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Laser-induced autoionization in the presence of radiative damping and transverse relaxation

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A model for laser-induced autoionization is considered which includes two qualitatively different relaxation mechanisms: radiation damping to the ground state and phase fluctuations. Analytic solutions are presented for the first- and second-moment correlation functions of the atomic and photon operators in the Heisenberg picture. We present an explicit analysis of the spectrum of emitted photons at long times and investigate the effect of both relaxation mechanisms, as well as the influence of detuning and laser intensity, on this photoemission spectrum.

I. INTRODUCTION

The purpose of this paper is to formulate a model for laser-induced autoionization, which includes the effects of radiation damping and transverse relaxation and still retains its simplicity. Some results have been reported in a previous brief publication.[1] Our model is an extension of that used by Rzążewski and Eberly[2] and, like theirs, our model can also be analytically solved. There are, of course, advantages to the present treatment which have not been previously investigated; we calculate the energy spectrum of emitted photons,[1,3] a fundamental quantum electrodynamic quantity, and we study the effect of two different relaxation mechanisms on the physical observables.

Recently, a similar investigation has been made by Agarwal *et al.*[3]; they included radiative damping to a third level which is not identical with the ground state. In order to discuss the case of radiative damping to the ground state of the atom, they introduced a scaling argument to calculate the total number of scattered photons. We find a significant difference between their published results, and when radiative recombination is neglected.[4]

Theoretical investigations on the subject of laser-induced autoionization have elucidated new features.[2,5] This is to be expected, since the interference effects giving rise to an asymmetric autoionization profile in absorption spectra[6] and the splitting of the levels in an intense field[7] influence one another. As a result of these coherence effects, there have been predictions of population trapping at the confluence of coherences[2,5,8] and the existence of a sharp maximum in the photoelectron spectra for sufficiently long measuring times near the confluence.

However, these initial studies did not include relaxation effects which become especially important in the neighborhood of the confluence of coherence. Radiation damping to a third level was considered by Agarwal, Haan, Burnett, and Cooper[3,9]; Rzążewski and Eberly[2(b)] have discussed transverse relaxation effects which appear when the exciting laser has phase fluctuations or when weak collisions between the atoms are significant. Haus, Rzążewski, and Eberly[10] have determined the effect which inhomogeneous broadening has on the photoelectron spectra. The physical properties are significantly altered by the inclusion of these effects, but in qualitatively different ways.[11] We complement these publications by simultaneously admitting two of these relaxation mechanisms.

Our discussion is restricted to a single continuum and we do not incorporate continuum-continuum transitions, such as have been discussed by Lambropoulos and Zoller,[5(b)] Bialynicka-Birula,[12] and Andryushin *et al.*[13] Also, we consider only the case in which the autoionizing resonance is sufficiently far from the edge so that its influence is negligible; however, edge effects have also been previously studied.[8(b),14]

Autoionizing resonances may be the result of mixing a bound two (or more) electron state with a continuum by Coulomb interaction between the electrons. This is not the only possibility; autoionizing-like resonances have been induced in the continuum by using a second laser to couple a discrete level to the continuum[15] and by inserting an atom in a static electric field, thereby mixing bound states with the continuum, very sharp Fano profiles have been observed.[16]

The plan of the paper is as follows: In Sec. II we present our analysis of the model and define physical properties of the system. Section III will be devoted to a discussion of our results; we first consider the case of a symmetric autoionizing resonance; the photoemission spectra derived by using an asymmetric Fano profile are then discussed in some detail; we also analyze the expression for the total number of emitted photons in this section. In the final section we present conclusions and a discussion of the results.

II. ANALYSIS OF THE MODEL

We consider a model with a bound state whose energy $-\hbar\omega_0$ lies below the edge of the continuum; the bound and continuum states are coupled by the electromagnetic field. The Hamilton operator is

$$H = H_A + H_F + H_{AF} \ . \tag{2.1a}$$

In this expression the electronic occupation-number operators for the ground state P_0 and for the continuum state $C_{\omega\omega}$ appear in the atomic Hamilton operator

$$H_A = -\hbar\omega_0 P_0 + \int_0^\infty d\omega\, \hbar\omega C_{\omega\omega} \ . \tag{2.1b}$$

In the absence of coupling between the atom and the electromagnetic field, the plane-wave mode $\vec{k}$ with polarization μ, the photon creation, $a^\dagger_{\vec{k}\mu}$, and annihilation, $a_{\vec{k}\mu}$, operators undergo free evolution

$$H_F = \sum \int d^3k\, c\hbar k a^\dagger_{\vec{k}\mu} a_{\vec{k}\mu} \ . \tag{2.1c}$$

The atomic and photon field operators are linearly coupled to one another

$$H_{AF} = \hbar \sum_\mu \int d^3k \int_0^\infty d\omega\, g(k) \times (\hat{\mathscr{E}}_{\vec{k}\mu}\cdot \vec{d}(\omega) a^\dagger_{\vec{k}\mu} B_\omega + \text{H.c.}) \ , \tag{2.1d}$$

$B_\omega, B^\dagger_\omega$ are related to the atomic polarization operators between the bound state $|0\rangle$ and the excited state in the dressed continuum $|\omega)$

$$B_\omega = |0\rangle(\omega| \ , \tag{2.2}$$

$g(k)$ is the form factor and $\vec{d}(\omega)$ is the coupling constant for the atom-field interaction. We assume this function has the following two-pole form:

$$\vec{d}(\omega) = \frac{\vec{d}}{\sqrt{4\pi\Gamma}} \left[\frac{\Gamma}{(\omega-\omega_1) - i\Gamma} + \frac{\Gamma_1}{(1-iq)(\omega-\omega_1+i\Gamma_1)} \right] , \tag{2.3}$$

where $\vec{d}$ corresponds to the dipole matrix element for the atomic transition between the ground state and the dressed continuum state. Since the ground and excited states have a definite angular momentum, $\vec{d}$ is independent of ω. q denotes the well-known Fano asymmetry parameter. For $q=\infty$, the amplitude $|\vec{d}(\omega)|$ is symmetric about the frequency ω_1.

We are interested in the properties of the radiated light. The Heisenberg equations of motion for the photon operators alone can be formally solved. This solution contains two contributions: a free oscillatory part of the photon field related to the photon operator at $t=0$ and a scattered part containing atomic polarization operators at previous times,

$$a_{\vec{k}\mu}(t) = e^{-ikct} a_{\vec{k}\mu}(0) - i\int_0^t dt' e^{-ikc(t-t')} \times \int_0^\infty d\omega\, \vec{d}(\omega)\cdot \hat{\mathscr{E}}_{\vec{k}\mu} g(k) B_\omega(t') \ , \tag{2.4}$$

where obviously $a^\dagger_{\vec{k}\mu} = (a_{\vec{k}\mu})^\dagger$.

The Heisenberg equations of motion for the atomic operators $B_{\omega'}$, $B^\dagger_{\omega'}$, $C_{\omega\omega'}$, and P_0 contain nonlinear terms coupling the atomic and photon operators. These contributions are treated perturbatively (Born approximation). In these terms the formal solution for the electric field operators, Eq. (2.4), are inserted into the equations of motion. The free evolution of the atomic polarization operators $B^\dagger_\omega(t'), B_\omega(t')$ for a time interval $(t-t')$ is inserted; this substitution produces errors of order $|\vec{d}|^3$ and is appropriate when the atom-field interaction energy is small compared to the atomic transition energy.[17]

We illustrate the method on the equation of motion for the polarization operator B_ω (the same procedure is used for the adjoint operator $B^\dagger_\omega$). Its equation of motion is

$$\frac{dB_\omega}{dt}=-i(\omega_0+\omega)B_\omega-i\sum_\mu \int d^3k\, g(k)\hat{\mathscr{E}}_{\vec{k}\mu}\cdot\vec{d}(\omega)P_0 a_{\vec{k}\mu}+i\sum_\mu \int d^3k \int_0^\infty d\omega' g(k)\hat{\mathscr{E}}_{\vec{k}\mu}\cdot\vec{d}(\omega')C_{\omega'\omega}a_{\vec{k}\mu}\ . \tag{2.5}$$

Inserting Eq. (2.4), and using the approximation described above, we have

$$\begin{aligned}\frac{dB_\omega(t)}{dt}=&-i(\omega_0+\omega)B_\omega(t)-i\int d^3k \sum_\mu g(k)\hat{\mathscr{E}}_{\vec{k}\mu}\cdot\vec{d}(\omega)P_0(t)a_{\vec{k}\mu}(0)e^{-ikct}\\&+i\sum_\mu \int d^3k \int_0^\infty d\omega' g(k)\hat{\mathscr{E}}_{\vec{k}\mu}\cdot\vec{d}(\omega')C_{\omega'\omega}(t)a_{\vec{k}\mu}(0)e^{-ikct}\\&-\int_0^t dt' \sum_\mu \int d^3k \int_0^\infty d\omega' g^2(k)\hat{\mathscr{E}}_{\vec{k}\mu}\cdot\vec{d}(\omega)\hat{\mathscr{E}}_{\vec{k}\mu}\cdot\vec{d}(\omega')B_{\omega'}(t)e^{-ikc(t-t')}\ .\end{aligned} \tag{2.6}$$

Note that normal ordering of the field operators has been used.

Finally, we introduce the Markov or single-pole approximation. The integrand of the time-dependent memory term in Eq. (2.6) is a rapidly oscillating function of time; hence, the integral over a finite interval can be extended to an infinite interval. The result is

$$\int_0^\infty dt' e^{-i(kc-\omega-\omega_0)t'} =\pi\delta(kc-\omega-\omega_0)-i\mathscr{P}\frac{1}{kc-\omega-\omega_0}\ , \tag{2.7}$$

where the $\mathscr{P}$ denotes the principal part.

Substitution of this result into Eq. (2.6) and integration over the photon momenta produces an ω-dependent frequency shift (Lamb shift) and spontaneous emission rate (delta function contribution). Furthermore, we neglect the Lamb shift, since it only leads to a small shift of the atomic frequency, and we neglect the ω dependence of the damping coefficient, since this dependence is weak in comparison to the rapid ω dependence of $\vec{d}(\omega)$

$$\gamma_s(\omega)=\tfrac{8}{3}\pi^2\frac{d^2}{4}g^2\left[\frac{\omega+\omega_0}{c}\right]\left[\frac{\omega+\omega_0}{c}\right]^2 \approx\frac{2}{3}\frac{d^2}{\hbar}\left[\frac{\omega_0+\omega_1}{c}\right]^3 . \tag{2.8}$$

The resulting equation is averaged with respect to the initial state of the electromagnetic field $|\mathscr{E}_L\rangle$. This state is assumed to be a coherent state with a slowly fluctuating phase, $\psi(t)$

$$\sum_\mu \int d^3k\, g(k)\hat{\mathscr{E}}_{\vec{k}\mu}a_{\vec{k}\mu}(0)e^{-ikct}|\mathscr{E}_L\rangle =\mathscr{E}_L\hat{\mathscr{E}}_{\vec{k}_L}e^{-i[\omega_L t+\psi(t)]}|\mathscr{E}_L\rangle\ . \tag{2.9}$$

The coherent radiation is assumed to be linearly polarized and $\hat{\mathscr{E}}_{\vec{k}_L}\cdot\vec{d}=d$. In order to simplify the equation of motion we introduce the following definitions:

$$\Omega(\omega)=\mathscr{E}_L d(\omega) =\frac{\Omega_0}{\sqrt{4\pi\Gamma}}\left[\frac{\Gamma}{\omega-i\Gamma}+\frac{\Gamma_1}{(1-iq)(\omega+i\Gamma_1)}\right], \tag{2.10}$$

$$\Delta(\omega)=\omega_L-\omega_0-\omega_1-\omega\ , \tag{2.11}$$

$$\langle B_\omega\rangle=\Omega(\omega)\langle D_\omega\rangle e^{-i[\omega_L t+\psi(t)]}\ , \tag{2.12}$$

and

$$\langle C_{\omega\omega'}\rangle=\Omega(\omega)\Omega^*(\omega')\langle E_{\omega\omega'}\rangle\ , \tag{2.13}$$

where the angular brackets denote an average over the initial photon state and Ω_0 is the Rabi frequency;

$$\begin{aligned}\frac{d}{dt}\langle D_\omega\rangle=&\ i[\Delta(\omega)+\dot\psi(t)]\langle D_\omega\rangle\\&-i\langle P_0\rangle+i\int d\omega'|\Omega(\omega')|^2\langle E_{\omega'\omega}\rangle\\&-\frac{4\gamma_s}{\Omega_0^2}\int d\omega'|\Omega(\omega')|^2\langle D_{\omega'}\rangle\ .\end{aligned} \tag{2.14}$$

The fluctuating phase $\psi(t)$ is a stochastic variable; it is assumed that $\psi(t)$ is characterized by a Gaussian, Markov, and stationary process (phase-diffusion model)

$$\langle\langle\dot\psi(t)\dot\psi(t')\rangle\rangle=2\gamma_T\delta(t-t')\ , \tag{2.15}$$

where γ_T is the laser bandwidth. Using the present scaling of the operators, there are direct contributions of these fluctuations only in the equation of motion for the polarization; thus, the effect which they produce on average properties has been termed transverse relaxation. The equations of motion for the remaining atomic operators $C_{\omega\omega'}$ and P_0 are similarly calculated

$$\frac{d}{dt}\langle P_0\rangle = -i\int d\omega'\,|\Omega(\omega')|^2(\langle D_{\omega'}\rangle - \langle D_{\omega'}^\dagger\rangle) + \frac{8\gamma_s}{\Omega_0^2}\int d\omega'\int d\omega''\,|\Omega(\omega')|^2 \times |\Omega(\omega'')|^2\langle E_{\omega'\omega''}\rangle\,, \tag{2.16}$$

$$\frac{d}{dt}\langle E_{\omega\omega'}\rangle = i(\omega-\omega')\langle E_{\omega\omega'}\rangle + i(\langle D_{\omega'}\rangle - \langle D_{\omega}^\dagger\rangle) - \frac{4\gamma_s}{\Omega_0^2}\int d\omega''\,|\Omega(\omega'')|^2 \times(\langle E_{\omega''\omega'}\rangle + \langle E_{\omega\omega''}\rangle)\,. \tag{2.17}$$

Note that these equations do not have the factorization property as in the work of Agarwal *et al.*[3,9] The equation corresponding to the adjoint operator $B_\omega^\dagger$ is similar to Eq. (2.14) and is not reproduced here. The equations of motion [Eqs. (2.14), (2.16), and (2.17)] are averaged over the stochastic process with variance, Eq. (2.15). The double average over the initial state and fluctuating phase is denoted by double angular brackets. The equation for $\langle\langle D_\omega\rangle\rangle$ after Laplace transforming the time variable is

$$\langle\langle D_\omega(z)\rangle\rangle = \frac{\mathscr{A}^-(z)}{z+\gamma_T - i\Delta(\omega)} + \frac{i}{z+\gamma_T-i\Delta(\omega)} \times\int d\omega'\,|\Omega(\omega')|^2\langle\langle E_{\omega'\omega}(z)\rangle\rangle. \tag{2.18}$$

We define

$$\mathscr{A}^-(z) = -i\langle\langle P_0(z)\rangle\rangle - \frac{4\gamma_s}{\Omega_0^2}\int d\omega'\,|\Omega(\omega')|^2\langle\langle D_{\omega'}(z)\rangle\rangle\,; \tag{2.19}$$

again the corresponding equation for $\langle\langle D_\omega^\dagger(z)\rangle\rangle$ is not written. This equation contains a function $\mathscr{A}^+(z)$ which is also similarly defined. The linear integro-differential equation (2.14) can be simplified by taking advantage of the separation property of $\langle\langle E_{\omega\omega'}(z)\rangle\rangle$,

$$[z-i(\omega-\omega')]\langle\langle E_{\omega\omega'}(z)\rangle\rangle = \xi_\omega(z) + \eta_{\omega'}(z)\,. \tag{2.20}$$

The new functions ξ_ω and η_ω satisfy the equations

$$\mathscr{H}^+(z,\omega)\xi_\omega = \frac{-i\mathscr{A}^+(z)}{z+\gamma_T+i\Delta(\omega)} - \frac{4}{\Omega_0^2}\left[\frac{\Omega_0^2/4}{z+\gamma_T+i\Delta(\omega)} + \gamma_s\right] \times\int\frac{d\omega''\,|\Omega(\omega'')|^2\eta_{\omega''}}{z+i(\omega''-\omega)} \tag{2.21}$$

and

$$\mathscr{H}^-(z,\omega)\eta_\omega = \frac{i\mathscr{A}^-(z)}{z+\gamma_T-i\Delta(\omega)} - \frac{4}{\Omega_0^2}\left[\frac{\Omega_0^2/4}{z+\gamma_T-i\Delta(\omega)} + \gamma_s\right] \times\int\frac{d\omega''\,|\Omega(\omega'')|^2\xi_{\omega''}}{z-i(\omega''-\omega)}\,, \tag{2.22}$$

where

$$\mathscr{H}^\pm(z,\omega) = 1 + \left[\frac{\Omega_0^2/4}{z+\gamma_T\pm i\Delta(\omega)} + \gamma_s\right] \times\left[\frac{(iq\pm1)}{(iq\mp1)(z+\Gamma\mp i\omega)} + \frac{1}{\Gamma(1+q^2)}\right]. \tag{2.23}$$

ξ_ω must be of the following form:

$$\xi_\omega = \frac{-i\mathscr{A}^+(z)}{\mathscr{H}^+(z,\omega)[z+\gamma_T+i\Delta(\omega)]} + \frac{D^+\left[\dfrac{\Omega_0^2/4}{z+\gamma_T+i\Delta(\omega)} + \gamma_s\right]}{\mathscr{H}^+(z,\omega)(z+\Gamma-i\omega)}\,, \tag{2.24}$$

and a similar equation is valid for η_ω with a, as yet unknown, complex amplitude D^-.

The complex amplitudes D^-, D^+ are determined by inserting these expressions in Eqs. (2.21) and (2.22). After some algebra, the equation for D^- is ($\Delta = \omega_0 + \omega_1 - \omega_L$)

$$D^- = \left[\frac{i\mathscr{A}^+(z)}{\mathscr{H}^+(z,i\Gamma)(z+\gamma_T+\Gamma-i\Delta)} - \frac{D^+[\mathscr{H}^+(z,i\Gamma)-1]}{\mathscr{H}^+(z,i\Gamma)g^+(z)(z+2\Gamma)} \right] \left[\frac{iq-1}{iq+1} \right] \tag{2.25}$$

and D^+ satisfies an analogous equation with $\mathscr{A}^+_{(z)} \to \mathscr{A}^-_{(z)}$, $\Delta \to -\Delta$, $\mathscr{H}^+ \to \mathscr{H}^-$, and $g^- \to g^+$, where

$$g^{\pm}(z) = \left[\frac{iq\pm 1}{iq\mp 1} \right] \frac{1}{z+2\Gamma} + \frac{1}{\Gamma(1+q^2)} \; . \tag{2.26}$$

Finally, $\langle\langle P_0(z)\rangle\rangle$, $\mathscr{A}^-$, and $\mathscr{A}^+$ can be expressed as functions of these amplitudes:

$$z\langle\langle P_0(z)\rangle\rangle = \langle\langle P_0(0)\rangle\rangle - i\frac{\Omega_0^2/4[\mathscr{A}^-(z)-\mathscr{A}^+(z)]}{\Gamma(1+q^2)\left[1+\dfrac{\gamma_s}{\Gamma(1+q^2)}\right]} - \frac{\Omega_0^2/4(D^-+D^+)}{\left[1+\dfrac{\gamma_s}{\Gamma(1+q^2)}\right]} , \tag{2.27}$$

and using Eq. (2.19)

$$h^{\mp}(z)\mathscr{A}^{\mp}(z) = \mp i\langle\langle P_0(z)\rangle\rangle \pm \frac{i\gamma_s\Omega_0^2/4}{z+\gamma_T+\Gamma\pm i\Delta} \times \left[\left[\frac{1\pm iq}{\pm iq-1} \right] \frac{D^{\mp}}{z+2\Gamma} + g^{\mp}(z)D^{\pm} \right] , \tag{2.28}$$

where

$$h^{\mp}(z) = 1+\gamma_s \left[\frac{1}{z+\gamma_T+\Gamma\pm i\Delta} \left[\frac{1\pm iq}{\pm iq-1} \right] + \frac{1}{\Gamma(1+q^2)} \right] . \tag{2.29}$$

The solution of the integro-differential equations [Eqs. (2.14), (2.16), and (2.17)] has thus been reduced to a problem requiring only algebraic methods. The occupation probability of the ground state $\langle\langle P_0(z)\rangle\rangle$ and the occupation probability of the continuum $\langle\langle C_{\omega\omega}(z)\rangle\rangle$ are determined from the solution of this problem. For instance, the latter function yields detailed information about the energy spectrum of the emitted electrons. However, we do not discuss these properties in this publication, although their expressions are provided or easily derived from expressions given in the Appendix.

We concentrate our attention on the properties of the radiated photons in the rest of this paper. The total number of scattered photons can be determined by considering the operator

$$\sum_\mu \int d^3k \; a^\dagger_{\vec{k}\mu} a_{\vec{k}\mu} - P_0 \; , \tag{2.30}$$

which commutes with the Hamiltonian (2.1), and its expectation value represents a conserved quantity. Equation (2.30) is used to determine the total number of scattered photons:

$$N_s = \lim_{t\to\infty} \sum_\mu \int d^3k \langle\langle a^\dagger_{s,\vec{k}\mu}(t) a_{s,\vec{k}\mu}(t)\rangle\rangle \; , \tag{2.31}$$

where $a_{s,\vec{k}\mu}(t)$ and $a^\dagger_{s,\vec{k}\mu}(t)$ are defined by the term proportional to $\vec{d}(\omega)$ in Eq. (2.3). For our initial state we have the following initial expectation values:

$$\langle\langle P_0(0)\rangle\rangle = 1 \; , \qquad \sum_\mu \int d^2k \langle\langle a^\dagger_{\vec{k}\mu}(0) a_{\vec{k}\mu}(0)\rangle\rangle = N_{\text{tot}} \; . \tag{2.32}$$

On the other hand, in the long-time limit the atom is ionized and the ground state is depopulated:

$$\lim_{t\to\infty} \langle\langle P_0(t)\rangle\rangle = 0 \; . \tag{2.33}$$

For the first term in Eq. (2.30), the solutions for $a^\dagger_{\vec{k}\mu}(t)$ and $a_{\vec{k}\mu}(t)$ are inserted from Eq. (2.4). The result in the long-time limit is

$$\lim_{t\to\infty} \sum_\mu \int d^3k \langle\langle a^\dagger_{\vec{k}\mu}(t) a_{\vec{k}\mu}(t)\rangle\rangle = N_{\text{tot}} + N_s - i\sum_\mu \int d^3k \int d\omega \int_0^\infty dt' g(k) \hat{\mathscr{E}}_{\vec{k}\mu} \times [\vec{d}^{\,*}(\omega) e^{ikct'} \langle\langle a^\dagger_{\vec{k}\mu}(0) B_\omega(t')\rangle\rangle - \vec{d}(\omega) e^{-ikct'} \langle\langle B^+_\omega(t') a_{\vec{k}\mu}(0)\rangle\rangle] \; . \tag{2.34}$$

This equation is further simplified by using the previous results for the electromagnetic and atomic operator averages

$$N_s=\frac{\Omega_0^2}{2\gamma_s}\mathrm{Im}[i\langle\langle P_0(z)\rangle\rangle+\mathscr{A}^-(z)]\,|_{z=0}-1\ , \tag{2.35}$$

where $\mathscr{A}^-(z)$ is defined in Eq. (2.19) and its solution is presented in the Appendix. This result, whose derivation is similar to the optical theorem in quantum mechanics, is one of our central results. It provides a stringent test for the normalization of our expression for the photoemission spectrum.

The first moments are not sufficient to determine the occupation of the photon plane-wave modes; this quantity is called the photoemission spectrum and is defined by the number of scattered photons with frequency ck integrated over the solid angle $\Omega_{\vec{k}}$:

$$S(kc,t)=\sum_{\mu}\int d\Omega_{\vec{k}}\langle\langle a^{\dagger}_{s,\vec{k}\mu}(t)a_{s,\vec{k}\mu}(t)\rangle\rangle\ . \tag{2.36}$$

We will only be interested in the long-time behavior of this quantity which we denote as $S(kc)$. The usual power spectrum for the steady state vanishes, since the atom eventually ionizes.

The photon spectrum is determined from the equation of motion;

$$\begin{aligned}\frac{d}{dt}\langle\langle a^{\dagger}_{\vec{k}\mu}(t)a_{\vec{k}\mu}(t)\rangle\rangle&=-ig(k)\int d\omega\,|\Omega(\omega)|^2\\&\quad\times[\langle\langle a^{\dagger}_{\vec{k}\mu}(t)D_{\omega}(t)e^{-i\psi(t)}\rangle\rangle\\&\qquad-\langle\langle D^{\dagger}_{\omega}(t)a_{\vec{k}\mu}(t)e^{i\psi(t)}\rangle\rangle]\ .\end{aligned} \tag{2.37}$$

The number of scattered photons in a particular mode is related to the averages of the operators $a^{\dagger}_{\vec{k}\mu}D_{\omega}$ and $D^{\dagger}_{\omega}a_{\vec{k}\mu}$. The equation of motion for these operators is derived from the Hamiltonian (2.1); the hierarchy of moments is closed by using a perturbation expansion in the atom-field coupling and thereby eliminating photon operators which couple plane-wave modes with unequal wave vectors $\vec{k}'\neq\vec{k}$. Consider first the operators $D_{\omega\vec{k}\mu}=a^{\dagger}_{\vec{k}\mu}D_{\omega}e^{-i\psi(t)}$, $P_{0\vec{k}\mu}=a^{\dagger}_{\vec{k}\mu}P_0e^{-i\psi(t)}$, $E_{\omega\omega'\vec{k}\mu}=a^{\dagger}_{\vec{k}\mu}E_{\omega\omega'}e^{-i\psi(t)}$, and $D^{\dagger}_{\omega\vec{k}\mu}=a^{\dagger}_{\vec{k}\mu}D^{\dagger}_{\omega}e^{-i\psi(t)}$. The equations of motion for these quantities, after averaging over the initial state and the phase fluctuations, are written in the following form:

$$\begin{aligned}\frac{d}{dt}\langle\langle D_{\omega\vec{k}\mu}\rangle\rangle&=i[kc+\Delta(\omega)]\langle\langle D_{\omega\vec{k}\mu}\rangle\rangle-i\left[\langle\langle P_{0\vec{k}\mu}\rangle\rangle-\int d\omega'\,|\Omega(\omega')|^2\langle\langle E_{\omega'\omega\vec{k}\mu}\rangle\rangle\right]\\&\quad-\frac{4\gamma_s}{\Omega_0^2}\int d\omega'\,|\Omega(\omega')|^2\langle\langle D_{\omega'\vec{k}\mu}\rangle\rangle+ig(k)\int d\omega'\,|\Omega(\omega')|^2\langle\langle E_{\omega'\omega}\rangle\rangle\ ,\end{aligned} \tag{2.38}$$

$$\begin{aligned}\frac{d}{dt}\langle\langle D^{\dagger}_{\omega\vec{k}\mu}\rangle\rangle&=\{i[kc-\Delta(\omega)]-4\gamma_T\}\langle\langle D^{\dagger}_{\omega\vec{k}\mu}\rangle\rangle+i\left[\langle\langle P_{0\vec{k}\mu}\rangle\rangle-\int d\omega'\,|\Omega(\omega')|^2\langle\langle E_{\omega\omega'\vec{k}\mu}\rangle\rangle\right]\\&\quad-\frac{4\gamma_s}{\Omega_0^2}\int d\omega'\,|\Omega(\omega')|^2\langle\langle D^{\dagger}_{\omega'\vec{k}\mu}\rangle\rangle\ ,\end{aligned} \tag{2.39}$$

$$\begin{aligned}\frac{d}{dt}\langle\langle P_{0\vec{k}\mu}\rangle\rangle&=(ikc-\gamma_T)\langle\langle P_{0\vec{k}\mu}\rangle\rangle+i\int d\omega'\,|\Omega(\omega')|^2(\langle\langle D^{\dagger}_{\omega'\vec{k}\mu}\rangle\rangle-\langle\langle D_{\omega'\vec{k}\mu}\rangle\rangle)\\&\quad+\frac{8\gamma_s}{\Omega_0^2}\int d\omega'\int d\omega''\,|\Omega(\omega')|^2\,|\Omega(\omega'')|^2\langle\langle E_{\omega'\omega''\vec{k}\mu}\rangle\rangle+ig(k)\int d\omega'\,|\Omega(\omega')|^2\langle\langle D^{\dagger}_{\omega'}\rangle\rangle\ ,\end{aligned} \tag{2.40}$$

and

$$\begin{aligned}\frac{d}{dt}\langle\langle E_{\omega\omega'\vec{k}\mu}\rangle\rangle&=[i(kc+\omega-\omega')-\gamma_T]\langle\langle E_{\omega\omega'\vec{k}\mu}\rangle\rangle-i(\langle\langle D^{\dagger}_{\omega\vec{k}\mu}\rangle\rangle-\langle\langle D_{\omega'\vec{k}\mu}\rangle\rangle)\\&\quad-\frac{4\gamma_s}{\Omega_0^2}\int d\omega''\,|\Omega(\omega'')|^2(\langle\langle E_{\omega''\omega'\vec{k}\mu}\rangle\rangle+\langle\langle E_{\omega\omega''\vec{k}\mu}\rangle\rangle)\ .\end{aligned} \tag{2.41}$$

The final contributions in Eqs. (2.38) and (2.40) are first moments. Their equations of motion have been solved previously. Equations (2.38)—(2.41) have the same analytical structure as in the previously discussed equations of motion, Eqs. (2.14), (2.16), and (2.17), the only difference being the previously mentioned inhomogeneities and the dependence on the transverse relaxation γ_T. The solution of these equations is obtained by paralleling the steps of the previous solution. The lengthy, but analytical, expression for the photoemission spectrum is given in the Appendix. These results are studied in detail in the following section.

III. RESULTS

A. Symmetric Fano profile

In this section we discuss the symmetric profile case, i.e., $q\to\infty$. The expressions for the number of scattered photons reduces to the simple result ($\gamma_T=0$), $N_s=\gamma_s/\Gamma$. If, as is usually the case, the resonance in the continuum is much broader than the spontaneous emission rate $\Gamma\gg\gamma_s$, then the yield of photons will be small and sensitive photon counting measurements may be required. Nevertheless, it is instructive to examine this situation, since the expression for the photon spectrum normalized to unity reduces to

$$S(kc)=\frac{1}{\pi}\mathrm{Re}\left[G_1(z)\left[z+\Gamma+\gamma_s+\gamma_T+\frac{i\Delta(\Gamma+\gamma_s)}{\Gamma+\gamma_s+\gamma_T}\right.\right.$$
$$\left.\left.\left.+\frac{\Omega_0^2}{2}\frac{(z+\Gamma+\gamma_T)}{(z+\Gamma+\gamma_s+4\gamma_T-i\Delta)[z+2(\Gamma+\gamma_s)+\gamma_T]}\right]\right]\right|_{z=-i(k-k_L)c} \tag{3.1}$$

with

$$G_1(z)=\frac{(z+\Gamma+\gamma_s+4\gamma_T-i\Delta)[z+2(\Gamma+\gamma_s)+\gamma_T]}{Q(z)}\,,$$

$$Q(z)=(z+\gamma_T)\left[[z+2(\Gamma+\gamma_s)+\gamma_T](z+\Gamma+\gamma_s+i\Delta)\right.$$
$$\left.\times(z+\Gamma+\gamma_s+4\gamma_T-i\Delta)+\frac{\Omega_0^2}{2}(z+\Gamma+\gamma_s+2\gamma_T)\right]$$
$$+\frac{\Omega_0^2}{2}(z+2\Gamma+\gamma_T)(z+\Gamma+\gamma_s+2\gamma_T)\,. \tag{3.2}$$

This result reduces to our previous one[1] when $\gamma_T=0$. We consider first the case of small Rabi frequency $\Omega_0<\Gamma$ and no detuning $\Delta=0$. Figure 1 typifies the effect which the radiative damping and transverse fluctuations have on the shape of the spectrum. For $\gamma_T=0$, increasing γ_s *decreases* the halfwidth[1]; the addition of transverse relaxation broadens the halfwidth, and in the presence of transverse broadening, the spectra are further broadened as γ_s is increased. The presence of inhomogeneous broadening and transverse broadening[10] has a similar effect on the electron emission spectrum. The width of this peak in the weak-field limit is obtained from the denominator of $G_1(z)$ [Eq. (3.2)]

$$z_0\simeq\frac{-\left[\frac{\Omega_0^2}{2}(2\Gamma+\gamma_T)(\Gamma+\gamma_s+2\gamma_T)+\gamma_T A\right]}{A+\gamma_T\{(\Gamma+\gamma_s)(\Gamma+\gamma_s+4\gamma_T)+[2(\Gamma+\gamma_s)+\gamma_T](2\Gamma+2\gamma_s+4\gamma_T)\}}\,, \tag{3.3}$$

where

$$A=[2(\Gamma+\gamma_s)+\gamma_T](\Gamma+\gamma_s)(\Gamma+\gamma_s+4\gamma_T)\,. \tag{3.4}$$

For large coherent fields, $\Omega_0\gg\Gamma$, γ_s, and resonant radiation, the spectrum remains symmetric. The three-peak structure can be considerably smeared by transverse relaxation effects (Fig. 2). Detuning the coherent field from the central frequency of the resonance, the spectrum becomes asymmetric and eventually, for large enough γ_T, a single peak occurs centered at the atomic frequency (Fig. 3). This inelastic peak appears also in weak fields and in this respect, the transverse relaxation differs from the previously mentioned relaxation mechanisms. Radiation damping and inhomogeneous broadening do not excite an inelastic peak.

In the regime, $\Omega_0\gg\Gamma$, γ_s, γ_T, there are three peaks centered at $ck=0$, $\pm(\Omega_0^2+\Delta^2)^{1/2}$, the widths of these peaks are

$$\alpha_0 = \frac{\Omega_0^2(\Gamma+\gamma_T)(\Gamma+\gamma_s+2\gamma_T)+\Delta^2\gamma_T(2\Gamma+2\gamma_s+\gamma_T)}{\Omega_0^2(2\Gamma+\gamma_s+3\gamma_T)+\Delta^2\gamma_T}$$

and

$$\alpha_\pm = \Gamma+\tfrac{3}{2}\gamma_s+\frac{\gamma_T}{2}-\frac{\Delta^2(\gamma_s-\gamma_T)}{2(\Omega_0^2+\Delta^2)} ,$$

respectively.

The photoemission spectrum is analogous to the fluorescence spectrum in resonance fluorescence of a two-level atom. As in resonance fluorescence, the peaks become asymmetric with the addition of phase fluctuations and detuning. However, the relationship between the widths and heights of the peaks differs between the two physical systems.[18] The widths of the peaks in the photon spectrum are determined by the autoionizing width Γ for γ_T, and $\gamma_s \ll \Gamma$; the widths of the peaks in fluorescence spectra are determined by the radiative damping γ_s for $\gamma_T \ll \gamma_s$.

B. Asymmetric Fano profile

We now investigate the photon spectrum for finite q. Although, the expressions do not allow the explicit analysis of the previous subsection, there is an interesting new feature, the confluence of coherences.

In contrast to the result previously found for the total number of radiated photons, for finite q this quantity is sensitive to the detuning and the Rabi frequency. When the asymmetry parameter $q=5$ and $\Delta=0$, the confluence of coherences is at $\Omega_c^2=104$. In Fig. 4 we semilogarithmically plot the total number of radiated photons versus Rabi frequency using these parameters. The maximum is near the confluence of coherences. When $\gamma_T=0$, spectra become sharper as γ_s is decreased; the maxima are only weakly dependent on the radiative damping for $\gamma_s < \Gamma$. The addition of transverse relaxation broadens the curves and reduces their maxima. This is not surprising since the transverse relaxation is equivalent to a spread of laser frequencies with respect to the atomic frequency; thus, the confluence of coherences is smeared.

In Fig. 5 we choose the detuning $\Delta=5$ and plot N_s versus Rabi frequency. This value of the detuning shifts the confluence of coherence to zero Rabi frequency. In the absence of transverse relaxation the number of emitted photons diverges as $\Omega_0 \rightarrow 0$. The transverse broadening reduces the total number

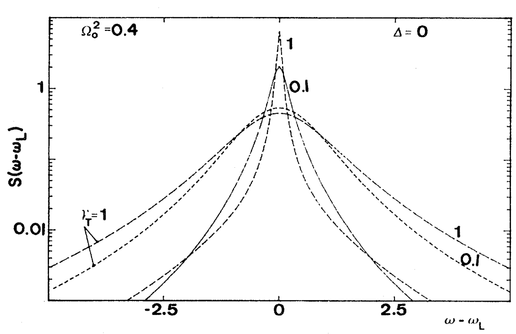

FIG. 1. The (normalized) photon spectrum for $q=\infty$, Rabi frequency $\Omega_0=0.4$, and detuning $\Delta=0$. Curves corresponding to spontaneous emission linewidths 1 and 0.1 are so indicated.

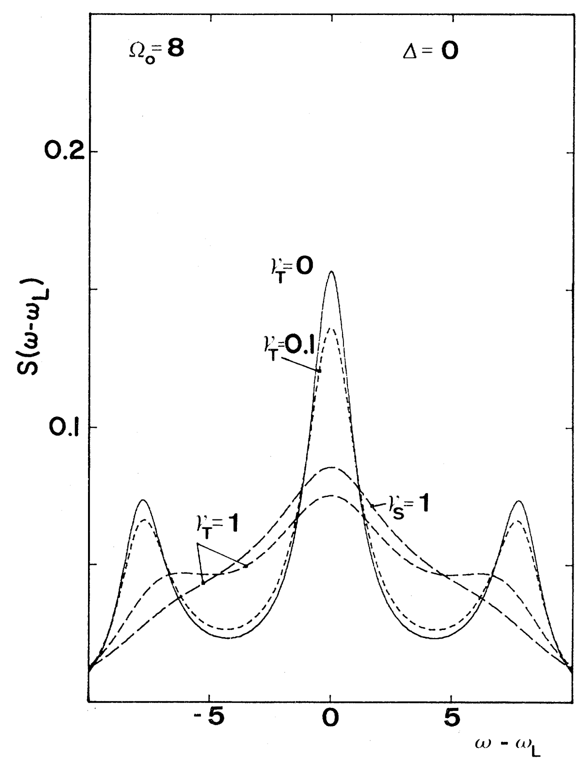

FIG. 2. The (normalized) photon spectrum for $q=\infty$, Rabi frequency $\Omega_0=8$, and detuning $\Delta=0$. Value of $\gamma_s=1$ is indicated, all other curves have $\gamma_s=0.1$.

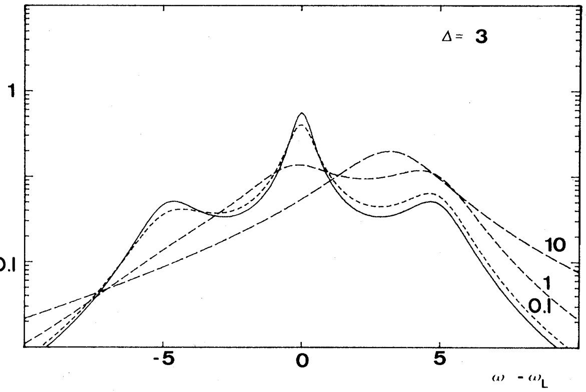

FIG. 3. The (normalized) photon spectrum for $q=\infty$, $\Omega_0=4$, $\Delta=3$, $\gamma_s=0.1$, and $\gamma_T=0$, 0.1, 1, and 10.

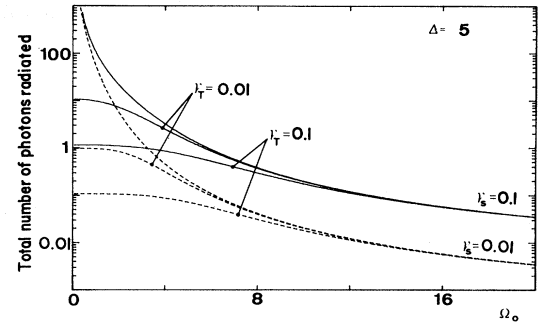

FIG. 5. The total number of photons radiated vs Rabi frequency. Parameters $q=5$, $\Delta=5$, $\gamma_s=0.01$ and 0.1, and $\gamma_T=0$, 0.01, and 0.1 are chosen.

of radiated photons radically in this regime of Δ.

More detailed information is obtained from the photon spectra. In a previous publication, results were presented for the photon spectra with finite q and no transverse relaxation. These spectra are asymmetric and at the confluence of coherences the central peak is narrowed to a width of order γ_s, since the radiation damping was the dominant relaxation mechanism. In Fig. 6 we consider the combined effect of transverse relaxation and radiative damping on the photon spectra. The Rabi frequency is chosen to be near the confluence of coherences and the laser is on resonance with the central frequency of Fano profiles. One spectrum is plotted with $\gamma_s=0.001$ and $\gamma_T=0$; we chose symmetric values of γ_s and γ_T in the remaining two curves in order to evaluate the quantitative difference between the two spectra.

Although the two spectra are qualitatively similar in appearance, the central peak is less affected by larger radiative damping and small transverse relaxation than by the converse situation.

The transverse relaxation has a redistributive effect on the photon spectra when the laser and atomic transition are detuned $\Delta\neq 0$. In Fig. 7 the Rabi frequency is $\Omega_0=\sqrt{2}$ and the radiative damping coefficient is $\gamma_s=0.1$. Increasing the transverse relaxation rate shows a shift of the maximum from the excitation frequency of the laser to the atomic frequency. In Fig. 8 the combined effects of large $\gamma_s=10$ and large transverse relaxation $\gamma_T=10$ are plotted. The extremely large value of the radiative damping increases the maximum, an effect related to the weak-field limit discussed in the previous subsection. The two damping mechanisms do not com-

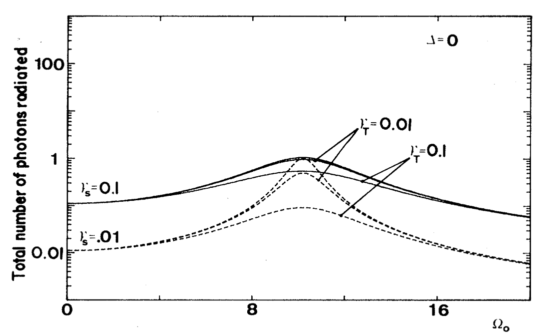

FIG. 4. The total number of photons radiated vs Rabi frequency. Parameters $q=5$, $\Delta=0$, $\gamma_s=0.01$ and 0.1, and $\gamma_T=0$, 0.01, 0.1 are chosen.

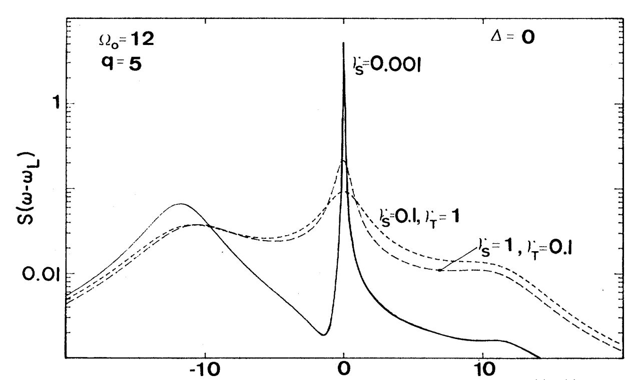

FIG. 6. The photon spectrum for $q=5$, $\Omega_0=12$, $\Delta=0$, and values of γ_s and γ_T as indicated in the figure.

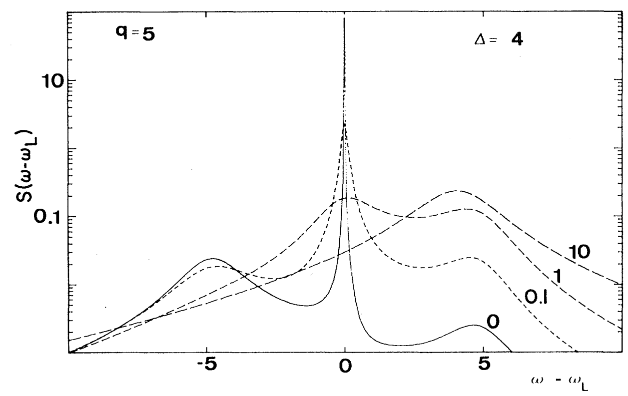

FIG. 7. The photon spectrum for $q=5$, $\Omega_0=\sqrt{2}$, $\Delta=4$, and $\gamma_s=0.1$. Effect of γ_T on the shape of the spectrum is demonstrated.

pete; in this case, when both are acting on the atom, the spectrum is very broad and the maximum lies between the laser and atomic frequencies.

IV. CONCLUSIONS AND DISCUSSION

We have analytically solved a set of coupled integro-differential equations relevant to autoionization in the presence of two relaxation mechanisms: transverse relaxation and radiation damping to the ground state. As in a previous publication on the photoelectron spectrum,[11] we note that the effect of transverse broadening differs from the effect which radiation damping has on the photon spectrum and we include both mechanisms in our solution.

The photon spectra of Agarwal *et al.*, differ significantly from ours since their calculation is specific for relaxation to a third electronic level, which is not identical to the ground state. Their photon spectra have two peaks in the strong field limit, $\Omega_0 \gg \Gamma$ and γ_s, since there is no elastic peak in their spectra. They also do not account for the transverse relaxation.

In their publication a scaling argument was used to formulate an expression for the total number of scattered photons. The probability of returning once to the final bound state, in their notation, is the total number of photons emitted N_f. Assuming now that this state is the ground state, the probability of returning a second time to the final bound state is N_f^2 and so forth. In their original paper,[3] the authors assume that the final-state Fano asymmetry parameter is infinite, $q_f=\infty$ (no radiative recombination). Our results fully corroborate their expression when radiative recombination is accounted for.[4] Figure 9 shows a comparison between the total number of scattered photons without radiative recombination[3] and that of this work. The translation of the notation from Ref. 3 to ours requires the following substitution ($q_f=\infty$): $\Gamma_{AHBC}=2\Gamma$, $\gamma_0=\gamma_s/2$, $\alpha=\Delta/\Gamma$, and $\Omega=\Omega_0^2/4(1+q^2)\Gamma^2$.

We calculated the total number of photons emitted using the conservation of the number of excitations. This not only provides a direct analytical expression for the total number of emitted photons, but also is a check of our expression for the photon spectrum. This provided an important consistency check for the complicated analytical calculations.

For $\gamma_s \ll \Gamma$ the integrated photon spectra have a sharp maximum near the confluence of coherences. This feature, also noted previously,[1,3] can be used to obtain detailed information about the autoionizing state. The effect of transverse damping is most dramatic when the detuning is chosen so that the

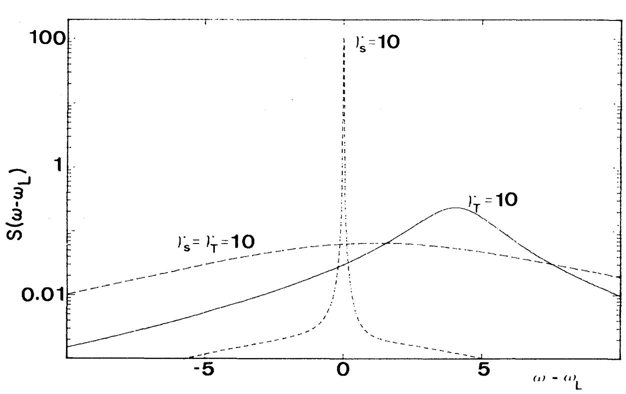

FIG. 8. The photon spectrum for extreme values of γ_s and γ_T. $q=5$, $\Delta=4$, and $\Omega_0=\sqrt{2}$.

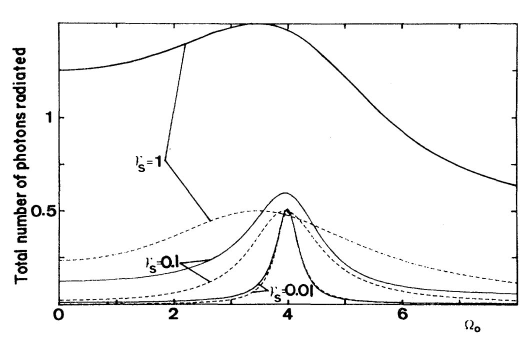

FIG. 9. Total number of photons radiated vs Rabi frequency for $q=1$ and $\Delta=1$. Dashed lines are the results from Ref. 3 and the solid lines are from Eq. (A6).

confluence of coherence appears near zero Rabi frequency. This is the regime where the Fermi golden rule decay rate of the ground state is largest and the transverse damping is the dominating relaxation mechanism controlling the lifetime of the ground state.

In the strong field limit, $\Omega_0 \gg \Gamma$, γ_s, and γ_T, the photon spectra exhibit three maxima. The similarity of these spectra to the power spectra in resonance fluorescence of a two-level atom has been commented on already. For finite q, the central peak shows a pronounced narrowing and heightening as the confluence of coherences is approached. These spectra should be more accessible to experimental verification (we envision here an experiment performed with an atomic beam) than the photoelectron spectra, since the techniques for obtaining high-resolution photoemission spectra are more advanced than the techniques for resolving the energy of emitted electrons. Thus the photoemission spectra provide an alternative to obtain detailed information about the autoionizing state.

Our treatment of the model does not use the quantum-regression hypothesis. We calculated equal-time correlation functions for a transient process and analyzed the photon spectra and number of radiated photons in the long-time limit in this article.[19] However, the solutions in Sec. II and the Appendix can be used to study the time dependence of these quantities, as well as, to investigate the population trapping and photoelectron spectra. A discussion of these interesting topics is reserved for the future.

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APPENDIX

The integro-differential equations derived in Sec. II have been already reduced to a set of algebraic equations. These equations are easily manipulated to obtain an explicit solution. We shall retain the notation used in that portion of the paper.

There are five amplitudes to be calculated; these equations are reduced to a set of two equations for $\mathscr{A}^-$ and $\mathscr{A}^+$ as a function of $\langle\langle P_0(z)\rangle\rangle$:

$$\begin{bmatrix} C_{11}(z) & C_{12}(z) \\ C_{21}(x) & C_{22}(z) \end{bmatrix} \begin{bmatrix} \mathscr{A}^- \\ \mathscr{A}^+ \end{bmatrix} = i\langle\langle P_0(z)\rangle\rangle \begin{bmatrix} -1 \\ 1 \end{bmatrix} . \tag{A1}$$

The coefficients of the 2×2 matrix are

$$C_{11}(z)=h^- + \frac{\Omega_0^2\gamma_s Q_1^-}{4(z+\gamma_T+\Gamma+i\Delta)} , \tag{A2a}$$

$$C_{21}(z)=\frac{\Omega_0^2\gamma_s Q_2^+}{4(z+\gamma_T+\Gamma-i\Delta)} , \tag{A2b}$$

$$C_{12}(z)=\frac{\Omega_0^2\gamma_s Q_2^-}{4(z+\gamma_T+\Gamma+i\Delta)} , \tag{A2c}$$

$$C_{22}(z)=h^+ + \frac{\Omega_0^2\gamma_s Q_1^+}{4(z+\gamma_T+\Gamma-i\Delta)} , \tag{A2d}$$

and

$$Q_1^\pm = \frac{g^\pm}{M(z+\gamma_T+\Gamma\mp i\Delta)} \left[\frac{iq\mp 1}{iq\pm 1}\right] \times[\mathscr{H}^\mp - 1 - \mathscr{H}^\pm g^+ g^-(z+2\Gamma)^2] , \tag{A3a}$$

$$Q_2^\pm = \frac{g^+ g^-(z+2\Gamma)}{M(z+\gamma_T+\Gamma\pm i\Delta)} , \tag{A3b}$$

$$M=\mathscr{H}^-\mathscr{H}^+ g^+ g^-(z+2\Gamma)^2-(\mathscr{H}^- - 1)(\mathscr{H}^+ - 1) . \tag{A3c}$$

The equations for D^- and D^+ are

$$D^- = \frac{i}{M}\left[\frac{(\mathscr{H}^+ - 1)g^-(z+2\Gamma)}{(z+\gamma_T+\Gamma+i\Delta)}\mathscr{A}^- + \frac{(iq-1)}{(iq+1)}\frac{\mathscr{H}^- g^+ g^-(z+2\Gamma)^2}{(z+\gamma_T+\Gamma-i\Delta)}\mathscr{A}^+\right] \tag{A4a}$$

and

$$D^+ = \frac{-i}{M}\left[\frac{(\mathscr{H}^- - 1)g^+(z+2\Gamma)}{(z+\gamma_T+\Gamma-i\Delta)}\mathscr{A}^+ + \frac{(1+iq)}{(iq-1)}\frac{\mathscr{H}^+ g^+ g^-(z+2\Gamma)^2}{(z+\gamma_T+\Gamma+i\Delta)}\mathscr{A}^-\right] . \tag{A4b}$$

In the above equations the arguments of the functions $\mathscr{H}^\pm(z,i\Gamma)$ and $g^\pm(z)$ are omitted.

The solution for $\langle\langle P_0(z)\rangle\rangle$ is

$$\langle\langle P_0(z)\rangle\rangle = \frac{1}{u(z)} \tag{A5a}$$

with

$$u(z)=z-\frac{\Omega_0^2[N^-(C_{12}+C_{22})+N^+(C_{21}+C_{11})]}{4(C_{11}C_{22}-C_{12}C_{21})} \tag{A5b}$$

and

$$N^{\pm}=\frac{\Gamma(1+q^2)}{[\gamma_s+\Gamma(1+q^2)]}\left[\frac{(\mathscr{H}^{\mp}-1)g^{\pm}(z+2\Gamma)-\left[\frac{iq\mp 1}{iq\pm 1}\right]\mathscr{H}^{\mp}g^{+}g^{-}(z+2\Gamma)^2}{M(z+\gamma_T+\Gamma\mp i\Delta)}-\frac{1}{\Gamma(1+q^2)}\right]. \tag{A5c}$$

The complete solutions for the ground-state occupation equation (A5) and the photoelectron emission spectrum are provided by the above results.

The total number of radiated photons is

$$N_s=\frac{\Omega_0^2}{2\gamma_s}\mathrm{Re}[\alpha^{+}(0)]-1\ , \tag{A6a}$$

where

$$\alpha^{+}(z)=\left[1-\frac{C_{11}+C_{21}}{C_{11}C_{22}-C_{12}C_{21}}\right]\frac{1}{u(z)}\ . \tag{A6b}$$

The photoemission spectrum normalized to unity is given by the expression

$$S(kc)=\frac{2}{\pi N_s}\mathrm{Re}\left[\frac{\Omega_0^2}{4\gamma_s}\tilde{\alpha}^{-}(z)\alpha^{+}(0)+\tilde{\beta}(z)J(0)\right]\Bigg|_{z=-i(k-k_L)c}\ . \tag{A7}$$

We define the following functions:

$$\alpha^{-}(z)=\left[1-\frac{(C_{22}+C_{12})}{C_{11}C_{22}-C_{12}C_{22}}\right]\frac{1}{u(z)}\ , \tag{A8a}$$

$$\beta(z)=\frac{\Omega_0^2}{4}\left[N^{-}R_1-N^{+}R_2-\frac{W}{\gamma_s}\left[\frac{iq-1}{iq+1}\right]\frac{\Gamma(1+q^2)}{\gamma_s+\Gamma(1+q^2)}\right]\frac{\alpha^{-}(z)}{z+\Gamma+\gamma_T+i\Delta}-\frac{R_1}{z+\Gamma+\gamma_T+i\Delta}\ , \tag{A8b}$$

$$W=(z+\Gamma+\gamma_T+i\Delta)\left[N^{-}+\frac{1}{\gamma_s+\Gamma(1+q^2)}\right]\ , \tag{A8c}$$

$$J(z)=\frac{(z+\Gamma+\gamma_T+i\Delta)}{u(z)}\left[1-\frac{h^{-}(C_{12}+C_{22})}{C_{11}C_{22}-C_{12}C_{21}}\right]\ , \tag{A8d}$$

$$R_1=\frac{\left\{-\frac{\Omega_0^2}{4}\left[\frac{iq-1}{iq+1}\right]Q_2^{-}C_{12}+\left[1+\frac{\Omega_0^2}{4}\left[\frac{iq-1}{iq+1}\right]Q_1^{-}\right]C_{22}\right\}}{C_{11}C_{22}-C_{12}C_{21}}\ , \tag{A8e}$$

and

$$R_2=\frac{\left\{\frac{\Omega_0^2}{4}\left[\frac{iq-1}{iq+1}\right]Q_2^{-}C_{11}-\left[1+\frac{\Omega_0^2}{4}\left[\frac{iq-1}{iq+1}\right]Q_1^{-}\right]C_{21}\right\}}{C_{11}C_{22}-C_{12}C_{21}}\ . \tag{A8f}$$

The tilde over the functions in Eq. (A7) denotes the corresponding solutions of the second-moment equations (2.38)–(2.41). A prescription for obtaining these solutions from the results given above is as follows: first set $\gamma_T=0$ in all equations, then take $z\to z+\gamma_T$, and where $z-i\Delta$ appears, let $-i\Delta\to -i\Delta+3\gamma_T$, and where $z+i\Delta$, let $+i\Delta\to +i\Delta-\gamma_T$.

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