Relativistic spin-dependent Compton scattering from electrons

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The differential cross section for photon scattering from relativistic polarized free electrons in motion is derived. The calculation extends early results from electrons at rest. It is argued that the present derivation is required for use in bound-state spin-dependent Compton scattering from polarized electrons. The results provide an alternative check on recent work carried out using a quasi-relativistic Hamiltonian.

Inelastic Compton scattering provides a useful method for probing the properties of particles in target materials.[1–5] Theoretical studies have been made within the framework of the so-called impulse approximation, in which one considers photon scattering from a stationary packet of plane electron waves, the packet being characterized by a distribution function which entails the momentum (and spin, if necessary) of the electrons in the target.[6] It is necessary, however, that the photon free-electron cross section be known. To be specific, if we consider the scattering of polarized photons from spin-polarized electrons present, for example, in a ferromagnetic target, then knowledge of the free cross section in terms of the electron momentum and spin and the photon wave vector and polarization is required. It is to this end that we present here an expression for this cross section.

In a recent publication on inelastic Compton scattering we proceeded by another method, utilizing a quasirelativistic Hamiltonian.[7] In that paper corrections to the cross section which involved the ratios $(\hbar\omega/mc^2)$ and (p/mc) were retained. Since that calculation was extremely complex we decided to ascertain its validity by utilizing the impulse approximation together with the completely relativistic Compton cross section. The present results confirm our earlier calculation.

In this paper we first derive the Compton cross section for the scattering of a photon from a free electron in motion. Since we are interested in polarization effects, only the spin-dependent part of the cross section is given in detail. Proceeding to the electron rest frame we show that our results are consistent with those given earlier.[8] However, the greater generality of our result (scattering of transversely polarized photons from moving electrons) allows us to compare our calculation in the small (but nonzero) momentum limit with that of our earlier work, and thus to confirm by a different method the conclusions previously reached. We argue that our general result for spin-dependent scattering from electrons *in motion* cannot be obtained from the cross section from electrons at rest by utilizing Lorentz and gauge transformations, unless the rest cross section is *explicitly* and uniquely expressed in terms of the photon polarization vectors. Since previous work is not presented in this form, it was necessary to carry out an independent calculation. For completeness, in an Appendix, we briefly discuss some aspects of the quasi-relativistic approach carried out in Ref. 7.

The Compton amplitude resulting from second-order interactions is[9]

$$S_{fi}^{(2)}=-\frac{e^2}{V^2}\left[\frac{m^2}{EE'}\right]^{1/2}\frac{1}{(4kk')^{1/2}}(2\pi)^4\delta^4(p'+k'-p-k)\bar{u}(p',s')\left[\not\epsilon'\frac{i}{\not p+\not k-m}\not\epsilon+\not\epsilon\frac{i}{\not p-\not k'-m}\not\epsilon'\right]u(p,s)\ . \tag{1}$$

Here p,s,k,ϵ refer, respectively, to the initial electron four-momentum and spin and the initial photon four-momentum and polarization.[10] The primed quantities refer to final electron and photon.

The resulting differential scattering cross section, following the standard procedure, has the form

$$\frac{d^2\sigma}{d\Omega_{k'}dk'}=\frac{e^4m^2}{(2\pi)^2 2kE}\int\left|\bar{u}(p',s')\left[\not\epsilon'\frac{1}{\not p+\not k-m}\not\epsilon+\not\epsilon\frac{1}{\not p-\not k'-m}\not\epsilon'\right]u(p,s)\right|^2\delta^4(p'+k'-p-k)\frac{d^3p'}{2E'}k'\ . \tag{2}$$

Using the properties of γ matrices and the Dirac spinors, together with the mass-shell conditions, we write (2) in a more convenient form as

$$\frac{d^2\sigma}{d\Omega_{k'}dk'}=\frac{e^4m^2}{(2\pi)^2 2kE}\int k'\left|\bar{u}(p',s')\left[\not\epsilon'\frac{(2p\cdot\epsilon-\not\epsilon\not k)}{2k\cdot p}+\not\epsilon\frac{(2p\cdot\epsilon'+\not\epsilon'\not k')}{-2k'\cdot p}\right]u(p,s)\right|^2\delta^4(p'+k'-p-k)\frac{d^3p'}{2E'}\ . \tag{3}$$

Since no information concerning either the final electron spin or the final photon polarization is sought, the sum over these final states must be carried out. The electron spin sum is most conveniently carried out by introducing a spin projection operator for the initial electron. Thus we find

$$\sum_{\pm s'} |\bar{u}(p',s')Tu(p,s)|^2 = \mathrm{Tr}\left[\frac{\not p'+m}{2m}T\frac{\not p+m}{2m}\frac{1+\gamma_5\not s}{2}\bar{T}\right] \tag{4}$$

with

$$T=\not\epsilon'\frac{(2p\cdot\epsilon-\not\epsilon\not k)}{2k\cdot p}+\not\epsilon\frac{(2p\cdot\epsilon'+\not\epsilon'\not k')}{-2k'\cdot p}$$

and

$$\bar{T}=\frac{2p\cdot\epsilon^*-\not k\not\epsilon^*}{2k\cdot p}\not\epsilon'^*+\frac{2p\cdot\epsilon'+\not k'\not\epsilon'^*}{-2k'\cdot p}\not\epsilon^* .$$

Finally, we carry out the sum on final photon polarization using the gauge invariance property of the transition amplitude. Note also, that the summing process allows for any convenient basis satisfying transversality. In particular, we will choose ϵ' to be real. The sum then gives

$$\begin{aligned} N &\equiv \sum_{\epsilon'}\sum_{\pm s'} |\bar{u}(p',s')Tu(p,s)|^2 \\ &= \sum_{\epsilon'} \mathrm{Tr}\left[\frac{\not p'+m}{2m}T\frac{\not p+m}{2m}\frac{1+\gamma_5\not s}{2}\bar{T}\right] \\ &= -\frac{1}{8m^2}\mathrm{Tr}\left[(\not p'+m)\left[\frac{\gamma^\mu}{2k\cdot p}(2p\cdot\epsilon-\not\epsilon\not k)+\frac{1}{-2k'\cdot p}\not\epsilon(2p^\mu+\gamma^\mu\not k')\right]\right. \\ &\qquad \left.\times(\not p+m)(1+\gamma_5\not s)\left[\frac{(2p\cdot\epsilon^*-\not k\not\epsilon^*)}{2k\cdot p}\gamma_\mu+\frac{(2p_\mu+\not k'\gamma_\mu)}{-2k'\cdot p}\not\epsilon^*\right]\right] . \end{aligned} \tag{5}$$

We can decompose N as $N=N_0+N_s$ where

$$\begin{aligned} N_0 = -\frac{1}{8m^2}\mathrm{Tr}&\left[(\not p'+m)\left[\frac{\gamma^\mu}{2k\cdot p}(2p\cdot\epsilon-\not\epsilon\not k)+\frac{1}{-2k'\cdot p}\not\epsilon(2p^\mu+\gamma^\mu\not k')\right]\right. \\ &\left.\times(\not p+m)\left[\frac{(2p\cdot\epsilon^*-\not k\not\epsilon^*)}{2k\cdot p}\gamma_\mu+\frac{(2p_\mu+\not k'\gamma_\mu)}{-2k'\cdot p}\not\epsilon^*\right]\right] \end{aligned} \tag{6}$$

is the spin-independent part of N, and

$$\begin{aligned} N_s = -\frac{1}{8m^2}\mathrm{Tr}&\left[(\not p'+m)\left[\frac{\gamma^\mu}{2k\cdot p}(2p\cdot\epsilon-\not\epsilon\not k)+\frac{1}{-2k'\cdot p}\not\epsilon(2p^\mu+\gamma^\mu\not k')\right]\right. \\ &\left.\times(\not p+m)\gamma_5\not s\left[\frac{(2p\cdot\epsilon^*-\not k\not\epsilon^*)}{2k\cdot p}\gamma_\mu+\frac{(2p_\mu+\not k'\gamma_\mu)}{-2k'\cdot p}\not\epsilon^*\right]\right] \end{aligned} \tag{7}$$

is the spin-dependent part. The cross section, resulting from N_0, for photon scattering from an unpolarized moving electron has been given by Jauch and Rohrlich,[11] and we will, therefore, consider only the initial electron spin-dependent part, N_s in the following discussion.

The equation for N_s presents traces containing up to nine γ matrices and in addition the γ_5 matrix is always present. The calculation involves straightforward algebra. At the end of this lengthy tedious work, we employ the four-momentum constraints $p'=p+k-k'$ to express N_s in terms of p, k, and k' as

$$\begin{aligned} N_s = \frac{1}{2m}\Bigg[\Bigg[\Bigg\{&-\frac{i}{k\cdot p}\left[p\cdot\epsilon\left[\frac{1}{k\cdot p}-\frac{1}{k'\cdot p}\right]+\frac{k'\cdot\epsilon}{k'\cdot p}\right](p+k')_\mu s_\nu k_\rho \epsilon^*_\sigma \epsilon^{\mu\nu\rho\sigma}\Bigg\}+\mathrm{H.c.}\Bigg] \\ &+i\left[\frac{p\cdot(p-k')}{k'\cdot p}\left[\frac{1}{k\cdot p}-\frac{1}{k'\cdot p}\right]s_\rho+\frac{1}{2}\left[\frac{k\cdot s}{(k\cdot p)^2}-\frac{k'\cdot s}{(k'\cdot p)^2}\right](p-k')_\rho\right]k_\mu\epsilon_\nu\epsilon^*_\sigma\epsilon^{\mu\nu\rho\sigma}\Bigg] . \end{aligned} \tag{8}$$

Accordingly, the spin-dependent cross section can now be written as

$$\frac{d^2\sigma_s}{d\Omega_{k'}dk'}=\frac{e^4m^2}{(2\pi)^2 2kE}\int k'\frac{d^3p'}{2E'}\delta^4(p'+k'-p-k)$$
$$\times\frac{1}{2m}\left[\left[\left\{-\frac{i}{\kappa}\left[p\cdot\epsilon\left[\frac{1}{\kappa}-\frac{1}{\kappa'}\right]+\frac{k'\cdot\epsilon}{\kappa'}\right](p+k')_\mu s_\nu k_\rho \epsilon^*_\sigma \epsilon^{\mu\nu\rho\sigma}\right\}+\text{H.c.}\right]\right.$$
$$\left.+i\left[\left[\frac{m^2}{\kappa'}-1\right]\left[\frac{1}{\kappa}-\frac{1}{\kappa'}\right]s_\rho+\frac{1}{2}\left[\frac{k\cdot s}{\kappa^2}-\frac{k'\cdot s}{\kappa'^2}\right](p-k')_\rho\right]k_\mu\epsilon_\nu\epsilon^*_\sigma\epsilon^{\mu\nu\rho\sigma}\right], \tag{9}$$

where $\kappa=k\cdot p$ and $\kappa'=k'\cdot p$.

Integration on p' is trivial, and we obtain

$$\frac{d^2\sigma_s}{d\Omega_{k'}dk'}=\alpha^2\left[\frac{m}{E}\right]\left[\frac{k'}{k}\right]\delta\{2[p\cdot(k-k')-k\cdot k']\}\theta(E+k-k')$$
$$\times\left[\left[\left\{-\frac{i}{\kappa}\left[p\cdot\epsilon\left[\frac{1}{\kappa}-\frac{1}{\kappa'}\right]+\frac{k'\cdot\epsilon}{\kappa'}\right](p+k')_\mu s_\nu k_\rho \epsilon^*_0 \epsilon^{\mu\nu\rho\sigma}\right\}+\text{H.c.}\right]\right.$$
$$\left.+i\left[\left[\frac{m^2}{\kappa'}-1\right]\left[\frac{1}{\kappa}-\frac{1}{\kappa'}\right]s_\rho+\frac{1}{2}\left[\frac{k\cdot s}{\kappa^2}-\frac{k'\cdot s}{\kappa'^2}\right](p-k')_\rho\right]k_\mu\epsilon_\nu\epsilon^*_\sigma\epsilon^{\mu\nu\rho\sigma}\right], \tag{10}$$

where

$$\Theta(E+k-k')=\begin{cases}1 & \text{for } E+k>k'\\ 0 & \text{for } E+k<k'\end{cases}$$

and ensures a positive energy final electron. On the other hand, the δ function representing the requirement of overall energy conservation entails the "Compton condition" for scattering.

Before we consider some limiting cases of this cross section and compare them with existing calculations, let us mention that the spin-independent part of the cross section which results from N_0 is given by[11]

$$\frac{d^2\sigma_0}{d\Omega_{k'}dk'}=\frac{\alpha^2}{2}\frac{k'}{\kappa}\delta(2[p\cdot(k-k')-k\cdot k'])$$
$$\times\Theta(E+k-k')X\,, \tag{11}$$

where

$$X=\frac{1}{2}\left[\frac{\kappa}{\kappa'}+\frac{\kappa'}{\kappa}\right]-1+2\left[\epsilon\cdot\epsilon'-\frac{\epsilon\cdot p\epsilon'\cdot p'}{\kappa}+\frac{\epsilon\cdot p'\epsilon'\cdot p}{\kappa'}\right]^2 \tag{12}$$

with $p'=p+k-k'$. The differential cross section can be obtained from Eqs. (10)–(12) by adding the spin-independent and spin-dependent parts, namely

$$\frac{d^2\sigma}{d\Omega_{k'}dk'}=\frac{d^2\sigma_0}{d\Omega_{k'}dk'}+\frac{d^2\sigma_s}{d\Omega_{k'}dk'}\,. \tag{13}$$

The resulting expression could then be integrated over the variables and under the limits governed by the particular experiment to be carried out. For example, if the final photon energy is not measured then the above is integrated over all values of k' and the Dirac δ function disappears. On the other hand, in experiments on bound-state Compton scattering, it is the double differential cross section integrated over the momentum distribution of bound electrons which is of interest.[7,12] For that case, the energy-conserving δ function is utilized to partially carry out the three-dimensional integration on the momentum $\vec{p}$. The remaining expression generally contains a two-dimensional integration in the plane perpendicular to the photon momentum transfer. This expression is a Compton profile function.

In the laboratory system, i.e., in the rest frame of initial electron, the differential cross section simplifies to

$$\left[\frac{d^2\sigma_s}{d\Omega_{k'}dk'}\right]_{\text{lab}}=\pm\frac{\alpha^2}{m^2}\left[\frac{k'}{k}\right](\cos\Theta-1)(\vec{k}\cdot\vec{S}\cos\Theta+\vec{k}'\cdot\vec{S})\delta(2[m(k-k')-kk'(1-\cos\Theta)])\Theta(m+k-k')\,. \tag{14}$$

Here we have chosen the incident polarization to be $\hat{\epsilon}_\pm=(\hat{x}\pm i\hat{y})/\sqrt{2}$, the plus and minus signs respectively designating right and left circular polarization. Also the incident propagation direction $\hat{k}$ is along the z axis whereas the final photon has $\hat{k}'=(-\sin\Theta\hat{x}+\cos\Theta\hat{z})$. We find that (14) agrees with the results found many years ago.[8]

On the other hand, (10) is exact and it allows us to correct (14) to include terms that are linear in the electron momentum. When this is done the differential cross section becomes

$$\left[\frac{d^2\sigma_s}{d\Omega_{k'}dk'}\right]_{\mathscr{L}} = \pm\frac{\alpha^2}{m^2}\left[\frac{k'}{k}\right]\Bigg[(\cos\Theta-1)(\vec{k}\cdot\vec{s}\,\cos\Theta+\vec{k}'\cdot\vec{s}\,)$$
$$+\frac{k}{m}\{\hat{k}\cdot\vec{p}[\hat{k}\cdot\vec{s}(2\cos^2\Theta-\cos\Theta+2)-\hat{k}'\cdot\vec{s}\,]$$
$$+\hat{k}'\cdot\vec{p}[\hat{k}\cdot\vec{s}(2\cos^2\Theta-4\cos\Theta+1)+\hat{k}'\cdot\vec{s}\,\cos\Theta]$$
$$+(\hat{k}\times\hat{k}')\cdot\vec{p}(\hat{k}\times\hat{k}')\cdot\vec{s}\;\}\Bigg]\delta(2[p\cdot(k-k')-k\cdot k'])\Theta(E+k-k')\,, \tag{15}$$

where $(\;)_{\mathscr{L}}$ designates the low-momentum approximation.

This result confirms the one obtained by considering the spin-dependent Compton scattering in quasirelativistic quantum electrodynamics, where the usual interaction Hamiltonian is expanded to the appropriate order (see Appendix A).[7]

One might think that cross-section results already appearing in the literature for electrons at rest could be utilized to determine the more general and required result for the scattering from electrons in motion. In Appendix B, we show that this is the case for the results of scattering from unpolarized electrons, but for the polarized electrons the expression given in the literature[8] [e.g., our Eq. (14)] cannot be readily generalized.

In summary, Eq. (10) provides a relativistic expression for the spin-dependent part of the cross section for scattering a polarized photon off a polarized electron in motion. In the final state only the photon energy and scattering angle are detected. Using this result we can obtain an approximate expression for the scattering cross section from bound electrons by using the impulse approximation together with a momentum distribution determined by the electron wave function. Our expression can most logically be considered as a generalization of Ribberfors's results for unpolarized scattering cross sections.[12] Our results provide a confirmation of earlier work based on the Hamiltonian given in the Appendix A.[7]

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APPENDIX A

In Ref. 7 we carried out the calculation of spin-dependent Compton scattering from bound electrons using a quasirelativistic Hamiltonian. This Hamiltonian was given by

$$H_{\rm QR}=\int d^3x\!:\!\psi^\dagger(x)\Bigg\{\frac{(\vec{p}-e\vec{A})^2}{2m}-\frac{(\vec{p}-e\vec{A})^4}{8m^3}-\frac{e}{2m}\vec{\sigma}\cdot\vec{B}+\frac{e}{8m^3}[\vec{\sigma}\cdot\vec{B},(\vec{p}-e\vec{A})^2]_+$$
$$+\frac{1}{8m^2}\nabla^2\left[-\frac{Z\alpha}{|\vec{x}|}\right]+\frac{1}{4m^2}\vec{\sigma}\cdot\left[\vec{\nabla}\left[-\frac{Z\alpha}{|\vec{x}|}\right]+e\dot{\vec{A}}\right]\times(\vec{p}-e\vec{A})+\frac{ei}{8m^2}\vec{\sigma}\cdot\dot{\vec{B}}-\frac{Z\alpha}{|\vec{x}|}\Bigg\}\psi(x)\!:$$
$$+\frac{\alpha}{2}\int d^3x\,d^3y\!:\!\psi^\dagger(x)\psi(x)\psi^\dagger(y)\left[\frac{1}{|\vec{x}-\vec{y}|}+\frac{1}{4m^2}\nabla_y^2\frac{1}{|\vec{x}-\vec{y}|}+\frac{1}{2m^2}\vec{\sigma}\cdot\left[\vec{\nabla}_y\frac{1}{|\vec{x}-\vec{y}|}\right]\times(\vec{p}-e\vec{A})_y\right]\psi(y)\!:$$
$$+\tfrac{1}{2}\int d^3x\!:\![\dot{\vec{A}}^2+(\vec{\nabla}\times\vec{A})^2]\!: \tag{A1}$$

We refer the reader to Ref. 13 for a discussion of the method used to derive (A1). This is based on a sequence of Foldy-Wouthuysen-type transformations of the relativistic quantum electrodynamic Hamiltonian.

We carried out these transformations and obtained Eq. (A1). This result differs from a corresponding expression of Ref. 13 in several respects: (1) It is more general since we have retained all terms quadratic in the radiation field and hence it is appropriate for use in Compton scattering. In Ref. 13 the author was primarily interested in radiative decays with the emission of a photon of frequency ω. Thus while we retain such forms as $\dot{\vec{A}}$ and $\dot{\vec{B}}$ these were replaced by $i\omega\vec{A}$ and $i\omega\vec{B}$ in Ref. 13. (2) In the electron-electron term of (A1) our expressions for the Darwin term and the spin-orbit term are twice as large. This affects the last two terms of Eq. (10) of Ref. 13 as well as Eq. (14). Lin has confirmed the existence of this numerical error in Ref. 13 in a recently published erratum.

APPENDIX B

Given a cross section in the electron rest frame for the Compton scattering of transversely polarized photons, one can always Lorentz transform that expression to boost the linear momentum of the electron. However, in the new coordinate system, the transformed polarization vectors e,e' are *no longer transverse.* The dependence of the cross section on the initial electron momentum and on the *physical* (transverse) polarization vectors in this new frame could, however, be obtained by carrying out a gauge transformation. Let ϵ,ϵ' be the transverse polarization vectors in the new frame. Consider the transformations,

$$e_\mu=\epsilon_\mu+k_\mu\Lambda$$
and
$$e'_\mu=\epsilon'_\mu+k'_\mu\Lambda' , \tag{B1}$$

where k and k' are the four momenta of the photons. The gauge choice of transverse photons in the electron rest frame restricts Λ and Λ' because $p\cdot e=p\cdot e'=0$ must be satisfied. Thus we readily find that

$$e_\mu=\epsilon_\mu-k_\mu\frac{p\cdot\epsilon}{p\cdot k}$$
and
$$e'_\mu=\epsilon'_\mu-k'_\mu\frac{p\cdot\epsilon'}{p\cdot k'} . \tag{B2}$$

Thus to recover the dependence of the cross section on the physical photon polarization vectors ϵ,ϵ' the prescription is that we have to replace the polarizations e,e' by using Eq. (B2).

However, for this prescription to be useful, we must have *exact* knowledge of the *functional* dependence on the polarization vectors of the rest frame cross section. For instance, the Klein-Nishina formula[14] for the scattering cross section from unpolarized electrons at rest is

$$x=\frac{1}{2}\left[\frac{k\cdot p}{k'\cdot p}+\frac{k'\cdot p}{k\cdot p}\right]_0-1+2(\epsilon\cdot\epsilon')_0^2 , \tag{B3}$$

where the subscript 0 means evaluation in the electron rest frame.

Terms of the type $p\cdot\epsilon$ or $p\cdot\epsilon'$ do not appear because of the transversality in the electron rest frame. Of course, in an arbitrary frame ϵ and ϵ' are replaced by e and e' and these do not represent transverse photon polarizations. To recover the dependence on the physical polarization vectors, we transform (B3) to an arbitrary frame and then use (B2) to obtain

$$\begin{aligned}x\to X&=\frac{1}{2}\left[\frac{k\cdot p}{k'\cdot p}+\frac{k'\cdot p}{k\cdot p}\right]-1+2\left[\left[\epsilon-\frac{kp\cdot\epsilon}{p\cdot k}\right]\cdot\left[\epsilon'-\frac{k'p\cdot\epsilon'}{p\cdot k'}\right]\right]^2\\&=\frac{1}{2}\left[\frac{k\cdot p}{k'\cdot p}+\frac{k'\cdot p}{k\cdot p}\right]-1+2\left[\epsilon\cdot\epsilon'-\frac{\epsilon\cdot p\epsilon'\cdot p'}{p\cdot k}+\frac{\epsilon\cdot p'\epsilon'\cdot p}{p\cdot k'}\right]^2 .\end{aligned} \tag{B4}$$

This is exactly the result given by Jauch and Rohrlich,[11] who carry out the algebra directly in a frame of reference in which the initial electron has a nonvanishing linear momentum.

Turning now to the spin-dependent cross section, as pointed out earlier, Ref. 8 does not explicitly provide the dependence on the photon polarization. Hence one cannot uniquely transform the result [see Eq. (14) of this paper] to recover the polarization dependence when the initial electron is in motion. We can easily show, however, that if the rest frame cross section is in the desired form we can generalize the result to an arbitrary frame. To show this consider Eq. (8) in the initial rest frame where $p\cdot\epsilon=0$. Under a Lorentz transformation, N_s would have the same form but the polarization vector is now replaced by e, which will not be transverse. Hence we find

$$\begin{aligned}N_s=\frac{1}{2m}&\left[\left\{\left[-\frac{i}{k\cdot p}\left[\frac{k'\cdot e}{k'\cdot p}\right](p+k')_\mu s_\nu k_\rho e^*_\sigma\epsilon^{\mu\nu\rho\sigma}\right]+\text{H.c.}\right\}\right.\\&\left.+i\left[\frac{p\cdot(p-k')}{k'\cdot p}\left[\frac{1}{k\cdot p}-\frac{1}{k'\cdot p}\right]s_\rho+\frac{1}{2}\left[\frac{k\cdot s}{(k\cdot p)^2}-\frac{k'\cdot s}{(k'\cdot p)^2}\right](p-k')_\rho\right]k_\mu e_\nu e^*_\sigma\epsilon^{\mu\nu\rho\sigma}\right] .\end{aligned} \tag{B5}$$

Again replace e according to Eq. (B2) in order to recover the dependence on the transverse polarization vector ϵ in the new coordinate system (where the initial electron has four-momentum p). When Eq. (B2) is used in Eq. (B5), only the term $k'\cdot e$ changes form under the replacement of e. All other polarization terms can be obtained by merely replacing e by ϵ since the gauge terms do not contribute as a consequence of the antisymmetry of the Levi-Civita symbols. The required term is

$$\frac{k'\cdot e}{k'\cdot p}=\frac{1}{k'\cdot p}k'\cdot\left[\epsilon-\frac{kp\cdot\epsilon}{p\cdot k}\right]=\frac{k'\cdot\epsilon}{k'\cdot p}-\frac{k'\cdot kp\cdot\epsilon}{k'\cdot pp\cdot k} .$$

Using the Compton condition $p\cdot(k-k')=k\cdot k'$ this can now be written as

$$\frac{k'\cdot\epsilon}{k'\cdot p}+p\cdot\epsilon\left[\frac{1}{k\cdot p}-\frac{1}{k'\cdot p}\right]$$

thus recovering Eq. (8) for the general case in which $p\cdot\epsilon$ is nonzero.

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[10]Notice that here $u(p,s)$ represents a positive energy electron spinor with arbitrary momentum p^μ and spin s^μ. The latter, however, is defined to be the Lorentz transform of the polarization vector $S^\nu=(0,\vec{S})$ in the electron rest frame. Thus $\vec{S}$ itself could be a unit vector in the positive or negative z direction corresponding to spin up or down. Similar remarks apply to s'^μ. Also here and in what follows, whenever k (or k') does not appear in a four-dimensional δ function or as one of the four vectors in a scalar product, then it represents the magnitude $|\vec{k}|$ (or $|\vec{k}'|$) which, for real photons in the problem, is also the zeroth component in k^μ (or k'^μ).

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