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Elastic scattering of electrons by the 2*s* state of atomic hydrogen at intermediate energies

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We present differential cross sections of the elastic scattering of electrons from the metastable 2*s* state of atomic hydrogen at 100 and 200 eV. The present calculations are compared with the available theoretical results.

The study of the scattering of electrons by excited states of atoms is of theoretical interest since the comparison of the scattering from an excited state and the ground state yields valuable information about the dynamics of the collisional process and has many applications in astrophysics and plasma physics.[1] Recently some calculations[1–4] have been reported for the elastic scattering of electrons from the metastable 2*s* state of atomic hydrogen at medium and high energies.

The convergence of the Born series for this elastic scattering process at intermediate energies has been shown to be poor[1] and it must be summed up in some sense. A reliable approach is obtained if the static interaction is treated exactly in partial-wave formalism, while the remaining part of the interaction is treated in second-order perturbation theory (corrected static approximation[5]). Recently this method has been improved upon by adding to it the contribution from the nonstatic parts of the third- and higher-order terms in the Glauber approximation.[6] The direct scattering amplitude in this approach is defined as[6]

$$f=f_{\mathrm{st}}+(f_{B2}-f_{B2}^{\mathrm{st}})+(f_G-f_{G2}-f_G^{\mathrm{st}}+f_{G2}^{\mathrm{st}}) \quad . \tag{1}$$

The static-interaction contribution is taken out and evaluated exactly and fairly good convergence is obtained. This approach is expected to be better than the eikonal-Born series (EBS) approach[7] and corrected static approximation.[5] We follow this approach to calculate the elastic differential cross sections (DCS) for e^--H(2*s*) scattering at 100 and 200 eV. In addition, we have also used a simple approach of Srivastava[8]:

$$f=f_{\mathrm{st}}+f_G-f_G^{\mathrm{st}} \quad . \tag{2}$$

The scattering amplitude f_{st} in the static approximation (SA) is obtained by solving the radial Schrödinger equation for the static interaction

$$V_{\mathrm{st}}=\langle 2s|V|2s\rangle=-\left(\frac{1}{r}+\frac{3}{4}+\frac{r}{4}+\frac{r^2}{8}\right)\exp(-r) \quad . \tag{3}$$

The phase shifts for the partial-wave amplitude are calculated up to $l=40$. The evaluation of the second Born term f_{B2} and the second Glauber term f_{G2} has been discussed in Ref. 6 (and the references therein). The exchange effects are taken into account in the Glauber-Bonham-Ochkur (GBO) approximation which is defined as[9,10]

$$g=\frac{2^{-i\eta}\Gamma(1-i\eta)}{k_i^2}\left[\left(\frac{\partial}{\partial\alpha}+\frac{\partial^2}{\partial\alpha^2}+\frac{\partial^3}{\partial\alpha^3}\right)I(\alpha;q)\right]_{\alpha=1} , \tag{4}$$

where

$$I(\alpha;q)=\frac{1}{\alpha^{i\eta}(\alpha^2+q^2)^{1-i\eta}} \quad . \tag{5}$$

Here k_i is the momentum of the incident electron, $\eta=1/k_i$, and $\vec{q}=\vec{k}_i-\vec{k}_f$ is the momentum transfer. Finally, the differential cross section (DCS) for the elastic scattering of electrons by H(2*s*) is given by

$$\frac{d\sigma}{d\Omega}=\tfrac{1}{4}|f+g|^2+\tfrac{3}{4}|f-g|^2 \quad . \tag{6}$$

The results of our calculations of the DCS at 100 and 200 eV are displayed in Figs. 1 and 2. They are compared with those in the simplified second Born (SSB), EBS,[1] and optical model[2] (OM) results at 200 eV. Some of the results have been omitted for clarity. The exchange effects are found to contribute less than 1% to the cross sections at 100 eV. We have also included the DCS from the ground state of atomic hydrogen at these energies (taken from Lal and Srivastava[11]). We observe that the elastic scattering from the metastable 2*s* state is larger compared to the elastic scattering from ground state in the angular region ≤50° at 100 eV and ≤40° at 200 eV. But at large scattering angles the elastic scattering from the H(1*s*) stands higher compared to scattering from H(2*s*). The difference in the cross sections at small scattering angles from the 1*s* and 2*s* states can be attributed to the different size of 1*s* and 2*s* electronic orbits—whereas at large scattering angles the scattering is dominated by the incoming electron-nucleus interaction.

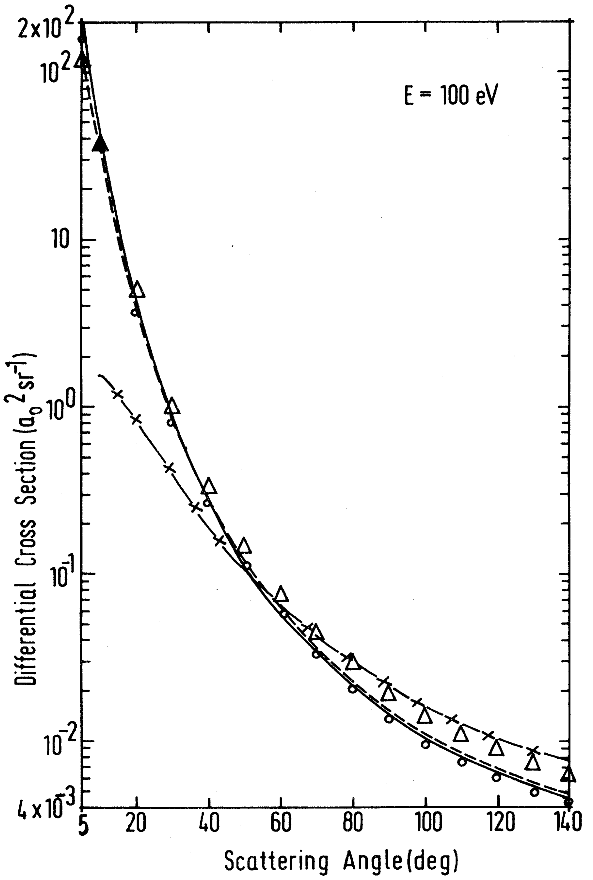

FIG. 1. Differential cross sections for e^--H($2s$) elastic scattering at 100 eV. Solid curve, present calculation using Eq. (1); dashed curve, present calculation using Eq. (2); Δ, SA calculation; ○, results of the Glauber approximation; dash-cross curve, e^--H($1s$) elastic scattering (Ref. 11).

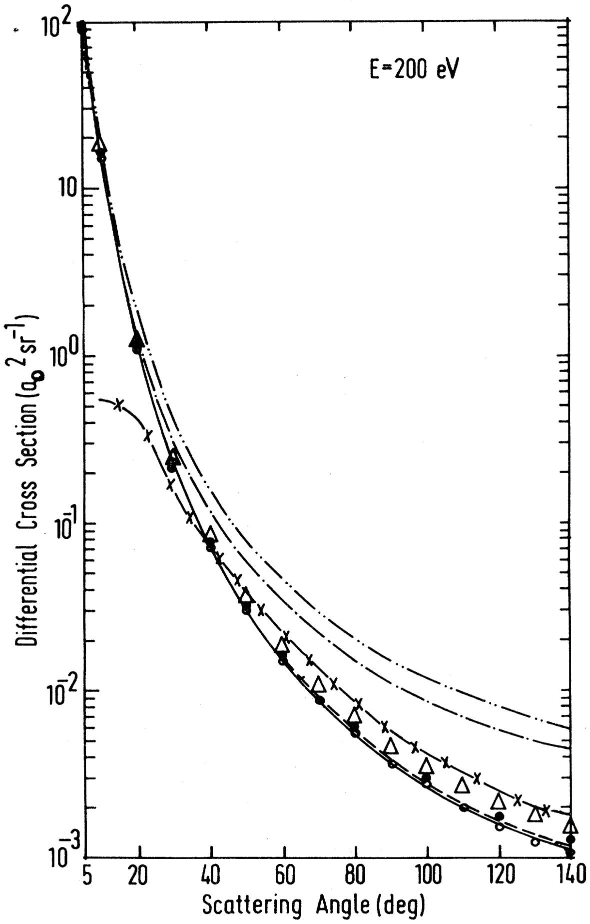

FIG. 2. Five of the curves are the same as for Fig. 1, but at 200 eV, and there are three additional curves. These are as follows: Dash-dot curve, EBS calculation (Ref. 1); dash-double dot curve, SSB calculations (Ref. 1); ●, OM calculation (Ref. 2).

The DCS obtained using Eqs. (1) and (2) are in good agreement with each other except in the forward direction and at large scattering angles. The results of Eq. (2) are about 25% lower than that of Eq. (1) in the angular region ~ 5° at 100 eV. The cross sections in the SA lie lower at small angles and higher at large scattering angles compared to the DCS obtained from Eq. (1). It is expected as the SA does not take into account the important second-order effects. We have also included in Figs. 1 and 2 the results obtained in the Glauber approximation. We notice that these lie close to the results of Eq. (1) in almost the entire angular range except at very small angles (< 5°) where the Glauber approximation is known to diverge. This agreement is due to the fact that the important absorption effect is well reproduced by the Glauber approximation.[2,12] The present results are in fairly good agreement with the OM results. The EBS and SSB calculations overestimate the DCS in the intermediate- and large-angle regions. This is due to the poor convergence of the multiple-scattering series. No experimental data are available for this process for comparison.

The present approach [Eq. (1)] has been found quite successful in predicting the elastic scattering cross sections from the ground state of hydrogen[11] and lithium,[6] where it provides a good agreement with experimental data and other theories. It is therefore reasonable to expect that this approach would provide a better description of this scattering process. In spite of the fact that Eq. (2) does not include the long-range polarization effect and suffers from a logarithmic divergence in the forward direction, it is expected to yield reasonable results for this process at intermediate and large angles with less computational efforts.

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