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Photon antibunching effect and statistical properties of single-mode emission in free-electron lasers

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It is shown that the field emitted by a free-electron laser oscillating on a single mode in the no-gain regime can exhibit antibunching. The quantum-statistical properties of field are also discussed.

I. INTRODUCTION

The radiation emitted in many nonlinear optical processes exhibits, under certain conditions, a negative Hanbury-Brown and Twiss (HBT) effect, i.e., photon antibunching or anticorrelation effects.[1-12] When even anticorrelation or antibunching effects occur, the field is characterized by a photon-counting distribution narrower than the Poisson distribution, corresponding to a coherent state, and by a negative intensity variance.

In the following we show that the nonlinear interaction which produces radiation emission in the FEL (free-electron laser)[13] can also give rise to antibunching in the output laser mode. Single-mode emission of a FEL amplifier is studied here in the particular case of zero gain (or very small gain). The statistical properties of radiation are also discussed, as shown by first- and second-order moments.

We have employed the coherent state technique and the q-c number correspondence, starting from the master equation and obtaining the generalized Fokker-Planck equation for the antinormal quasidistribution function. The solutions of the Fokker-Planck equation provide the photon-counting distribution and its factorial moments. In the following we do not include losses. We confirm in this way the results obtained by other authors using different approaches,[14,15] and show that antibunching can arise; an effect which could not be predicted in the calculation performed by Becker.[14]

II. MASTER EQUATION FOR THE RADIATION FIELD

We start from the quantum description of a FEL in a moving frame in which the frequencies of laser and Wiggler coincide (with the use of the Weizsäcker-Williams approximation).

The free radiation Hamiltonian is given by

$$H_{\text{rad}} = \sum_{j=1}^{2} \hbar\omega_j \hat{a}_j^\dagger \hat{a}_j \;, \tag{1}$$

where $j=1=L$ (laser frequency), $j=2=W$ (Wiggler frequency), and $\hat{a}^\dagger$ and $\hat{a}$ are creation and annihilation field operators.

The free-electron system is described by the following Hamiltonian:

$$H_{\text{el}} = \sum_{p,\sigma} \epsilon(\hat{c}_{p,\sigma}^\dagger \hat{c}_{p,\sigma} - \hat{d}_{p,\sigma}\hat{d}_{p,\sigma}^\dagger) \;, \tag{2}$$

where the sum ranges over the momentum and spin values, $\hat{c}^\dagger, \hat{c}$ are fermion operators for the particle, and $\hat{d}^\dagger, \hat{d}$ are fermion operators for the antiparticle.

It is possible to show[16] that starting from the interaction Hamiltonian (in the nonrelativistic approximation)

$$H_{\text{int}} = \frac{e^2}{2mc^2} A^2 \;, \tag{3}$$

the following "effective Hamiltonian" is found[11]:

$$H_{\text{int}} = \sum_{p,p'} \sum_{n} \hbar K^{(2)} \hat{c}_{pn}^\dagger \hat{c}_{p'n} \hat{O} + \text{c.c.} \;, \tag{4}$$

where $\sum_n$ is the sum on the particles, $K^{(2)}$ is the coupling constant proportional to the transition matrix element, and $\hat{O}$ contains the radiation field operators.

We remember that in the effective Hamiltonian the virtual electronic transitions are taken into account, and real transitions are neglected.

Statistical properties of the radiation-electron system are described by the density operator $\hat{\rho}(t)$ which satisfies the following equation of motion:

$$i\hbar\frac{\partial\hat{\rho}}{\partial t}=[H_T,\hat{\rho}] , \tag{5}$$

where $H_T=H_{\rm rad}+H_{\rm el}+H_{\rm int}$, in the Schrödinger picture (SP).

We are interested only in the radiation properties, therefore, we eliminate the reservoir variable (electron system), obtaining the density operator for the boson field alone: $\hat{\rho}_F(t)$.

The motion equation for the density operator may be described with the use of the Markoff approximation and standard techniques. In our case the temporal condition for the Markoff approximation is given by

$$t_c \ll t-t_0 \ll \gamma^{-1} , \tag{6}$$

i.e., the interaction time must be smaller than the radiation damping time γ^{-1} and larger than the reservoir correlation time t_c (t_c is the time between electron collisions). Thus we obtain the master equation for the reduced density operator in the interaction picture (IP)[17]:

$$\frac{\partial\hat{\rho}_F}{\partial t}=-\sum_i\sum_K\delta(\omega_i-\omega_K)\times[(\hat{O}_i\hat{O}_K\hat{\rho}_F-\hat{O}_K\hat{\rho}_F\hat{O}_i)W_{i,K}^{+}-(\hat{O}_i\hat{\rho}_F\hat{O}_K-\hat{\rho}_F\hat{O}_K\hat{O}_i)W_{K,i}^{-}], \tag{7}$$

where $i(K)=1,2$ because they refer to second order in perturbation theory. Moreover, $W_{i,K}^{\pm}$ are the "reservoir spectral densities" which contain the electron system operators[16]

$$W_{i,K}^{\pm}\sim\sum_n K^{(2)}(\vec{x}_n)\int_0^\infty\int_0^\infty dp\,dp'f(p)f(p')\times\langle F_i(\tau)F_K\rangle_R I \tag{8}$$

with

$$F_i\sim\hat{c}_p^{\dagger}\hat{c}_{p'} ,\quad F_K=F_i^{\dagger} ,$$

$$I\sim\int_0^\infty \exp[i(\Delta\omega_e-\Delta\omega_f)\tau]d\tau ,$$

$$\tau=(t-t_o),\quad \Delta\omega_e=\omega_p-\omega_{p'},\quad \Delta\omega_F=\omega_L=\omega_W ,$$

and $f(p)$ represents the electron momentum distribution, where in the very small gain limit $\Delta\omega_e=\Delta\omega_f$. Now, in the considered frame the fields are at the same frequency, and the whole radiation field gives rise to a stationary field (collinear structure of the laser and Wiggler). For a stationary field we can write the operator $\hat{O}$ as

$$\hat{O}=\hat{O}^{\dagger}=(\hat{a}_L^{\dagger}+\hat{a}_W^{\dagger})(\hat{a}_L+\hat{a}_W) , \tag{9}$$

and the master equation finally becomes

$$\frac{\partial\hat{\rho}_F}{\partial t}=K(\hat{O}\hat{\rho}_F\hat{O}-\tfrac{1}{2}\hat{O}^2\hat{\rho}_F-\tfrac{1}{2}\hat{\rho}_F\hat{O}^2) , \tag{10}$$

where

$$K\simeq W_{i,K}^{\pm} .$$

III. FOKKER-PLANCK EQUATION

Let us now introduce the following variables:

$$\hat{A}_1=\frac{1}{\sqrt{2}}(\hat{a}_L+\hat{a}_W),\quad \hat{A}_2=\frac{1}{\sqrt{2}}(\hat{a}_L-\hat{a}_W) . \tag{11}$$

Using the q-c correspondence, we obtain the equation of motion for the quasidistribution ϕ_A related to the antinormal ordering of field operators[18,19]:

$$\frac{\partial\phi_A}{\partial t}=-2K\left[A_1\frac{\partial}{\partial A_1}-|A_1|^2\frac{\partial^2}{\partial A_1\partial A_1^*}+A_1^2\frac{\partial^2}{\partial A_1^2}+\text{c.c.}\right]\phi_A , \tag{12}$$

where $A_1=(\alpha_L+\alpha_W)/\sqrt{2}$ and $A_2=(\alpha_L-\alpha_W)/\sqrt{2}$, provided that α_L and α_W are eigenvalues of $\hat{a}_L$ and $\hat{a}_W$ in the coherent state $|\alpha_L,\alpha_W\rangle$.

The quasidistribution is defined as

$$\phi_A=\phi_A(\{\alpha_j\},t)=\rho^n(\{\hat{a}_j\to\alpha_j\},\{\hat{a}_j^{\dagger}\to\alpha_j^*\})\frac{1}{\pi^M} ,$$

where ρ^n is the equivalent normal form of the density matrix obtained with the help of the commutation rules; α_j are eigenvalues of $\hat{a}_j$ in the coherent state $|\{\alpha_j\}\rangle$, and M represents the number of modes. Performing the Fourier transform

$$\phi_A=\frac{1}{\pi^2}\int C_A(\{\beta_j\},t)\times\prod_j\exp(-\beta_j\alpha_j^*+\beta_j^*\alpha_j)d^2\beta_j , \tag{13}$$

where β_j are conjugated to α_j and using the variables γ_j that are conjugated to A_j

$$\gamma_1=\frac{(\beta_L+\beta_W)}{\sqrt{2}}\ ,\quad \gamma_2=\frac{(\beta_L-\beta_W)}{\sqrt{2}}\ , \tag{14}$$

we obtain the Fokker-Planck equation for the antinormal function

$$\frac{\partial C_A}{\partial t}=-2K\left[\gamma_1\frac{\partial}{\partial\gamma_1}-|\gamma_1|^2\frac{\partial^2}{\partial\gamma_1\partial\gamma_1^*}+\gamma_1^2\frac{\partial^2}{\partial\gamma_1^2}+\text{c.c.}\right]C_A\ . \tag{15}$$

It is possible to verify that a solution of Eq. (15) can be[20]

$$C_A(\gamma_1,\gamma_2,t)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}C_{nm}(\gamma_2)\gamma_1^n\gamma_1^{*m}\exp[-2Kt(n-m)^2]\ , \tag{16}$$

where $C_{nm}(\gamma_2)$ are arbitrary functions of γ_2. If the initial field is in a coherent state $|\xi_L\rangle|\xi_W\rangle$, a final solution for the normal and antinormal characteristic functions can be found in the form

$$\begin{aligned}C_N(\beta_L,\beta_W,t)&=C_A(\beta_L,\beta_W,t)\exp(|\beta_W|^2+|\beta_L|^2)\\&=\exp[\tfrac{1}{2}(\beta_L-\beta_W)(\xi_L^*-\xi_W^*)-\text{c.c.}]\\&\quad\times\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m}{2^{n+m}n!m!}(\beta_L+\beta_W)^n(\beta_L^*+\beta_W^*)^m(\xi_L^*+\xi_W^*)^n(\xi_L+\xi_W)^m\exp[-2Kt(n-m)^2].\end{aligned} \tag{17}$$

The statistical properties of radiation are obtained from the β derivatives of the $C_N(\{\beta_j\},t)$ functions taken at $\{\beta_j\}=0$. Then the mean integrated intensity in each mode reads

$$\langle W_L\rangle=\left.\frac{\partial^2C_N(\{\beta_j\},t)}{\partial\beta_j\partial(-\beta_j^*)}\right|_{\{\beta_j\}=0}=\tfrac{1}{2}(|\xi_L|^2+|\xi_W|^2)+\tfrac{1}{2}(|\xi_L|^2-|\xi_W|^2)\exp(-2Kt)\ , \tag{18}$$

$$\langle W_W\rangle=\tfrac{1}{2}(|\xi_L|^2+|\xi_W|^2)-\tfrac{1}{2}(|\xi_L|^2-|\xi_W|^2)\exp(-2Kt)\ , \tag{19}$$

while the variance is given by

$$\langle(\Delta W)^2\rangle=\left.\frac{\partial^4C_N(\{\beta_j\},t)}{(\partial\beta_j)^2\partial(-\beta_j^*)^2}\right|_{\{\beta_j\}=0}-\left[\left.\frac{\partial^2C_N(\{\beta_j\},t)}{\partial\beta_j\partial(-\beta_j^*)}\right|_{\{\beta_j\}=0}\right]^2\ , \tag{20}$$

$$\begin{aligned}\langle(\Delta W_L)^2\rangle&=\langle(\Delta W_W)^2\rangle=-\langle\Delta W_L\Delta W_W\rangle\\&=\tfrac{1}{8}[|\xi_L|^4+|\xi_W|^4-(\xi_L^{*2}\xi_W^2+\text{c.c.})]-\tfrac{1}{4}(|\xi_L|^2-|\xi_W|^2)^2e^{-4Kt}\\&\quad+\tfrac{1}{8}[(|\xi_L|^2-|\xi_W|^2)^2+(\xi_L^*\xi_W-\text{c.c.})^2]e^{-8Kt}\ ;\end{aligned}$$

the negative value of $\langle\Delta W_L\Delta W_W\rangle$ being connected to the presence of anticorrelation between the modes W and L.

If the intensities are equal,

$$|\xi_L|=|\xi_W|\ ,$$

then $\langle W_L\rangle=\langle W_W\rangle=|\xi|^2$ and

$$\langle(\Delta W_L)^2\rangle=\langle(\Delta W_W)^2\rangle=0,$$

i.e., the field turns out to be coherent at all times. In general, the whole radiation field remains coherent in the interaction:

$$\langle(\Delta W)^2\rangle=\langle(\Delta W_L)^2\rangle+\langle(\Delta W_W)^2\rangle+2\langle\Delta W_L\Delta W_W\rangle=0\ . \tag{21}$$

Moreover, if $|\xi_W|=0$ we have

$$\langle W_L\rangle=\tfrac{1}{2}|\xi_L|^2(1+e^{-2Kt})\ , \tag{22}$$

$$\langle(\Delta W_L)^2\rangle=\tfrac{1}{8}|\xi_L|^4(1-e^{-4Kt})^2\ . \tag{23}$$

Saturation values are (for $|\xi_W|\neq0$ and $t\to\infty$)

$$\langle W_L\rangle=\tfrac{1}{2}(|\xi_L|^2+|\xi_W|^2)\ , \tag{24}$$

$$\langle(\Delta W_L)^2\rangle = \tfrac{1}{8}[\,|\xi_L|^4+|\xi_W|^4 - 2|\xi_L|^2|\xi_W|^2\cos(2\phi_L-2\phi_W)] \,. \quad (25)$$

From these expressions we see that the statistics of the laser mode differ from the Poissonian statistics [with $\langle(\Delta W)^2\rangle=0$]. This behavior depends upon the values of initial phase fields.

We observe that the lowest value of Eq. (25) is

$$\langle(\Delta W_L)^2\rangle=\tfrac{1}{8}(|\xi_L|^2-|\xi_W|^2)^2$$

and, therefore, no antibunching is possible in the L mode at long time. Instead, in the small time limit (i.e., $8Kt\leq 1$) from Eq. (20) we have

$$\langle(\Delta W_L)^2\rangle=\tfrac{1}{2}|\xi_L|^2|\xi_W|^2[1-\cos(2\phi_L-2\phi_W)] - Kt(|\xi_L|^4+|\xi_W|^4) + 2Kt\,|\xi_L|^2|\xi_W|^2\cos(2\phi_L-2\phi_W)\,, \quad (26)$$

where we have assumed

$$\exp(-8Kt)\sim 1-8Kt\,,$$
$$\exp(-4Kt)\sim 1\,.$$

The mode exhibits antibunching if $2\phi_L-2\phi_W=2\pi n$ because Eq. (26) becomes

$$\langle(\Delta W_L)^2\rangle\simeq -Kt(|\xi_L|^2-|\xi_W|^2)^2\,. \quad (27)$$

IV. CONCLUSIONS

By the use of the standard master equation approach, it is possible to obtain some information on the statistical properties of a process involving the interaction between radiation and free electrons.

It is interesting to observe that the coherence properties, as expressed by the value of the variance of the amplified laser mode, depend upon the phase relation between the laser and the virtual Wiggler mode. This is connected with the fact that emission of real photons in the Wiggler modes, which contain only virtual photons at time $t=0$ takes place due to the scattering process.

With respect to previous treatments,[15,14] our technique allows us to observe the presence of antibunching of the emitted radiation as a function of the values of the initial phase of the fields, and also could be generalized in a straightforward way to the multimode high-gain case.

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