Theory of optical multistability and chaos in a resonant-type semiconductor laser amplifier

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Multistable light amplifications and self-pulsations in a resonant-type semiconductor laser diode (LD) amplifier are predicted. A basic idea is derived from the active layer refractive index dependence on carrier density. An LD amplifier is shown to act as a high-Q nonlinear Fabry-Perot interferometer with true optical gain and nonlinear refraction. Periodic and chaotic self-pulsations are shown to occur in the regime where the delay time of the feedback is smaller than the carrier lifetime.

In semiconductor lasers, the active layer refractive index near the gain spectrum peak varies in an approximately linear fashion with injected carrier density, as a result of the anomalous dispersion effect.[1] The strong carrier density dependence of the refractive index has been found to result in various peculiar phenomena, such as double-lobed far-field patterns,[2] lateral mode instability[3] in stripe geometry lasers, carrier-modulation contribution to laser linewidth,[4] and asymmetric tuning curves associated with injection locking.[5]

This Communication predicts optical multistable operation and chaotic behavior in a semiconductor Fabry-Perot–type laser amplifier. The basic idea is derived from the carrier depletion-induced refractive index change in the active layer due to external light injection.

Figure 1(a) conceptually illustrates the model of the semiconductor laser diode (LD) amplifier used for the following analysis. A coherent optical beam with frequency ν_i is injected into the LD amplifier through one of the facets. The other facet is assumed to be antireflection coated, and the Fabry-Perot resonator consists of one of the facets and an external mirror. The LD is driven by dc injection current below the threshold and acts as a resonant-type amplifier for input laser light. The injected carrier density decreases as a result of light injection, while the refractive index in the active layer increases accordingly. This is because of the negative proportionality constant.[1,6] Therefore this system is considered to be a nonlinear Fabry-Perot interferometer, with true optical gain and nonlinear refraction.

When the spatial dependence of the population difference, coming from standing-wave effects, and absorption is neglected,[7,8] the result is the mean-field model.[9] Most analytic work in optical bistability has been carried out within the framework of the mean-field model.[9,10] In the LD amplifier system, the spatial diffusion of carriers in the longitudinal direction is fast enough to allow neglect of standing-wave modulation of carrier density. In addition, it is assumed for brevity that the length of the LD, l, is much shorter than that of the external cavity, L, and that the reflectivities of mirrors are high enough to allow the condition of G (gain) $\simeq 1$. In this case, we can employ the average field distributed over the length of the LD[11] (mean-field approximation) and the propagation effect has been neglected.[7] If the field changes faster as compared with the transit time through LD or $G \gg 1$, mean-field approximation is no longer valid.

The response of the active medium (LD) can be described by the conventional rate equation for carrier density. The rate equation is subsequently converted to the differential equation for field's phase shift across the LD adiabatically, through the carrier density dependence of the refractive index. Applying the mean-field approximation, the dynamics of the system is governed by the following difference-differential equations[12]:

$$E(t,l) = [\eta(1-r_1^2)G]^{1/2}|E_i| + r_1 r_2 G \exp(\phi+\phi_0) E(t-t_R,l) \ , \quad (1)$$

$$\tau_s \frac{d\phi}{dt} = -\left[1 + \left|\frac{E(t-t_R,l)}{E_s}\right|^2\right]\phi - g_0 R l \left|\frac{E(t-t_R,l)}{E_s}\right|^2 \ , \quad (2)$$

$$G = \exp[(\Gamma g - \alpha)l] \ , \quad (3)$$

$$g = g_0/(1+|E/E_s|^2) \ , \quad (4)$$

$$E_s^2 = J/ed\Gamma g_0(1+r_2^2) \ . \quad (5)$$

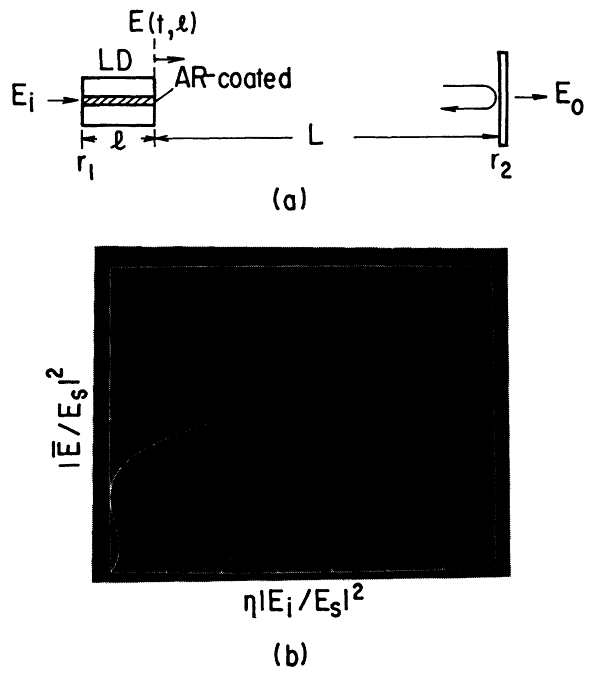

FIG. 1. (a) Conceptional model of multistable semiconductor laser amplifier with external mirror. (b) Output $|\bar{E}/E_s|^2$ vs input $\eta|E_i/E_s|^2$ relation. Numerical values for calculations are described in the text. The positive branch unstable regions were obtained for $t_R/\tau_s = 0.05$ numerically, with use of Eqs. (1) and (2). 0.05/div for both axes.

Here, E is the complex mean-field amplitude, η the coefficient for light coupling into active layer, r_1, r_2 the amplitude reflectivity, ϕ the roundtrip phase shift across LD, ϕ_0 the cavity mistuning parameter, τ_s the carrier lifetime, t_R the delay time of the feedback, g_0 the small signal gain coefficient, Γ the optical confinement factor, α the loss coefficient, g the mean-field gain coefficient, E_s^2 the effective saturation intensity, J the injection current density, e the electronic charge, and d the active layer thickness. The factor R can be expressed as

$$R = \frac{4\pi}{\lambda} \frac{\partial n}{\partial N} \Big/ \frac{\partial g}{\partial N} ,$$

with n being the active layer refractive index. This is the ratio of the derivatives with respect to the carrier density N of the real part to the imaginary part of the dielectric constant.

The steady-state solution of Eqs. (1) and (2), denoted by $\bar{E}$, can be given as a multivalued function of E_i such that

$$|E_i/E_s|^2 = |\bar{E}/E_s|^2 \frac{1 + r_1^2 r_2^2 G^2 - 2 r_1 r_2 G \cos(\bar{\phi} + \phi_0)}{(1 - r_1^2)\eta G} , \tag{6}$$

$$\bar{\phi} = -g_0 R l |\bar{E}/E_s|^2 / (1 + |\bar{E}/E_s|^2) , \tag{7}$$

which is illustrated in Fig. 1(b). Here, the adopted parameter values are $\Gamma = 0.2$, $g_0 = 115$ cm^{-1}, $\alpha = 20$ cm^{-1}, $l = 300$ μm, $r_1^2 = r_2^2 = 0.9$, $\phi_0 = -0.5$, and $R = -6$.

These $|\bar{E}|^2$ are not always stable. A linear stability analysis reveals that the stationary solutions of Eq. (6) are stable only in the regions satisfying the condition $0 < S < S_c$, where S is defined as $S \equiv d|E_i|^2/d|\bar{E}|^2$, and S_c is given by

$$2(1 + r_1^2 r_2^2 G^2)/(1 - r_1^2)\eta G ,$$

assuming $r_1 r_2 G \simeq 1$. For $S < 0$ (negative branch), the system is always unstable, and this region is depicted by the dotted line in the figure. Using a computer we have found that the "Ikeda unstable" region appears in the positive branch for a comparatively small t_R value.[13,14] This positive branch unstable region is shown by the dashed line in the figure for $t_R/\tau_s = 0.05$.

Figures 2(a)–2(d) show the numerical results for Eqs. (1) and (2) with different $\eta|E_i/E_s|^2$ values, assuming $\Gamma = 0.2$, $g_0 = 115$ cm^{-1}, $\alpha = 20$ cm^{-1}, $l = 300$ μm, $r_1^2 = r_2^2 = 0.9$, $\phi_0 = -0.5$, $R = -6$, and $t_R/\tau_s = 0.2$. These figures indicate the period-doubling (or successive subharmonic) bifurcations of self-pulsations. It is interesting to point out that self-pulsations occur in the regime where the delay time t_R is much smaller than the carrier lifetime τ_s.[14,15] In the LD amplifier system, the lifetime of the cavity, which is approximately given by $\tau_p \simeq t_R/(1 - r_1 r_2 G)$, becomes longer than the carrier lifetime in the regime $t_R/\tau_s \ll 1$, because of its low dissipation rate (high-Q factor) as a result of optical gain, i.e., $1 - r_1 r_2 G \ll 1$. This situation is brought about when the LD is biased just below the threshold for lasing. Figure 3 shows (a) time-dependent trajectory and (b) corresponding trajectory in the $[\mathrm{Re}(E/E_s) - \mathrm{Im}(E/E_s)]$ phase space for chaotic self-pulsations, where $t_R/\tau_s = 0.5$ and $\eta|E_i/E_s|^2 = 0.3$.

These positive branch instabilities, however, disappear for a conventional LD resonant-type amplifier without external mirror, i.e., $L = 0$, since t_R/τ_s is as small as 10^{-3}. In order to further investigate optical bistable properties we will re-

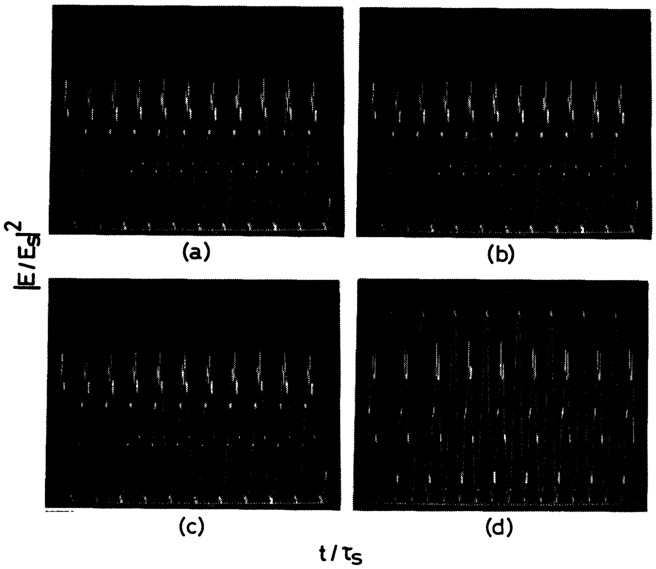

FIG. 2. Period-doubling bifurcations of self-pulsation in a LD amplifier, assuming $t_R/\tau_s = 0.2$. Adopted parameter values are shown in the text. Horizontal axes: t/τ_s (0.2/div). Vertical axes: $|E/E_s|^2$ (0.1/div). (a) period 1 ($\eta|E_i/E_s|^2 = 0.1$); (b) period 2 (0.2); (c) period 4 (0.32); (d) period 8 (0.35). A corresponding cavity length for t_R/τ_s is $L = 6$ cm when $\tau_s = 2$ ns is assumed.

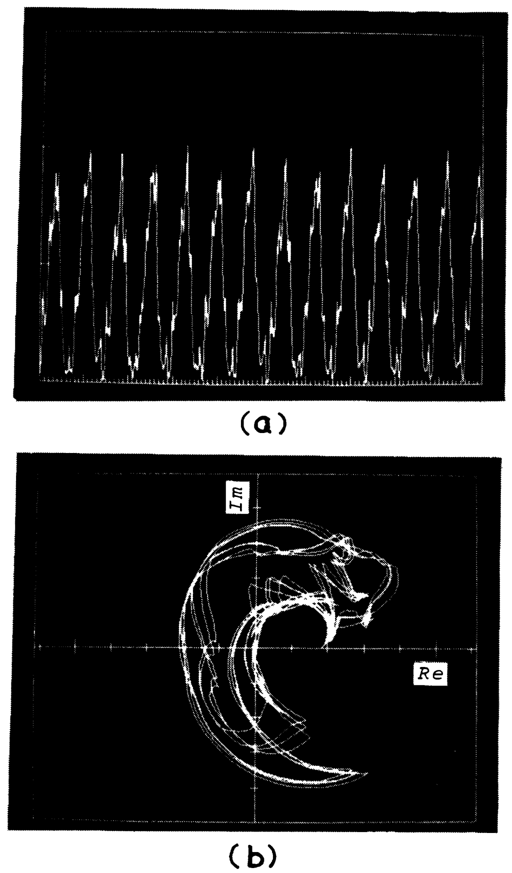

FIG. 3. Long-term periodic or chaotic self-pulsation in a LD amplifier, assuming $t_R/\tau_s = 0.5$ and $\eta|E_i/E_s|^2 = 0.3$. Other parameters used are same as in Fig. 2. (a) Time-dependent trajectory, t/τ_s (0.4/div) $|E/E_s|$ (0.1/div). (b) Trajectory in complex phase space, 0.1 div, with origin at (0,0).

strict the analysis to the lowest branch in Fig. 1(b) and $L=0$ case.[16]

Figure 4(a) illustrates $|E_0/E_{so}|^2$ vs $\eta|E_i/E_{so}|^2$ for various R values [$|E_0|^2 \equiv (1-r_2^2)|\bar{E}|^2$: steady-state output power; $E_{so}^2 \equiv (1-r_2^2)E_s^2$: saturation output power[16]], assuming $\Gamma=0.2$, $g_0=240$ cm^{-1}, $\alpha=20\ \text{cm}^{-1}$, $l=300\ \mu\text{m}$, $r_1^2=r_2^2=0.3$, and $\phi_0=0$, 4.5. It can be seen that bistability arises more easily when R becomes large. The R factor has been reported to vary between -0.5 and -6.2 in the case of a GaAs active layer.[1,6,17,18]

Figure 4(b) shows $|E_0/E_{so}|^2$ versus nominal frequency detuning $(\nu_i-\nu_0)/\Delta\nu$ ($\Delta\nu=c/2nl$, axial mode spacing; c, velocity of light), where ν_0 is the cavity resonance frequency in the absence of light injection. Calculations were carried out for various R values, assuming the same parameter values as in Fig. 3(a) and $\eta|E_i/E_{so}|^2=0.01$. It can be seen in the figure that the detuning curve becomes very asymmetrical with respect to $\nu_i=\nu_0$ when $|R|$ takes place on the order of 1–2. In particular, if R is a negative value, the maximum output power can be obtained at a negative value of nominal detuning. Furthermore, hysteresis properties coming from the multivalued function, Eq. (6), appear in negative detuning. This nonlinear resonance has been observed experimentally for a GaAlAs/GaAs resonant-type amplifier.[16] The result is shown in Fig. 4(c). Similar asymmetrical tuning curves have been reported by Lang[5] for an injection-locked LD oscillator. However, peculiar nonlinear resonance properties *with hysteresis* as shown in Fig. 4(c) were not observed for locking with low power light injection.[19,20]

To summarize, multistable light amplifications as well as periodic and chaotic self-pulsation in a resonant-type semiconductor laser amplifier have been theoretically predicted. Previously reported carrier depletion-induced refractive index change in the active layer has been found to be large enough for multistability as well as self-pulsations at a realistic input light power level. Peculiar nonlinear resonance properties have been also shown to exist with hysteresis. The present theory can be generally applicable to any other laser amplifier system which has asymmetry of the derivative with respect to population density, i.e., nonzero R value, at the lasing frequency.[21]

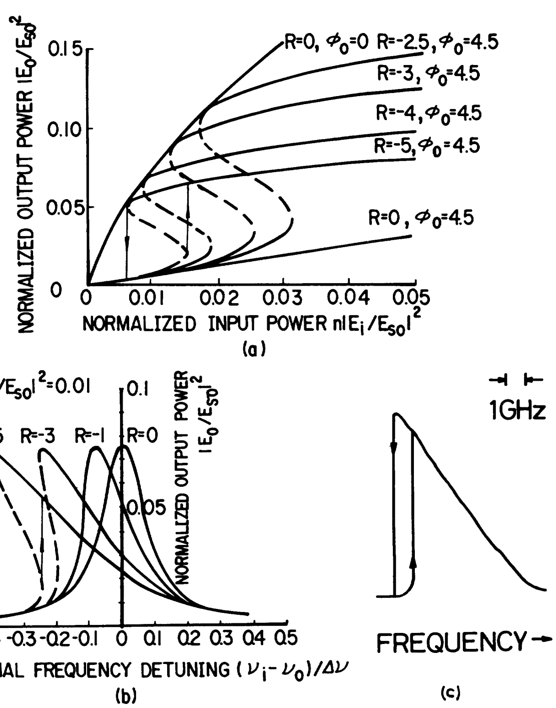

FIG. 4. (a) $|E_0/E_{so}|^2$ vs $\eta|E_i/E_{so}|^2$ for different R values, assuming the same parameter values as Fig. 2 (see the text). (b) $|E_0/E_{so}|^2$ vs $(\nu_i-\nu_0)/\Delta\nu$ for different R values, assuming $\eta|E_i/E_{so}|^2=0.01$. (c) Nonlinear resonance phenomenon observed in a 300-μm-long chaotic self-pulsations GaAlAs/GaAs resonant-type amplifier, where bias injection current is 81.8 mA and ηP_{in} is 0.1 mW (P_{in}, injected light power; η, light coupling coefficient into the active layer).

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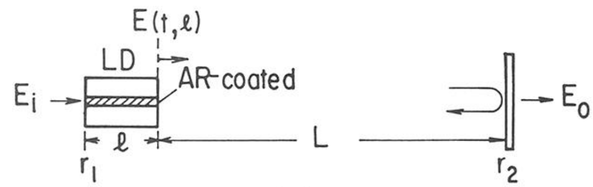
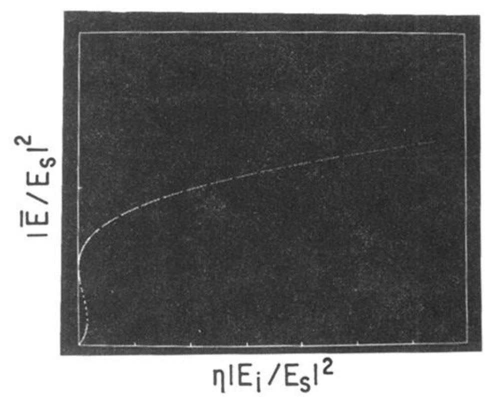


(a)



(b)

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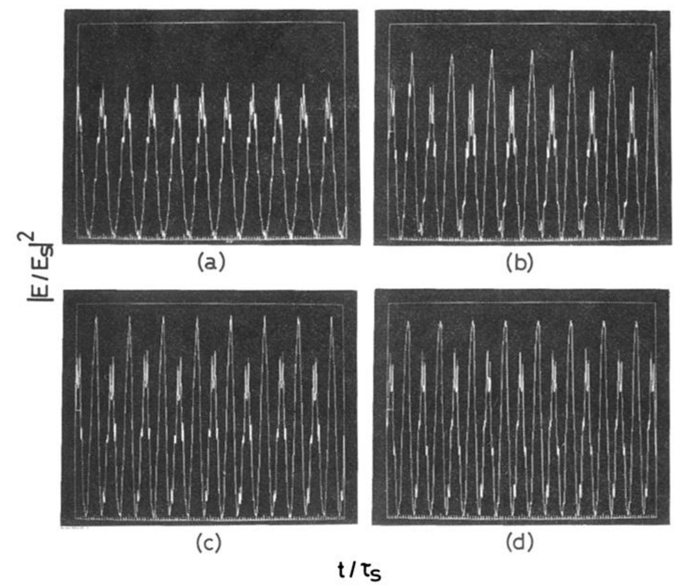

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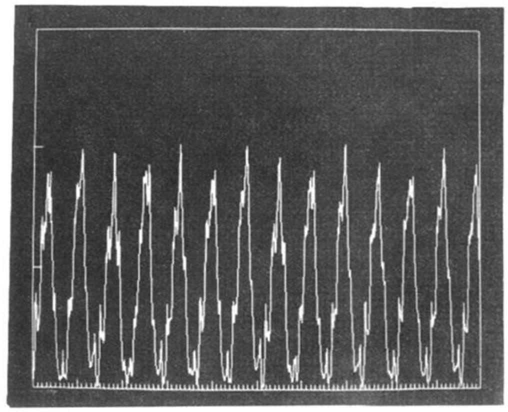
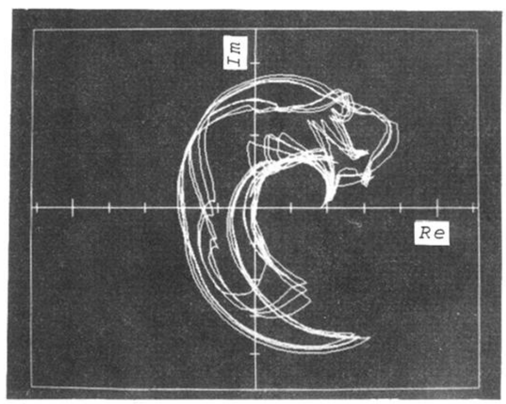


(a)



(b)

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