{3}$$

where $f_{L,V}(\Delta',x)$ is either the Landau or the Vavilov distribution and $\Delta$ is the actual energy loss.

The term $\delta_2=\sigma^2$ in Eq. (3) has been computed by Shulek *et al.*, who derived the following equation:

$$\delta_2=\tfrac{8}{3}\xi\sum_i l_i(Z_i/Z)\ln(2m_e c^2\beta^2/l_i)\ , \tag{4}$$

where $Z_i$ is the number of electrons in the $i$th shell of the stopping material, $l_i$ is the ionization potential of the $i$th shell, and the summation is performed over the shells for which $l_i<2m_e c^2\beta^2$. These ionization potentials have been computed by Sternheimer[15-17] and enter into the density effect correction to the mean energy loss. Only the tails of the actually measured spectra are expected to differ systematically from those given by Eq. (3), because $\delta$ rays produced in high-energy transfer interactions may escape from the detector.

## III. EXPERIMENTAL METHOD

In the present investigation, energy loss has been measured for pions and protons in the momentum range between 0.7 and 115 GeV/$c$ (see Table I). The momentum

 

TABLE I. Momentum spread of the beam either at the Centre d'Etudes Nucléaires de Saclay or at CERN.

| Momentum (GeV/$c$) | Particle | $\Delta P/P$ (%) | Laboratory |
|---|---|---|---|
| 0.736 | $p$ | $<\pm 0.1$ | Saturne II-Saclay |
| 1.000 | $p$ | $<\pm 0.1$ | Saturne II-Saclay |
| 1.916 | $p$ | $<\pm 0.1$ | Saturne II-Saclay |
| 30.0 | $\pi$ (>95%) | $<\pm 1.0$ | CERN-SPS |
| 45.0 | $\pi$ (>95%) | $<\pm 1.0$ | CERN-SPS |
| 115.0 | $p$ (>95%) | $<\pm 1.0$ | CERN-SPS |

spread $\Delta P/P$ was never more than 0.1% for the measurements performed at Saturne in Saclay, and never more than 1% at the CERN Super Proton Synchrotron (SPS). A well-defined momentum is important, particularly at low values, since any momentum spread broadens the energy-loss distribution and thereby masks the effect due to long-distance collisions.

Two Enertec ion-implanted passivated-silicon junction detectors with an active area of 1.0 $cm^2$ and a thickness of $300\pm5$ $\mu$m were used.[18] Total depletion of the detectors was achieved at reverse biases of about 100 V. The initial leakage currents were 16 and 20 nA.

The signal pulses from these detectors were sent to standard ORTEC-125 charge preamplifiers. The preamplified signals were then sent to a modified version of an ORTEC-472 spectroscopy amplifier, in the case of the more upstream of the two detectors, and to an EG&G-474 timing filter amplifier, in the case of the downstream detector. The ORTEC amplifier produced an output pulse with a 600-nsec base and a 200-nsec rise time, while the timing filter amplifier produced a pulse with a 100-nsec base and a 50-nsec rise time. The instantaneous counting rate never exceeded 3000 events per second in order to avoid pile up.

The spacing between the two detectors was 4 cm, with their active areas well aligned. A scintillator with an area of $0.5\times0.5$ $cm^2$ and a thickness of 0.5 cm was positioned 1 cm from the downstream detector. The beam was defined by the coincidence of the downstream detector and this small scintillator, the latter generating a jitter of only a few nsec. This beam-particle trigger was shaped to provide a 40-nsec gate for a LeCroy 2249A analog-to-digital

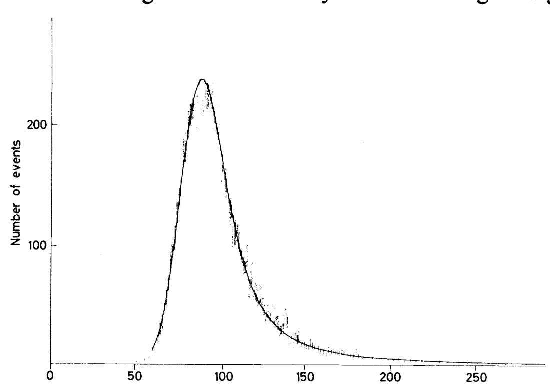


FIG. 1. Energy-loss spectrum for $\beta^-$ coming from a Ru source. Kinetic energies are selected to be greater than 1 MeV. Continuous line represents the fitted Landau distribution convolved by a Gaussian function.

converter (ADC), where the pulse coming from the upstream detector was recorded.

The upstream detector, together with its special purpose electronics, had a Gaussian noise distribution with an independently measured standard deviation $\sigma_{noise}$, whose typical value was $4.0\pm0.4$ keV. The exact value was determined separately before each data taking.

The initial calibration of the energy scale for the entire system was performed using a 30-GeV $\pi^-$ beam. The most probable energy loss for pions has no significant momentum dependence for $\beta\gamma>50$.[11] We assumed a value of $84.0\pm2.8$ keV for the most-probable energy loss ($\Delta_{mp}$ defined in Sec. IV) in 300 $\mu$m of silicon.[19] Thereafter a Ru $\beta^-$ source has been used for calibration purposes once it had been determined that the value of $\Delta_{mp}$ for relativistic electrons is $86.8\pm2.8$ keV on the same scale that yields the canonical value of $84.0\pm2.8$ keV for relativistic pions.

The source was placed in front of the upstream detector and a trigger formed for electrons crossing both of the detectors and the scintillator. This way only relativistic electrons, namely, those with a kinetic energy between about 1 and 3 MeV were selected. Figure 1 shows the resulting energy-loss spectrum and the corresponding fitted curve. This calibration was performed after each measurement at a new beam momentum in order to monitor very closely the stability of the electronics.

## IV. DATA ANALYSIS AND DISCUSSION

In the limit as $k=\xi/\epsilon_m\rightarrow0$, the Vavilov distribution approaches that of Landau theory. It has been shown[1] that for $k\approx0.06$ (corresponding to 65.3-MeV positive pions crossing a 0.216-cm-thick lithium-drifted silicon detector), the Vavilov and Landau spectra are practically indistinguishable.

The lowest momentum used in our investigation corresponds to $k\approx0.02$. Consequently, the observed spectra are expected to be well represented by Eq. (3), in which a Gaussian function convolves a Landau distribution. The standard deviation $\sigma_{tot}$ of the Gaussian part can be defined to also take into account the detector and electronic noise with

$$\sigma_{tot}=(\delta_2+\sigma_{noise}^2)^{1/2},$$

where $\sigma_{noise}$ is the standard deviation of the Gaussian noise distribution (see Sec. III).

The Landau probability density function is given by

$$f_L(\Delta,x)=(1/\xi)\phi(\lambda),$$

$$\phi(\lambda)=(1/2\pi i)\int_{c-i\infty}^{c+i\infty}\exp(u+\ln u+\lambda u)\,du,$$

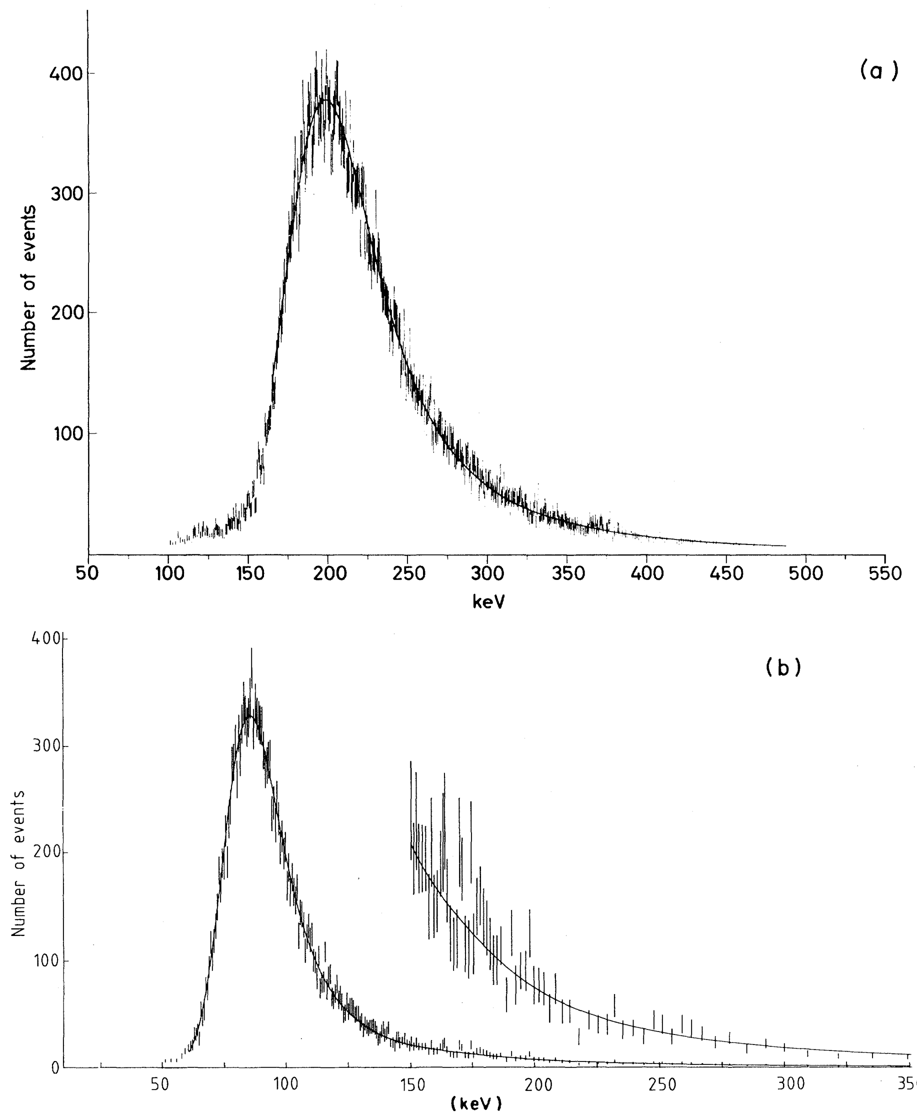


FIG. 2. (a) and (b) show the energy-loss spectra at 0.736 and 115 GeV/$c$ of incoming proton momenta, respectively. Continuous curves are the complete fits to the experimental data. Values of $\xi$ are $14.9\pm0.8$ and $5.5\pm0.3$, respectively (the corresponding values of the best fit are $15.0\pm0.8$ and $5.6\pm0.3$). Values of $\sigma=\sqrt{\delta_2}$ are $11.3\pm1.4$ and $5.5\pm1.1$, respectively (the corresponding values of the best fit are $10.2\pm1.3$ and $5.7\pm1.1$). In (b) the tail is also shown with the vertical scale multiplied by ten.

where

$$\lambda=(\Delta-\langle\Delta\rangle)/\xi-1-\beta^2+C-\ln(\xi/\epsilon_m)$$
$$=[\Delta-(\Delta_{\rm mp}-\xi\lambda_0)]/\xi\ ,$$

$\langle\Delta\rangle$ is the mean energy loss, the Euler constant $C\approx0.577\,215$, $\lambda_0\approx-0.225$ is the value for which $\phi$ is a maximum, $\Delta_{\rm mp}$ is the most-probable energy loss of the Landau distribution, i.e., $f_L(\Delta_{\rm mp},x)=(1/\xi)\phi(\lambda_0)$, and where $c$ is an arbitrary real positive constant.

The experimental energy-loss distributions for each momentum and type of particle were fitted to the Landau probability density convolved with a Gaussian as described above, with free parameters $\delta_2$, $\xi$, and $\Delta_{\rm mp}$. The fitting procedure is described in the Appendix to this paper. The good fits obtained for all the data samples give us confidence in the overall appropriateness of Eq. (3) and in the values of the fitted parameters. However, some deviation from the model is observed, as expected, in the tail due mainly to the neglect of escaping $\delta$ rays. In order to avoid a possible bias in the fitted parameters, we have varied the upper limit to which each experimental distribution was fitted. The best fits had $\chi^2$ probabilities of the order of 50% or better and were obtained by fitting up to $\Delta_{\rm max}=\Delta_{\rm mp}+\alpha W$, where $W$ is the full width at half maximum, and the values of $\alpha$ are between 2.5 and 3.5. However, the fitted values of $\delta_2$, $\xi$, and $\Delta_{\rm mp}$ determined at the best $\chi^2$ did not differ, within the error of their determination, from those of the complete fit.

In Fig. 2 the energy-loss distributions for the lowest and highest proton momenta are shown. The continuous curves are the least-squares fits, with all of the tail fitted. The curves obtained by using the parameters at the best $\chi^2$

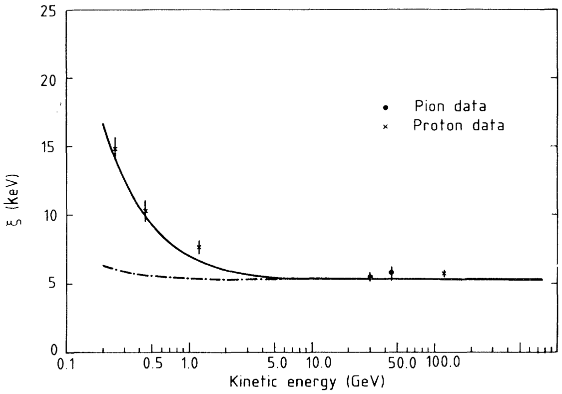


FIG. 3. Determined values of the parameter $\xi$. Continuous and broken curves are computed from Eq. (2) for proton and pion, respectively. General agreement between the fitted value and the predicted one indicates that both the energy calibration had been correctly performed and that the shape of the energy-straggling spectrum is well represented by the improved energy-loss distribution.

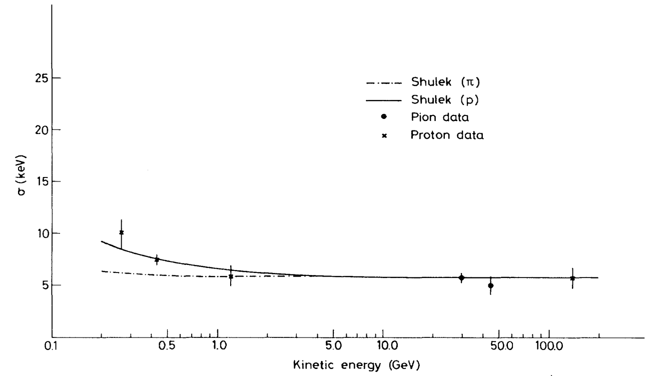


FIG. 4. Determined values of $\sigma=\sqrt{\delta_2}$. Continuous and broken lines are the Shulek *et al.* (Ref. 9) predictions for protons and pions, respectively. Pion (●) and proton (×) data are in general agreement with the Shulek *et al.* predictions.

are, by eye, indistinguishable from those given in Fig. 2. In Fig. 3 the fitted values of the parameter $\xi$ are shown. The continuous and broken lines are the calculated curves for incoming protons and pions, respectively, in 300 $\mu$m of silicon. The agreement between experiment and Eq. (2) seen in Fig. 3 indicates that our calibration of the energy scale for the data is correct to within 6%.

Table II gives a list of the fitted values of $\xi$, $\sigma=\sqrt{\delta_2}$, and the most-probable energy loss of the Landau distribution $\Delta_{mp}$. The observed values of $\Delta_{mp}$ are in general agreement, within the experimental errors, with Landau theory provided $\langle\Delta\rangle$ are computed using the Bethe-Bloch formula and taking into account the correction for the density effect. The shell correction term of the Bethe-Bloch formula is negligible over the momentum range of the present investigation.[20–22] The most-probable energy loss $E_{mp}$ of the overall straggling distribution defined by Eq. (3), is about 3% higher than $\Delta_{mp}$ due to the folding of the Gaussian distribution (Table II). There is no significant difference between $E_{mp}$ for $\pi^-$ at a momentum of 45 GeV/$c$ ($\beta\gamma\approx 330$) and protons at 115 GeV/$c$ ($\beta\gamma\approx 123$). Esbensen *et al.*[11] have already shown that there is no relativistic rise of the most-probable energy loss for $50<\beta\gamma<120$. The value of $E_{mp}$ determined at the lowest momentum is in agreement with the value quoted by Aitken *et al.*,[1] once it is extrapolated to a detector thickness of 0.216 cm.

In Fig. 4 we show a plot of the values obtained for the parameter $\sigma=\sqrt{\delta_2}$ together with the corresponding theoretical predictions made by Shulek *et al.* for protons and pions, respectively (see Sec. II). The quoted errors take into account the uncertainty in determining the noise contribution to $\sigma_{tot}$ (see the Appendix). Both the pion and proton data seem to be in general agreement with the model of Shulek *et al.* As $\beta\to 1$, $\sigma$ becomes a constant to within the error of its determination.

The results of the present investigation confirm the collision-loss theories of Landau and Vavilov. The effect of atomic binding of electrons has been clearly observed and seems to be well described by Eq. (3), and the standard deviation of the Gaussian contribution is compatible with the computations of Shulek *et al.*

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TABLE II. Values of $\xi$, $\sigma=\sqrt{\delta_2}$, $\Delta_{mp}$, and $E_{mp}$ vs the incoming momentum.

| Momentum (GeV/$c$) | $\xi$ (keV) | $\sigma=(\delta_2)^{1/2}$ (keV) | $\Delta_{mp}$ (keV) | $E_{mp}$ (keV) |
|---|---|---|---|---|
| 0.736 | 15.0±0.8 | 10.2±1.3 | 194.8±5.8 | 196.2±5.8 |
| 1.000 | 10.4±0.8 | 7.4±0.6 | 127.5±3.8 | 130.8±3.8 |
| 1.916 | 7.6±0.4 | 5.9±1.0 | 94.0±3.0 | 97.2±3.0 |
| 30.0 | 5.5±0.3 | 5.7±0.5 | 84.0±2.8 | 85.6±2.8 |
| 45.0 | 5.8±0.4 | 5.0±0.9 | 86.0±2.8 | 88.8±2.8 |
| 115.0 | 5.6±0.3 | 5.7±1.0 | 83.2±2.8 | 85.5±2.8 |

## APPENDIX: FITTING PROCEDURE

The large amount (between 20000 and 30000 pulse heights per data sample) and high resolution of data provide a very sensitive test of the energy-loss model, but also require considerable care in the fitting procedure in order not to introduce numerical and statistical bias.

The parameter fitting and goodness-of-fit testing were done with the usual least-squares techniques. The function minimized was

$$\chi^2(\alpha)=\sum_i \{[t_i(\alpha)-n_i]^2/d_i{}^2\}\ , \tag{A1}$$

where $\alpha$ are the free parameters of the fit, namely, $\xi$, $\delta_2$, and $\Delta_{mp}$, and a normalization factor; the sum is over all histogram bins, $t_i(\alpha)$ and $n_i$ are, respectively, the expected and observed numbers of counts in the $i$th bin, and $d_i$ is the standard deviation of the numerator. The histogram bins were constructed as follows: starting from the raw data histogram (ADC channels of a width of about 0.5 keV), bins with less than a minimum number of events were joined with neighboring bins until each new bin contained at least that minimum number [the resulting unequal bin widths in the tail are clearly visible in Fig. 2(b)]. The minimum bin content could be varied to make sure that it was large enough not to influence the fit, and the value finally used was 20.

The expected number of events in the $i$th bin $t_i(\alpha)$ is just the integral of Eq. (3), properly normalized, over the bin. It is customary to approximate this by the value of the function at the center of the bin, multiplied by the bin width. We have used both methods and find no significant difference (due to the extremely narrow bins in the peak region).

As the universal Landau function is evaluated very often during the fitting, a special subroutine was prepared which calculated this function using four-point interpolation in a table of 200 very accurate values calculated by Fourier series and checked against both published values[23,24] and those calculated by contour integration. The convolution with the Gaussian was done numerically using usually 20 points to evaluate the convolution integral for each data bin, but his number could be varied to make sure that it was large enough not to influence the fit.

The variance $d_i^2$ is usually taken as the observed number of events $n_i$, since this is indeed the variance of a Poisson distribution of mean $n_i$. However, under the null hypothesis, the expected value is $t_i(\alpha)$, not $n_i$. We find significant (although small) differences between fits done using the two techniques. An intermediate method is to use the average number of events observed over three bins, which gives essentially the same fits as using the expected number of events. This intermediate method has the advantage that the denominator in each term is constant (independent of the fit parameters) so there is no tendency to try to reduce the $\chi^2$ by making the variances larger.

Finally, the entire fitting procedure has been tested by generating Monte Carlo data with known parameter values and fitting it through the same procedures. In order to do this we have developed a very accurate Landau-distributed random number generator [this generator, as well as our Landau density function, are available from the CERN Program Library as GENLAN (G903) and FUNLAN (G112)]. The Monte Carlo procedure helped us to choose the best fitting procedure and verified that our method did not introduce any significant bias in the fitted parameter values. The parameter errors given in Table II result from combining the statistical error calculated from our fits with a systematic error, largely calibration uncertainty (which is the dominant one) and the error in measuring $\sigma_{\text{noise}}$.

Good fits were obtained for all data samples, apart from a slight depression of the tail for reasons described above. The data taken at the SPS energies (see Table I) required adding about 1% contamination from two particles simultaneously traversing the silicon, which is consistent with the expected interaction probability in the upstream beam-defining scintillators. The effect is included in the curve shown in Fig. 2(b), but the resulting contribution (between 150 and 250 keV) is too small to be noticed by the eye. Two-particle contamination was neither expected nor observed for the other data.

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to be in agreement with the value of $\sim 83.75$ keV for the most-probable energy loss of the Landau distribution. The accuracy of the measurement was estimated to be of the order of 3%.

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