Route to mode locking in a three-mode He-Ne 3.39-μm laser including chaos in the secondary beat frequency

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A helium-neon laser at 3.39 μm has been operated under cavity and discharge conditions permitting three modes to simultaneously oscillate for some regions of cavity tuning. Both mode-locked and -unlocked regions have been found. For unlocked three-mode operation, the laser output may include a secondary beat frequency. As the cavity length is tuned to the mode-locked region, this secondary beat frequency approaches zero and displays critical fluctuations at the transition to mode-locked operation. As the discharge current is increased, for fixed cavity length, the peaks in the power spectrum representing the secondary beat and its harmonics undergo a 100-fold broadening enroute to a broad chaotic spectrum before the laser reaches a mode-locked, low-noise configuration. This appears to be the onset of a deterministically chaotic behavior of the coupled modes that is distinct from the critical fluctuations. Details of cavity tuning and current variation on the laser output, including hysterisis effects, are reported here.

I. INTRODUCTION

Combination tones have been a standard feature of the analysis of multimode lasers from the first study by Lamb of the three-mode laser.[1] When three frequencies, $\nu_1 < \nu_2 < \nu_3$, coexist in a cavity with a nonlinear medium, the third-order susceptibility generates combination tones at $2\nu_2 - \nu_3$, $\nu_1 + \nu_3 - \nu_2$, and $2\nu_2 - \nu_1$, among others. Of particular interest is the case when ν_1, ν_2, and ν_3 are nearly equally spaced. Then the three combination tones listed are approximately equal to the three original frequencies, respectively.

Two interesting phenomena can occur when the combination tones are close to oscillating frequencies. Mode-locking results when the combination tones, acting as injected signals, pull the modes into exactly equal spacing.[1] This case is of considerable practical interest as the output becomes regular pulsing. When a large number of modes is involved, ultrashort pulsing results.[2] Thus mode-locked operation has been studied intensively in both three-mode[1–28] and many-mode[1–4,6–10,12,13,18,20,24,29–38] cases.

Contrary to some initial expectations, it has been shown that modes having different transverse indices could also lock together resulting in both spatial and temporal beat patterns.[24,39–41] For any combination of modes it has also been shown that mode competition can lead to suppression of certain modes by the dominant mode or modes.[7,8,32]

In general, modes are not equally spaced because of dispersive effects or differences in mode patterns. Mode locking thus requires sufficiently strong third-order coupling which seems to be provided only in a specific range of excitation conditions. The modes must be sufficiently far above threshold, but if the excitation is increased too far, the mode locking can be disrupted.[2] Also, because of symmetries in the medium, the required nearly equal spacing of the frequencies is best achieved when ν_2 is tuned close to the resonance peak of the medium.[1–5,29]

Alternatively, the combination tone may generate a beat frequency with the nearby oscillating frequency, and the system may lock all of these "secondary beat frequencies" to a common value. These "secondary beats" (or "beat-beat frequencies[18]") have received less study than the mode-locked operation, but they have been reported in several experimental systems[5,7,8,11,13,14] and in various theoretical analyses.[6–8,10,15,16,21,23,34]

Much of the work on multimode lasers has been to establish the operating conditions and characteristics of mode-locked operation. Some careful work has also gone into learning the features of the unlocked operation as well, though the mathematics becomes very difficult at this point, particularly for a many-mode case.[34] By far the most common mode of operation of three-mode lasers is an unlocked state which may result in broad low-frequency noise in the laser power output as there are both phase jitter in the modes and a variety of unstable secondary beat notes. Mode locking has thus been observed to quiet the random noise in multimode lasers.[6,31]

Recently, special interest in the three-mode case has been revived because of links to the theory of chaos in deterministic systems.[25,26] Quite generally, three degrees of freedom are needed to observe chaos, so the three-mode laser has seemed a perfect model. Several distinct routes to chaos[42] have been derived for nonlinear systems including a sequence of period doublings,[43] a single period doubling,[44] and intermittency.[26,45] As a general feature of the resulting deterministic chaos, the overall power spectrum of the signal features a broadband noise level (perhaps in addition to some spectral peaks) in contrast to the prechaotic periodic behavior in which the power spectrum shows only peaks without the broadband noise.

The early experimental studies of multimode systems concentrated on fluctuations before the mathematical signature of deterministic chaos was well known. It was frequently reported that in unlocked multimode operation, the separate mode intensities fluctuated more widely than the total intensity.[22,33,37,46] Only a few detailed characterizations of the fluctuations of separate modes have been reported. These show out-of-phase fluctuations for adjacent modes and in-phase fluctuations for alternate modes.[38,46]

Looking back at the earlier studies of unlocked operation, there is considerable evidence of "low-frequency noise" which was variously attributed to random-phase fluctuations,[6] combination tones,[7] or noise from mode competition.[31,33] Experimental studies of three-mode operation were carried out by Allen, Jones, and Sayers[10,11] and more recently by Gonchukov *et al.*,[14] Ermachenko *et al.*,[16] and Gelikonov *et al.*[19] Their results in studies of the 0.6328-μm line in He-Ne showed regions of locked and unlocked operation. Generally locked operation occurred when the central mode was tuned to the peak of the atomic resonance. Two self-mode-locked regimes were observed, one involving equal strength of the side modes and one in which one of the modes was suppressed to almost noise level.[19] In the unlocked case, "oscillations due to the nonequidistant nature of the spectrum" were reported.[19] Also recorded in the locking interval was a "small subinterval where shallow fluctuations of (the total intensity) were observed."[19]

From our current studies we will report particular details of the transition from unlocked to mode-locked behavior. These features seem to have gone unobserved, or at least unreported, in previous studies.

II. EXPERIMENT

The He-Ne laser used in these studies had a 30-cm-long cavity formed by a 90%-reflecting, plane output-coupling mirror and a 98%-reflecting, spherical mirror with a 60-cm radius of curvature forming an essentially confocal geometry. The He-Ne discharge was dc excited using a cold cathode design and a ballast resistor of 150 kΩ. The active region of the discharge was confined to a 4-mm-i.d. capillary tube 16-cm long. The ends of this part of the tube were closed by quartz windows epoxied on at Brewster's angle for high transmission of the vertical polarization.

The laser was filled with 140 mTorr of research-grade neon (natural isotopic mixture) and 1.6 Torr of research-grade helium. Given the natural linewidth and the pressure broadening coefficients,[47] this mixture resulted in a homogeneously broadened linewidth of 63 ± 6 MHz full width at half maximum (FWHM) compared to the Doppler broadening of 290 ± 10 MHz FWHM.[48]

The output of this laser was focused on a high-speed InAs diode reversed biased for improved frequency response. While the 3-dB point of the detector and electronics was approximately 120 MHz, response was possible out to approximately 220 MHz. The free cavity mode spacing between longitudinal modes was 500 MHz, but with mode pulling the modes appeared to be spaced at about 470 MHz, with a maximum tuning range of about 300 MHz. The beats between longitudinal and transverse modes appeared in the vicinity of 100 MHz.

A second He-Ne laser was operated on a single longitudinal mode to provide a reference frequency. This second laser was heterodyned with the first to provide moving intermode beat frequencies as the cavity length of the first laser was varied. The optical bench setup is shown schematically in Fig. 1.

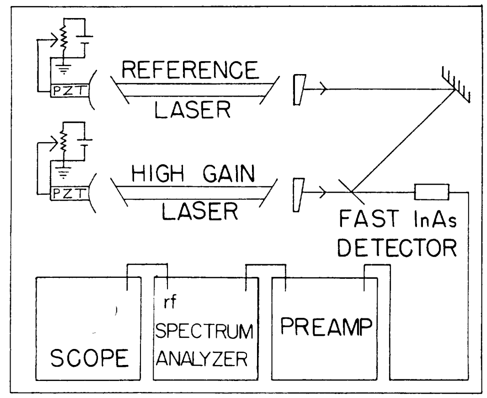

FIG. 1. Optical bench setup showing single-mode reference laser and high-gain laser under study which supported up to three modes simultaneously.

Both interlaser and intralaser beat frequencies were observed on a Tektronix Model-1401A rf spectrum analyzer following amplification by a Hewlett-Packard Model-461A wideband video amplifier. With this analyzer, power spectra could be obtained with a resolution of 10 kHz.

The lasers were mounted on a vibration-isolated table and were kept in a stable temperature environment to reduce mechanical and thermal drifts. As these could not be completely eliminated over long times, a video recording system was used to acquire data as the cavity length was slowly increased and then decreased and as the gain was slowly increased, then decreased.

III. RESULTS

Figure 2 shows output traces of the rf power spectrum of the photocurrent. In this case the three intralaser beat frequencies indicate the presence of three unequally

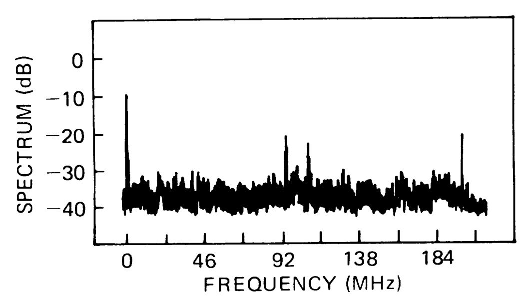

FIG. 2. Intralaser beat frequencies in power spectrum of detector photocurrent. Vertical scale is logarithmic, horizontal scale is 23 MHz/div, spectral resolution is 10 kHz. Beat pattern indicates three free-running modes with spacings of 92 and 106 MHz.

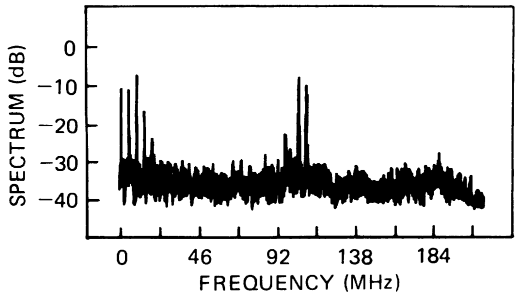

FIG. 3. Intralaser beat frequencies for phase-locked unevenly spaced frequencies. Mode separations are 105 and 109.5 MHz and phase locking produces secondary beat frequency at 4.5 MHz and its harmonics. Note that the 214 MHz intermode beat has disappeared or been reduced below the electronic noise limit.

spaced modes. Figure 3 shows an example of phase-locked operation of the unequally spaced modes which generated a secondary beat frequency (at approximately 4.5 MHz) and its harmonics. The highest intermode beat frequency (between the outer two optical modes) has vanished as often occurred when the secondary beat note appeared.

Figure 4 shows a sample summary of peaks in the power spectrum that were observed as the length of the laser cavity was increased. The horizontal axis is the frequency of one of the modes which could be continued since mode variation with length was highly linear. The vertical axis shows both the beats between the modes of the three-mode laser and the single-mode reference laser (slanted lines), as well as the intralaser beat frequencies and secondary beat frequencies (nearly horizontal lines).

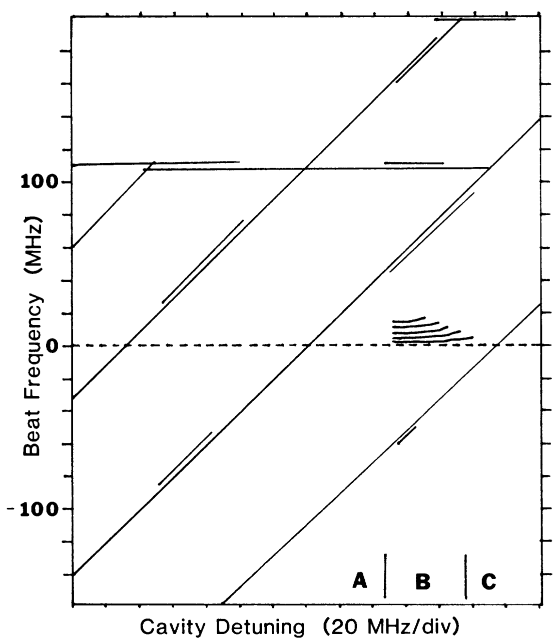

FIG. 4. Intralaser (nearly horizontal lines) and interlaser (slanted lines) beat frequencies and secondary beat frequencies vs cavity detuning for a relatively large excitation current (8.0 mA). Regions of one-, two-, and three-mode operation are shown with small windows of phase-locked, unequally spaced frequency operation (B) adjacent to mode-locked regions (A). Scan is taken so that the horizontal axis is varied from right to left progressing from phase-locked to mode-locked regimes.

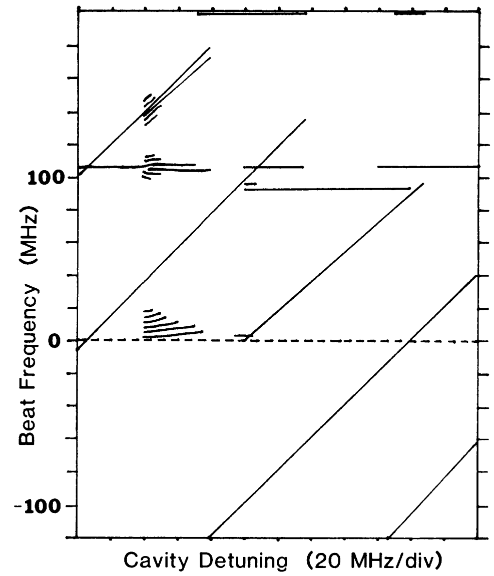

FIG. 5. Patterns as in Fig. 4 for a lower excitation current (3.7 mA). Phased-locked region occurs with only two apparent modes and with sidebands on only one of the two interlaser beat notes.

In region C we see uncoupled three-mode operation. In region B we see that the unlocked modes have generated a secondary beat frequency and its harmonics. This secondary beat goes to zero as the center of the three modes is tuned close to the atomic resonance (region A). There we see the onset of mode locking. The disappearance of the 200 MHz beat note (between the two off-line center modes) may be attributed to the "π" mode locking observed by Gelikonov *et al.*[19]

In Fig. 5 we show similar data to that of Fig. 4 taken for a lower excitation current. In this case only two modes were observed in the region of the secondary beat notes and only one of those developed the complicated pattern of sidebands. This behavior was commonly observed for lower excitation conditions, and it may represent a special two-mode couplng to a third "image frequency" developed because a detuned mode in a partially inhomogeneously broadened line burns two holes in the gain and polarization of the medium, or it may be interaction with a third mode that is just below threshold.

Some of these features are captured in the photographs of the spectrum-analyzer traces shown in Fig. 6. As the modes were tuned to more symmetrical locations about the central mode, the secondary beat note moved toward zero. As this transition took place, the stable secondary beat-note pattern (stable in frequency and amplitude) became noisy, displaying a type of critical fluctuations as the transition to mode-locked operation was approached.

In general, not all combinations of three modes would

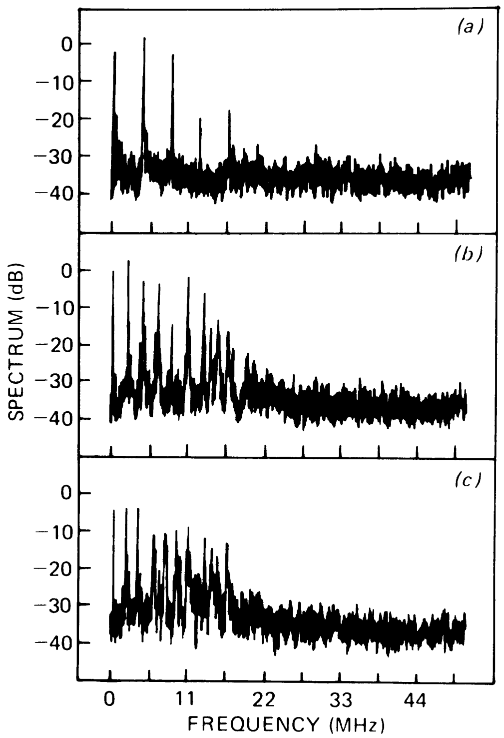

FIG. 6. Power spectra of phase-locked operation of the three-mode laser showing progression of secondary beat frequencies as the center mode approached the atomic resonance, bringing the laser to a mode-locked condition. Spectra displayed logarithmically at 10 kHz resolution vs frequency at 5.5 Mhz/div. Stable peak heights and frequency locations in (a) give way to unstable amplitudes and frequencies similar to critical fluctuations near the mode-locking threshold.

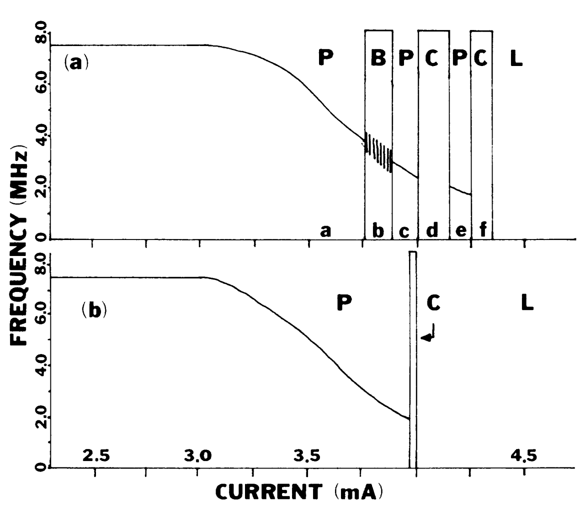

FIG. 7. Summary of secondary beat frequency spectra as laser gain (discharge current) is increased (a) and decreased (b). Frequency of primary peak is plotted vs current. Regions of operation include periodic (P) with peaks limited by 10-kHz resolution, broadened (B) with peaks widened to 1 MHz FWHM, chaotic (C) with broadband power spectra displaying little or no discrete spectral peaks, and mode locked (L) with no spectral noise. Sample spectra are shown in Fig. 8 as indicated by sublabels a–f on Fig. 7(a).

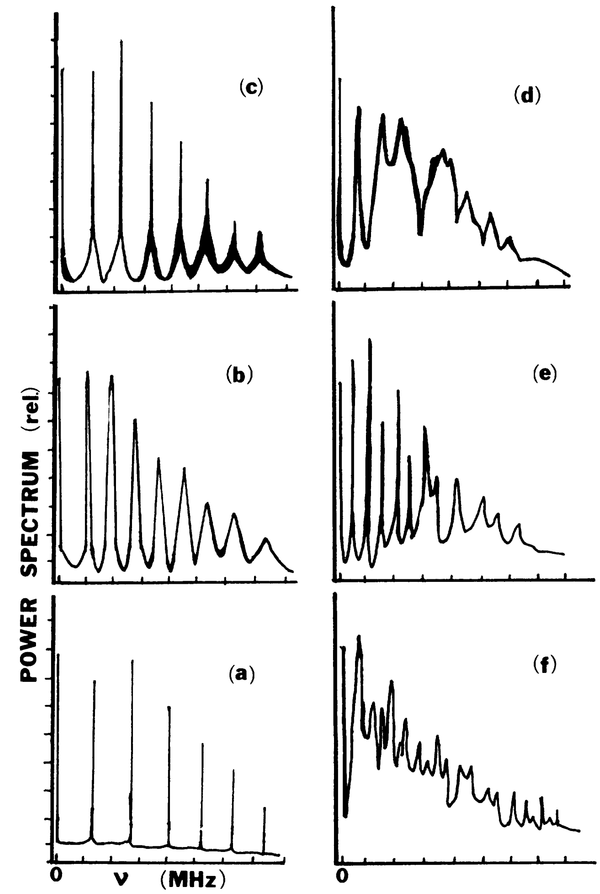

FIG. 8. Sequence of power spectra showing representative samples from the data summarized in Fig. 7(a). Power spectra are displayed on a linear vertical scale vs frequency at 3.8 Mhz/div. Spectra are selected from regions shown as indicated by sublabels a–f on Fig. 7(a).

generate the secondary beat frequencies enroute to mode locking. This may be attributed to the different combinations of transverse and longitudinal modes in the threesomes. In some cases the combination tones did not come close enough to oscillating frequencies to cause locking in any form. We do note several anomalous tuning ranges where beat frequencies are missing. Whether these are due to mode phasing effects, weak mode amplitudes (making intermode beats too weak to observe), or some more-subtle phenomena has not yet been determined.

The secondary beat frequency was also observed under variation of the discharge current (roughly proportional to the gain) for fixed cavity length. The cavity length was tuned until the secondary beat note appeared, then the power spectrum of the output was recorded as the discharge current was increased and then decreased. A summary of some of the observed features is given in Fig. 7 and in the sequence of power spectra shown in Fig. 8.

As the current was increased, the secondary beat frequency and all other low-frequency noise would ultimately vanish as the system went into the mode-locked state. Upon decreasing the current, it was found that the mode-locked state persisted until a considerably lower value of the current was reached.

The peaks in the power spectrum of the secondary beat frequency and its harmonics occasionally showed more power at the first harmonic than in the fundamental pulsing frequency. Because this evolution proceeded gradually and because the lowest frequency observed remained at the difference between the primary intermode beat frequencies, we concluded that this was usually not evidence of a period-doubling phenomenon but rather evidence for strong nonlinear coupling.

As the current was increased, the secondary beat frequency moved toward zero. Approximately midway to mode locking, the secondary beat-note region rather abruptly went from a 10 kHz bandwidth to a 1 MHz bandwidth. This broadened state persisted over a narrow range of increasing current and then more stable pulsing was observed, followed by a broadband spectrum typical of deterministic chaos, which appeared close to the mode-locking threshold. A brief window of stable pulsing [Fig. 8(e)] was observed within the chaotic region before the onset of relatively noise-free mode locking. Within this region the spectrum suggests a period-two behavior of the pulsations.

As the current was decreased, part or all of the chaotic or broadened regimes were skipped as the system persisted in the mode-locked condition before abruptly making the transition to secondary beats. The appearance of quasiperiodic behavior in the form of broadening of the peaks in the power spectrum rather than through multiple cycles (two frequencies or subharmonics) is possible evidence of a tangent bifurcation.[42,45,49] However, the abruptness of the transition leaves that open to some question.

IV. CONCLUSION

We have observed chaos in an all-optical system involving the nonlinear coupling of three modes in a laser. When the modes in a three-mode laser are not locked, secondary beat frequencies have been observed to undergo a transition to chaos with increasing gain or as the cavity is tuned toward the mode-locked conditions. These observations are similar to the all-optical chaos observed in optically bistable systems,[50] in single-mode inhomogeneously broadened lasers,[51] in Q-switched lasers,[52] and in lasers with injected signals.[53]

It has recently been shown that an initally single-mode, partially homogeneously broadened, He-Ne laser such as ours may display complicated mode-coupling effects as one of the mirrors is misaligned.[54] Both a distinct sequence of period doublings and a Ruelle-Takens sequence leading to chaos have been observed in such experiments. As the frequency of the instability which period doubled into chaos was much less than the mode-spacing frequency, we suggest that the instability was similar to the one observed here—having its origin in the phase-locked coupling between unequally spaced transverse modes.

Note added. It now appears that these authors have reached the same conclusion.[54] The single-mode instability in He-Ne has recently been observed[55] and does represent a different phenomenology.

This low-frequency modulation by the secondary beat frequency may also explain the so-called "breathing" observed in theoretical models of unstable optical systems.[25,56,57] Above the instability threshold for regular pulsations a low-frequency modulation has been observed. Sometimes this is locked to a subfrequency of the pulsing frequency, sometimes it is independent. In light of our observations, the modulation may arise when the sideband frequencies which give rise to the instability become unequally spaced about the strong mode.

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