Electrostatic waves in a relativistic and anisotropic plasma stream, and electron waves and ion-acoustic modes: Fusion instabilities

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The electrostatic dispersion and instabilities for an anisotropic relativistic plasma stream are analytically studied for various regimes with streaming dissipation relevant to the analysis of fusion and turbulence in beam-plasma and wave-plasma interaction. It reveals the modified spectrum of streaming dispersion and onset of instability at nonrelativistic temperatures suitable for space, at moderately relativistic temperature including fusion, and at ultrarelativistic temperature limit relevant to astrophysical plasmas and pulsars.

I. INTRODUCTION

In order to investigate the high current driven microinstability and heating of the plasma mechanism in the wake of anisotropy in particle momentum distribution in a relativistic high-temperature plasma beam, it is imperative to choose the relativistic Vlasov equation including a corresponding unapproximated distribution function with streaming dissipation loss, etc. Early results[1-3] do not reveal the effect of streaming turbulence in plasma instabilities, particularly at different temperature limits which explain fusion and astrophysical plasmas including the pulsars. Even for the nonrelativistic temperature limits or regimes, our present analysis shows marked innovations in the upper range of the species, i.e., $T_i < 10^{13}$ K and $T_e < 10^8$ K. The information is really important in explaining the thermal modes and finite-temperature oscillations for the field-free case of Bernstein modes. We illustrate in detail the modified aspect of streaming dispersion and Landau damping for various limits or particle regimes.

The conventional analysis of fusion plasmas and their drift-type instabilities in stellarator, tokamak, and the like[4-6] have received much attention. Another kind of motivation lies in the attainment of fusion and high temperature in a relativistic plasma beam, preferably showing the effects of streaming anisotropy and thermal dissipative modes covering turbulence and instabilities.

Early works on relativistic electron beams contain interactions between the beam and the plasmas and such beams also generate return-current instability in the plasma species[7]; however, the fusion aspect and the high-temperature streaming effect have found little relevance. We propose here to clear up the issue which reveals the modified frequency spectrum in a relativistic plasma stream valid for all limits of temperatue and thermal modes.

Previous works based on the approximations of Maxwellian equilibrium distribution functions relate to the isotropic relativistic plasmas. Similarly, extensive isotropic works on magnetized relativistic plasma have been referred to in a recent paper (Mohanty and Misra,[8]) and in Papuashvili, Tsikamshvili, and Tsintsadze[9] and Inoue, Itoh, and Yoshikawa.[10] Buti[3,11] reported the results of nonstreaming electrons and delineated the stability zones on a counterstreaming magnetized relativistic plasma stream. Mikhailovskii[12] correlated the early results of isotropic relativistic plasmas, whereas he later[13,14] dealt with the Čerenkov growth rate of instability of the beam with thermal spread and density inhomogeneity in an ultrarelativistic plasma. The same Čerenkov aspect for longitudinal waves was analyzed by Onischenko.[15] However, the effects of streaming anisotropy and dissipative thermal modes have not been considered at all in obtaining information on streaming and turbulent plasmas undergoing fluctuations in the frequency mode in terms of the refractive index (kc/ω) and streaming parameter $[(u_0/c)(mc^2/k_BT)]$.

Similar to the study of electromagnetic waves for the relativistic plasma stream,[8] we present here the analytic results of excitation of electrostatic waves driven by microinstabilities valid for the limiting cases of regimes.

In Sec. II, we have analytically derived the expressions for the imaginary and real parts of the Lindhard tensor K_{33} or K_L and evaluated the series of integrals for the principal value and the pole contribution in the complex analytic plane. Sections III and IV deal with the nonrelativistic temperature limit which finds relevance to space and galactic plasmas including solar flares, etc., and moderately relativistic temperature conditions displaying electron fusion processes. Section V, which is more esoteric, covers the importance of plasma in cosmology, astrophysics, exploding black holes, and other processes linked with the expansion of the universe and unstable protons in high energies. In Sec. VI, a brief discussion is added and the results and specific applications are summed up.

II. THEORY AND COMPUTATION OF THE RESPONSE FUNCTION

Introducing an anisotropy of the streaming loss of energy in the form $V_0 \cdot p_j$ in the energy-momentum relativistic equation

$$E = m_j c^2 \left[1 + \frac{p_j^2}{m_j^2 c^2}\right]^{1/2} - V_0 \cdot p_j \ , \tag{1}$$

we make the distribution anisotropic in particle momentum and streaming velocity. The thermodynamic equilibrium distribution function assumes the unapproximated form as

$$f_0(p_j) \sim \frac{Z_j}{4\pi m_j^3 c^3 \gamma_0^3 K_2(Z_j/\gamma_0)} \times \exp\left\{-Z_j\left[\left[1 + \frac{p_j^2}{m_j^2 c^2}\right]^{1/2} - \frac{V_0 \cdot p_j}{m_j c^2}\right]\right\} \ , \tag{2}$$

where $Z_j = m_j c^2/k_B T_j$, $\gamma_0 = (1 - V_0^2/c^2)^{-1/2}$, and $K_2(Z_j/\gamma_0)$ is the modified Bessel function of second order with imaginary argument (Z_j/γ_0) and $V_0 \cdot p_j$ is the streaming dissipation of energy in the relativistic anisotropic distribution function.

Using the above function, one can compute the Lindhard tensor K_{33} or K_L for the longitudinal part of the response function as

$$K_{33},K_L=-ie\omega\sum_{j=+,-} j\omega_{0j}^2 m_j\int_0^\infty\int_0^1 \frac{dp_j dx' p_j^3(p_j^2+m_j^2c^2)^{1/2}x'^2}{\omega^2(p_j^2+m_j^2c^2)-c^2p_j^2k^2x'^2}\frac{df_{0j}}{dp_j}\ , \tag{3}$$

where we define the dimensionless parameters as

$$\alpha=1-\frac{a\omega}{ck},\quad x=\frac{p_j}{m_jc}\ ,$$

$$y=\pm\frac{\omega}{(c^2k^2-\omega^2)^{1/2}},\quad a=\frac{V_0}{C}\ .$$

Solving the series of integrations in x and x', which are of the form

$$\ln\frac{\omega(1+x^2)^{1/2}-ckx}{\omega(1+x^2)^{1/2}+ckx}$$

and binomial in

$$[\omega(1+x^2)^{1/2}/ckx\pm 1]^n$$

and evaluating the rest of the integrals using the form (Gradeshteyn and Ryzhik[16])

$$\int_0^\infty d\theta\sinh^{2n}\theta\exp(-\alpha Z_j\cosh\theta)=\frac{\Gamma(n+\frac{1}{2})}{\Gamma(\frac{1}{2})}\frac{K_n(\alpha Z_j)}{(\alpha Z_j/2)^n}\ , \tag{4}$$

one can get an expression for $\mathrm{Im}K_L$ after lengthy simplifications in terms of the modified Bessel function of the second kind, $K_n(\alpha Z_j)$,

$$\begin{aligned}\mathrm{Im}K_L=\sum_{j=+,-}\frac{\omega_{0j}^2Z_j^2}{8\pi ck\gamma_0^2K_2(Z_j/\gamma_0)}\Bigg\{&\frac{2\omega}{ck\alpha Z_j}+K_0(\alpha Z_j)+\frac{2K_0(\alpha Z_j)}{(\alpha Z_j)^2}\\&+\sum_{n=1}^\infty\frac{1\times3\times5\times\cdots\times(2n-1)}{(\alpha Z_j)^n y^{2n}}\left\langle K_{n+2}(\alpha Z_j)+\frac{K_{n+1}(\alpha Z_j)}{\alpha Z_j}+\frac{2K_n(\alpha Z_j)}{(\alpha Z_j)^2}\right\rangle\\&+\frac{2\omega}{ck}\left[\frac{K_2(Z_j)}{Z_j}-\frac{K_2(\alpha Z_j)}{\alpha Z_j}\right]-\frac{1}{2}\frac{a\omega^2}{c^2k^2}[K_3(Z_j)+3K_1(Z_j)]\\&+\sum_{n=0}^\infty\frac{(aZ_j)^{2n}}{n!2^n(\alpha Z_j)^n}\Bigg[K_{n+3}(\alpha Z_j)\frac{2\omega^2a}{c^2k^2(2n+1)}+\frac{2k_{n+2}(\alpha Z_j)}{\alpha Z_j}\left\{a-\frac{\omega}{ck}\left\langle\frac{1+2(a\omega/ck)}{2n+1}+\frac{a\omega}{ck}\right\rangle\right\}\\&+\frac{2K_{n+1}(\alpha Z_j)}{\alpha Z_j}\left\{-\frac{2n}{aZ_j}\frac{2n+3}{2n+1}+\frac{\omega}{ckZ_j}\frac{2n}{2n+1}\right\}\Bigg]\Bigg\}\ ,\end{aligned} \tag{5}$$

and solving for the singularity across the pole, one finds

$$\mathrm{Re}K_L=\sum_{j=+,-}\frac{Z_j\omega_{0j}^2y^2}{8K_2(Z_j/\gamma_0)\gamma_0^2kc}\left[1+\frac{2\omega}{\alpha Z_jcky}+\frac{2\omega^2}{\alpha^2Z_j^2c^2k^2y^2}\right]\times\exp[-\alpha Z_j(1+y^2)^{1/2}]\ . \tag{6}$$

III. NONRELATIVISTIC TEMPERATURES: APPLICATIONS TO PLASMA

It is our aim to illustrate the streaming anisotropic results for the nonrelativistic range of temperatures for the species. Electrons and ions become moderately relativistic particles at 10^8 and 10^{13} K, respectively, so the plasmas characterized by them below these limits reveal interesting results. Usually these plasmas occur in ionospheres, equatorial electrojets, intergalactic or interestellar plasma radiation belts, and solar corona and flares or shock waves, etc.

Using the desired approximations, viz.,

$$c\to\infty;\ \alpha=1 \tag{7}$$

and

$$K_n(Z_j)\simeq\left[\frac{\pi}{2Z_j}\right]^{1/2}\exp(-Z_j)\ , \tag{8}$$

one obtains an infinite series in the relevant expression for $\mathrm{Im}K_L$ which becomes asymptotic for the low-wavelength limit and low streaming energies of the species. One can easily find the modified frequency spectrum,

$$\omega=\sum_{j=+,-}\omega_{0j}^2\left[\frac{m_j\omega}{k^2k_BT_j}+\frac{1}{\omega}+\frac{m_jV_0}{kk_BT_j}\right]\ , \tag{9}$$

and the Landau term for the growth rate of instability,

$$\gamma=\sum_{j=+,-}\frac{m_j\omega^2}{k^2k_BT_j}\left[1+\frac{u_0}{c_{\rm th}}\right]\left[\frac{\pi}{8}\right]^{1/2}\frac{\omega_{0j}}{k^3\lambda_{Dj}^3}\exp\left[-\frac{1}{2(k\lambda_{D1})^2}\right], \tag{10}$$

where

$$c_{\rm th}=\left[\frac{k_BT_j}{m_j}\right]^{1/2}\ .$$

Both ω and γ increase as V_0 ranges from 0 to 0.001 °C in the temperature limit up to $T_e<10^8$ K and $T_i<10^{13}$ K. The early results of Bell and Buneman[17] and Sudan[18] do not display these aspects and chiefly nonstreaming characters are indicated, and our results conform to isotropic results in the limit of vanishing V_0.

Quantitatively, the streaming introduces a factor 1.2×10^{-3} in (9) for the electron species with $V_0\sim10^{-3}$°C, $T_e\sim10^7$ K, and $\lambda=10$ cm and $\omega_{0e}\sim10^4$ rad/sec, while the corresponding estimate for the ion component is 1.2×10^{-4} with $\omega_{0i}\sim10^2$ rad/sec.

However, for small wave numbers $k\lambda_D\ll1$, damping is negligibly small. For ion temperature at 10^8 K, this limiting case is significant to explain fusion instability, whereas for electrons we have to resort to the moderately relativistic temperature limit.

The modified frequency dispersion for streaming and the growth term of instability reduce to the classical results of Bernstein and Landau terms in the limit of vanishing streaming or drift. Such oscillations and thermal modes are readily applied to the plasmas occurring in ionospheres and equatorial electrojets, where artificial control of irregularities is desirable. Further, the satellite probes in space yield valuable insight into the waves and instabilities which nature furnishes in coronal discharge, solar flares, shock waves, and other kinds of galactic plasmas below the range $T_e \lesssim 10^8$ K (10 keV) and $T_i \leqslant 10^{13}$ K (1 GeV). The numerical estimates have been shown for varying degrees of streaming, viz., low $V_0 = 0.001c$ and high $V_0 = 0.01c$. It is interesting that the limit or regime displays the effects of field-free toroidal plasma around the electron temperature 10^7 K.

IV. MODERATELY RELATIVISTIC TEMPERATURE AND FUSION

To deal with the effect of streaming at moderately relativistic temperatures ($T_e > 10^8$ K, $T_i > 10^{13}$ K), we assume $Z_j \geqslant 1$. For streaming parameters $a = 10^{-2}$ and $Z = 10^2$, one has to retain the terms of the order aZ, $1/Z$ and neglect higher-order terms such as $1/Z^2$, etc. Since the expression for $\mathrm{Im}K_L$ contains terms up to $(\alpha Z_j)^2$ in the numerator, it is imperative to retain the $1/(\alpha Z_j)^3$ term in the approximated expression for $K_n(\alpha Z_j)$, otherwise the analysis cannot explain the novel physics inherent in the limit.

Choosing

$$K_n(\alpha Z_j) \sim \left[\frac{\pi}{2\alpha Z_j}\right]^{1/2} \exp(-\alpha Z_j)\left[1 + \frac{4n^2-1}{8\alpha Z_j} + \frac{(4n^2-1)(4n^2-3^2)}{2!(8\alpha Z_j)^2} + \frac{(4n^2-1)(4n^2-3^2)(4n^2-5^2)}{3!(8\alpha Z_j)^3}\right] , \tag{11}$$

one finds, after some lengthy algebra and rearrangement of terms, an approximate expression for $\mathrm{Im}K_L$ and, writing $1/\alpha^n \simeq (1 + na\omega/ck)$ for "a" small, we can obtain another expression, which for brevity is not included here. We next combine these results to obtain the dispersion relation with negligible damping for the spectrum of electrostatic waves in a two-component streaming plasma:

$$\omega^2 = \sum_{j=+,-} \omega_{0j}^2 \left[1 - \frac{5}{2Z_j} + \frac{3}{Z_j}\frac{c^2k^2}{\omega^2} - 7a\frac{\omega^3}{c^3k^3} + \frac{13}{2}\frac{a\omega}{ck} - 10aZ_j\frac{\omega^3}{c^3k^3} + \frac{aZ_j\omega}{ck} - 4a^2Z_j^2\frac{\omega^3}{c^3k^3} \right.$$
$$\left. + 3a\frac{ck}{\omega} - 6a\frac{\omega^2}{c^2k^2} - \frac{14}{3}aZ_j\frac{\omega^4}{c^4k^4} + a^2Z_j^3\frac{\omega^6}{c^6k^6} - \frac{3}{4}a^2Z_j^3\frac{\omega^4}{c^4k^4}\right] . \tag{12}$$

However, for the low-frequency limit $\omega < ck$, the streaming correction terms reduce to three only, the frequency of oscillation increases, and fluctuations are minimized:

$$\omega^2 = \sum_{j=+,-} \omega_{0j}^2 \left[1 - \frac{5}{2Z_j} + \frac{3}{Z_j}\frac{c^2k^2}{\omega^2} + \frac{13}{2}\frac{a\omega}{ck} + aZ_j\frac{\omega}{ck} + 3a\frac{ck}{\omega}\right] . \tag{13}$$

Let us discuss the numerical estimates. For the low-frequency limit, the frequency ratio is estimated for the following parameters: $Z_j = 50$, $ck/\omega = \frac{1}{2}$. It is seen that $(\omega/\omega_{0j})^2 \sim 1.79$ for $a = 0.01$, whereas $(\omega/\omega_{0j})^2 \sim 0.83$ for $a = 0.001$. The fluctuations are prominent. Again for $n \sim 1$, $(\omega/\omega_{0j})^2$ rises to 6 when $a = 0.1$.

Such fusion processes in an unmagnetized beam are unique in the relativistic plasma stream, where drift or streaming velocity is conspicuously dominant. The frequency spectrum displays the fluctuations in the excitation modes in terms of streaming parameters aZ_j and ω/ck (reciprocal or refractive index) and contains the amplification or the growth of wave-gaining energy in the form $(\omega/\omega_{0j})^2$. The effect of turbulence as a sequel to aZ_j terms in the plasma instabilities for the species is well marked. The above results display the fusion instability at electron temperature whereas for ion fusion Eqs. (9) and (10) are referred to.

The streaming fluctuations in the wave spectrum with almost no damping in the temperature limit or regime find numerous practical applications in the laboratory and heating devices as well. One can cite the following heating mechanisms governed by the temperatures or corresponding particle energies $T_e > 10^8$ K (10 keV) and $T_i > 10^{13}$ K (10 GeV): (a) rf heating of plasmas, (b) laser-induced heating and fusion pellets, (c) deuterium-tritium implosion and laser heating, (d) fusion reaction chamber or fusion reactor, (e) neutral beam injection device and plasma heating, (f) irradiation of hot gas or relativistic electron beams by fusion pellets, (g) intense relativistic electron beam of high current (REB) in diodes, (h) attainment of fusion temperature in unmagnetic relativistic plasma stream and open-ended fusion systems, and (i) deuterium plasma in collisionless shock tubes ($T_i > T_e$).

As discussed earlier both the electron waves and ion-acoustic waves exhibit the streaming dissipative modes and marked fluctuations in relative strength, i.e., $(\omega/\omega_{0j})^2$.

V. ULTRARELATIVISTIC TEMPERATURES AND APPLICATIONS

For ultrarelativistic temperature limits $k_BT_j >> m_jc^2$ which usually occur in pulsars and other astrophysical plasmas of low density and REB experimental simulations in laboratory and other devices, the investigations with streaming effect can account for the turbulence in plasma flow and beam plasma or wave plasma interactions. The constituent species in the plasma beam are known as superthermal electrons and ions and generate waves in plasma having anisotropy in particle momentum distribution. Such waves are called electron waves and ion-acoustic waves. The temperatures in the regime cover up to 10^{32} K which correspond to particle energies of 10^{19} GeV called Planck's temperature. This explains the plasma properties observed in the sporadically exploding black holes, cosmological phenomena in the universe, stellar bodies, and high-energy particles.

With

$$Z_j << 1, \quad K_0(\alpha Z_j) = -\ln(\alpha Z_j) , \tag{14}$$

and

$$K_n(\alpha Z_j) \simeq \frac{\Gamma(n)}{2(\alpha Z_j/2)^n} \simeq 2^{n-1}\frac{(n-1)!}{(\alpha Z_j)^n} , \tag{15}$$

one obtains the imaginary K_L for anisotropic oscillations as

$$\mathrm{Im}K_L = \sum_{j=+,-} \frac{\omega\omega_{0j}^2 Z_j}{8\pi c^2 k^2}\left(1 - 11\frac{a\omega}{ck} + 2a\frac{ck}{\omega} - 8\frac{a\omega^2}{c^2k^2}\right) \quad (16)$$

and

$$\mathrm{Re}K_L \simeq 0 \;; \quad (17)$$

using the above values, of real and imaginary K_L, one finds the frequency of oscillation, which is real as

$$\omega^2 = \sum_{j=+,-} \frac{\omega_{0j}^2 Z_j}{3}\left(1 - 11\frac{a\omega}{ck} + 2a\frac{ck}{\omega} - 8\frac{a\omega^2}{c^2k^2}\right) . \quad (18)$$

Let us estimate the numerical values for $(\omega/\omega_{0j})^2$ in the limit. We choose the parameters in the following way:

$$Z_j = 10^{-3}; \text{ and } \left(\frac{\omega}{\omega_{0j}}\right)^2 \sim 1.013 \times 10^{-3}$$

when $V_0 \sim 0.001c$, whereas $(\omega/\omega_{0j})^2$ falls to 0.5×10^{-3} when the streaming is strong or near relativistic, i.e., $V_0 \sim 0.1c$.

In view of the academic and esoteric significance of the limit, it encompasses the mode of oscillations and triggering of instabilities up to 10^{32} K, corresponding to the particle energies up to 10^{19} GeV in the universe. It also explains the heating of the universe which consists of plasmas at 10^{15} GeV or 10^{28}K, apart from finding applications in stellar plasmas, stars, astrophysical plasmas covering quasars, pulsars, and sporadically exploding black holes, where dissipative processes and turbulent heating either in streaming or drift form are dominant.

It should be emphasized that applications to Monte Carlo simulation can yield valuable information on the probability of distribution and random behavior of astrophysical objects having ultrarelativistic plasma matters.

Early results of Buti[3] on the ultrarelativistic limit only do not find a modified dispersion relation for the streaming species. On the contrary, these conclude only for nonstreaming electron species. Owing to mathematical inhibitions, it has been valid for $\omega > ck$ limit. Our dispersion results contain the above analysis as a special case and is valid both for the low- and high-frequency limit showing correspondence between nonstreaming and streaming propagation.

Besides, other isotropic results of Imer[19] and Silin[20] as discussed in Mikhailovskii[12] follow from the present analysis in the limit of vanishing V_0 $(a = V_0/c)$.

VI. DISCUSSION

We have investigated the influence of streaming velocity of the plasma species as a whole in a relativistic plasma stream instability both for $\omega > ck$ and $\omega < ck$, suitable for varying thermal modes and particle regimes. At nonrelativistic temperatures, the results of excitation of electrostatic waves yield unique dispersion relations and Landau damping indicating the growth rates of instability for spectrum of waves $(m_jc^2 \gg k_BT_j)$. The results are significant for the propagation of waves and instabilities in space plasma, ionospheres, equatorial electrojets, intergalactic plasma, stellar plasmas, and other types of solar coronal discharge, shock waves, and solar flares, etc. We have indicated quantitative estimates both for ions and electron species for low streaming $V_0 \sim 10^{-3}c$ in Sec. III. Besides, these results find possible applications in space simulations in laboratory, satellite-based observations for space studies in the plasmospheres, and artificial control of equatorial irregularities.

In Sec. IV, we have discussed the analytic results of dispersion with negligible damping for moderately relativistic plasma ($T_e > 10^8$ K, $T_i > 10^{13}$ K), where there is perceptible streaming in various ranges, viz., low and strong streaming parameters. These results are widely significant for the fusion of nuclei in a fusion reactor in the presence of a strong repulsive force or Coulomb barrier particularly at $T_e \sim 10^8$ K, while for $T_i - 10^8$ K we have referred to the results of Sec. III qualitatively. It explains the waves in a plasma suitable for a fusion reaction chamber, high temperature, relativistic plasma beam, relativistic ions in a diode, laser-induced irradiation, and REB irradiations on a fusion pellet, etc. Other possible applications include deuterium plasmas in a collisionless shock tube, where $T_i > T_e$ is specifically retained.

In Sec. V, we have dealt with the extremely high-temperature plasma species $(m_jc^2 \ll k_BT_j)$ up to 10^{32} K or 10^{19} GeV which usually occur in low-density pulsars, quasars, and stars of various kinds including exploding black holes and other bursts at high temperature. Unique dispersion terms with modified streaming have been derived with negligible damping.

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