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Analysis of proton-He resonances in very-low-energy collisions

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Energy dependence of the H^+-He differential cross section is analyzed for a fixed scattering angle. The energy range is between 0.1 and 0.7 eV. Rather complex resonance structure is found in which it is difficult to isolate the contribution of a single resonance.

I. INTRODUCTION

The quasibound states of the proton-He ground state have been analyzed in great detail, both experimentally and theoretically.[1–4] This work has also prompted the calculation of a very accurate ground-state potential[5] from which a complete spectrum of these states has been obtained. Most of the quasibound states have been identified spectroscopically and their experimental positions are in reasonably good agreement with theoretical predictions. However, besides using spectroscopy for identification of these quasibound states one can also try an alternative idea, namely, these states will appear as resonances in molecular-beam experiments. Using such a method for analyzing H^+-He quasibound states will serve two purposes: provide a direct way of measuring the width of quasibound states and contribute to our understanding of resonance phenomena in atomic collisions.

Indeed such an experiment was recently carried out,[6] and the first results appeared encouraging. However, such experiments are not without difficulties, which can be summarized in two points: the contribution of resonances is difficult to distinguish from other contributions in the cross sections and the theoretical interpretation of the observed data is not simple. Let us briefly discuss these two points.

The usual way to study the resonance phenomena is by analyzing their contribution in the total cross section, and indeed the first attempts with the neutral atoms have been carried out in this way.[7,8] However, the results were not as spectacular as in nuclear- or electron-atom collisions, where the resonance effect is dominant in the cross section. In atom-atom collisions this effect appeared in the integral cross section as an almost insignificant perturbation. The reason why this is so is very well understood and can be summarized in two points: atom-atom potential is of long range and the wavelength of atoms is short compared to the effective radius of atoms. Both of these factors contribute to the fact that the integral cross section is dominated by diffraction scattering.[9] The contribution of resonances in the total cross section is only a few percent of the diffraction scattering, therefore, it is difficult to distinguish them from the background.[10] Also, the short wavelength of atoms implies that resonances are narrow, hence, a small uncertainty in the collision energy can average out most of the resonance contribution.[10]

In the case of ion-atom collisions, the ratio of the resonance to diffraction contributions in the total cross section is even less favorable for the resonances. Therefore, observation of ion-atom resonances would not be easy in this way, and for this purpose Konrad *et al.*[6] used a novel idea which they applied to H^+-He scattering. The idea is to measure the energy dependence of the differential cross section for a fixed scattering angle. The scattering angle can be conveniently chosen away from the forward direction, i.e., in the region where diffraction scattering is negligible; in which case the resonance contribution will be quite prominent. As in the total cross section, formation of a resonance will be noticed as a rapid variation in intensity of the differential cross section around some well-defined collision energies.

Although the major problem of detection of resonances is overcome in this way, there remains another difficulty: interpretation of cross sections. Because of the relatively large number of quasibound states, it is very often the case that few of them will be very close for a given collision energy, therefore they will appear as overlapping resonances. The interference between these overlapping resonances can produce patterns which are not easily recognizable as individual resonances. Furthermore, if one includes interference with the background term (which will be discussed later) the resonance effects can be really difficult to observe. Therefore, before attempting to interprete the differential cross section

one should have a good understanding of the mechanisms which produce such patterns, and for this purpose we have several methods at our disposal: the partial-wave method, the complex-energy and complex-angular-momentum methods, and the semiclassical method. As it turns out, the most convenient is the complex-angular-momentum method (usually referred to as the Regge theory) since other methods have certain inherent weaknesses when applied to the problem of resonance scattering.

The obvious disadvantage of the partial-wave method is that in its foundations it is not designed to prefer the contribution of one partial wave over the other. All partial waves have equal weight, and therefore the resonance partial wave (i.e., the partial wave in which a resonance occurs) will not have any preferential treatment over the other partial waves. A resonance will only appear as a rapid change of the phase shift in the resonance partial wave when energy is varied. Also, it will appear as a rapid change of the phase shift for a fixed energy when partial waves are scanned. However, once a resonance is located in a certain partial wave, there is no way of telling how it will appear in the cross section. The problem becomes even more serious when few resonances are very close, or when interference effects with the other partial waves can produce nonstandard patterns of differential cross sections.

The complex-energy formalism does not solve the essential problems of the partial-wave method. It only offers a way of parametrizing phase shifts (the S matrix) in the vicinity of a resonance.[11] Furthermore, it is assumed that a resonance only contributes to the resonant partial wave, and that the neighboring partial waves are not affected by its presence. This is far from being true since a presence of a resonance in a certain partial wave can also greatly affect the neighboring partial waves. Nevertheless, the formalism is quite useful for the description of cross sections where only few partial waves are involved (e.g., nuclear collisions, electron-atom collisions, etc.), or for analyzing quasibound states in spectroscopy (e.g., in the case of H^+-He).

The semiclassical method was shown to be quite useful for the description of elastic collisions, and in fact most of the features of the elastic cross sections have been analyzed in this way.[12] However, resonance phenomena have no simple description using this method. In order to understand why, we should briefly mention how resonances are formed in elastic collisions. There are two mechanisms of their formation: one when the collision energy is higher than the centrifugal barrier and the other when it is lower. In the first case, resonance is formed by the interference of two waves: one reflected from the top of the centrifugal barrier and the other from the hard core of potential. Mutual interference of these two waves can cancel the wave outside the centrifugal barrier so that a standing wave is formed inside the potential. Therefore, it is essential that any theory of resonances incorporates the possibility of reflection of the waves from the top of the centrifugal barrier, which is not the case in the semiclassical method. One should go beyond the ordinary semiclassical approximation[12] in order to incorporate this effect, but then the simplicity of this method is lost. In the second case, resonances are formed by tunneling through the centrifugal barrier and formation of a standing wave inside the potential. Of these resonances, the most dominant are those for which their energy is slightly smaller than the height of the centrifugal barrier. In such a case, ordinary semiclassical treatment fails because two turning points of kinetic energy on the centrifugal barrier are very close. Therefore, other approximate techniques should be considered, such as replacing the centrifugal barrier near the top by an inverted parabola.[13]

The complex-angular-momentum method (the Regge theory) is a compromise between the semiclassical treatment and a full quantum treatment. It will be shown that the elastic scattering amplitude in this method is given as a sum of two terms: one describing the contribution of resonances (and orbiting) and the other which describes the direct reflection. The latter contribution can be analyzed using the semiclassical method, but for the calculation of the resonance terms we will require the Regge poles (complex-angular-momentum poles of the S matrix), which are calculated from the numerical solution of the radial Schrödinger equation for complex angular momenta. Several methods have been developed for this purpose,[14–17] which are no more difficult to use than methods used for calculation of phase shifts.[18] Another advantage of describing resonance cross sections by the Regge method stems from relatively simple energy behavior of the Regge poles. They are nearly a linear function of energy and this fact can be used for a simple location of resonances. Also, from the knowledge of the poles and residues, it is possible to obtain the contribution of resonances in cross sections in a simple way.

Representation of the scattering amplitude in terms of the complex-angular-momentum poles was first suggested by Regge,[19] but it was only suitable for the use in nuclear collisions because of the tacit assumption that the potential does not have a hard core. The theory was modified so that it is applicable in atom-atom (ion-atom) collisions,[20] where the potential for small internuclear distances can be replaced by a hard core.[21] In this work we will use this representation for analyzing the energy depen-

dence of the differential cross section for a fixed scattering angle.

II. THEORY AND PROPERTIES OF REGGE POLES

It was shown for potentials with a hard core that the scattering amplitude, in the representation of the complex-angular-momentum poles of the S matrix (Regge poles),[20] is

$$f(\theta)=-\frac{\pi i}{k}\sum_n \lambda_n\beta_n\frac{P_{\lambda_n-1/2}(-\cos\theta)}{\cos(\pi\lambda_n)}+\frac{1}{k}\int_0^\infty d\lambda\,\lambda S(\lambda)e^{-i\pi\lambda}P_{\lambda-1/2}(-\cos\theta)$$

$$=f_R+f_B\ , \tag{2.1}$$

where λ_n and β_n are the Regge poles and the appropriate residues of the S matrix $S(\lambda)$, respectively. The poles are complex and for atom-atom potential the poles of physical significance are located in the first quadrant of the complex λ plane.

The integral in (2.1) can be evaluated analytically by the stationary phase method. If we designate by η the phase $\eta=\arg(S)-\pi\lambda$ and by λ_0 the angular momentum $\lambda_0=l_0+\frac{1}{2}$ for which $\partial\eta/\partial\lambda=\theta-\pi$, then f_B in (2.1) is[22,23]

$$f_B\sim-\frac{i}{k}\left[\frac{\lambda_0}{|\eta_0''|\sin\theta}\right]^{1/2}\times\exp[i\lambda_0(\pi-\theta)+i\eta_0]\ , \tag{2.2}$$

where $\eta_0=\eta(\lambda_0)$ and $\eta_0''=\partial^2\eta_0/\partial\lambda^2$. The approximate form (2.2) of f_B is quite accurate in most cases of relevance in atomic collisions. Therefore, if the poles λ_n of the S matrix are known, the scattering amplitude (2.1) is relatively simple to calculate, since in the orbiting region only a few poles are necessary to achieve convergence in the series for f_R.[24]

Part of the scattering amplitude, designated by f_R, is associated with the orbiting, and hence, also with the resonance phenomena. Therefore, in this way we have isolated in the scattering amplitude the contribution of resonances in a separate term, in contrast with the partial-wave method where this contribution is located in one partial wave. The advantages of this are manifold: rather complex resonance structure can be interpreted as an interference of a few terms, a single pole describes a series of resonances, location of resonances becomes simple, etc. For a more detailed discussion of this topic see Refs. 20 and 24; however, we would like to mention briefly how resonances are classified.

The energy dependence of λ_n is monotonic, i.e., $\mathrm{Re}(\lambda_n)$ and $\mathrm{Im}(\lambda_n)$ are increasing for increasing energy, hence a single pole obtains several times the half-integer value, while its imaginary part is small. At such energies a resonance appears in the cross section. Therefore, labeling of resonances can be done with only one index, corresponding to the label of the pole. Since there is only a finite number of poles which produce resonances, and this number equals the number of bound states of the system, labeling of resonances can be achieved with the same number of indices.

Location of resonances can also be done with relative ease. If the value of λ_n is known for several energies, then using a simple interpolation-extrapolation procedure position of resonances can be obtained with relatively high precision. The same technique applies also for interpolation and/or extrapolation of the cross section. In fact, the analysis of the cross sections in this work was done using such a technique. The values of λ_n and β_n were calculated for only few energy values and by interpolation we obtained all the other values of λ_n and β_n. The cross section was then obtained from (2.1) without resorting to additional calculation of phases and poles from the Schrödinger equation.

Let us now discuss the potential and properties of the Regge poles for H^+-He. The best available H^+-He potential was given by Kolos,[5] but only as a set of points at some internuclear distances. For large internuclear separations the potential was given in the analytic form of dispersion expansion. However, for calculation of the poles one also needs the interpolated values of potential and this can be achieved in several ways: through the use of (1) a straightforward parabolic fit through three points, (2) splines,[25] or (3) an analytic fit through all points. All of these procedures have been tried and in the end we decided to use the analytic fit through all points. Let us briefly mention why other methods did not work. In all fitting procedures based on splines (parabolic fit is a special case of splines), one makes an approximation that some higher derivatives of splines are not continuous at the end points of intervals. It can be shown that such a discontinuity, although negligible for many purposes, produces spurious Regge poles which behave nonuniformly. Such poles usually have large imaginary parts, indicating that they represent short-lived states, something like surface waves. But their contribution in the differential cross section is quite important, especially for small scattering angles. Be-

TABLE I. Parameters for the H^+-He potential given in the analytic form (2.11). The internuclear distance is in units of angstroms while the coefficients are in units which give V in eV.

r_0	0.7743 Å	a_3	4.258 754
r_1	0.525 Å	a_4	8.767 504
a	−1.301 58	a_5	6.858 96
b	−5.015 42	a_6	3.087 388
a_1	4.879 129	a_7	0.708 278
a_2	16.37		

cause of these poles and their nonuniform behavior, we cannot make a simple analysis of differential cross sections.

For the reason just mentioned we have given up using splines for interpolating the potential. Instead, we have looked at the analytic forms which will accurately reproduce the H^+-He potential, including its tail. After a few trials we have decided on the ratio of two polynomials and found that V is given quite accurately by

$$V(r)=(r-r_1)\frac{b+ar^2}{x^7+a_1x^6+a_2x^5+a_3x^4+a_4x^3+a_5x^2+a_6x+a_7}\ , \tag{2.3}$$

where $x=r-r_0$. The numerical values for the coefficients in (2.3) are given in Table I, in such units that V is in eV. This form also reproduces the tail of V, but is singular for $r=0.3869$ Å. However, this singularity has no effect on the phase shifts, at least for the collision energies below a few eV. For all relevant values of r, the potential (2.3) reproduces the calculated points by less than 1%, except for $r>2.1$ Å, where this deviation is larger ($\sim$5–13 %).

Using (2.3) the Regge poles and their respective residues were calculated in the energy range 0.1 eV $<E<$ 0.7 eV. Altogether ten poles were analyzed, although at a given energy not all of them significantly contribute to the cross section. The energy dependence of these poles, usually referred to as the pole trajectories, is shown in Fig. 1.

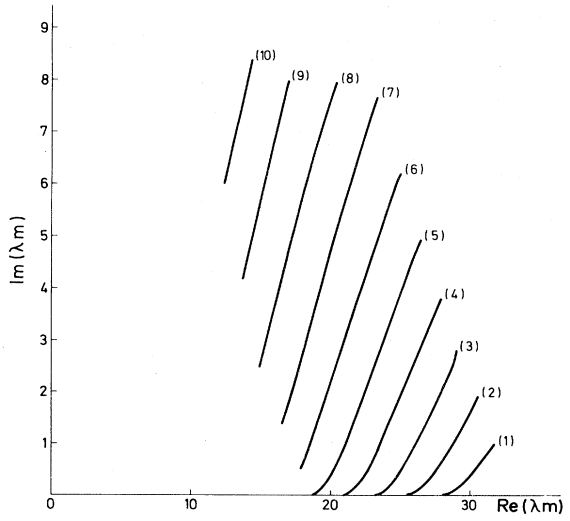

FIG. 1. Regge poles for the H^+-He system in the energy range 0.1 eV $<E<$ 0.7 eV. Poles correspond to the potential given by (2.11).

III. ANALYSIS OF CROSS SECTIONS

In order to obtain the energy dependence of the differential cross section, for a fixed scattering angle, we should first analyze in more detail the energy dependence of the poles and residues. In Fig. 1 we showed the pole trajectories, which appear to be almost linear, except when the imaginary part of the poles becomes small. To confirm this, in Fig. 2 we show the energy dependence of the real and imaginary part of the poles separately. The poles were

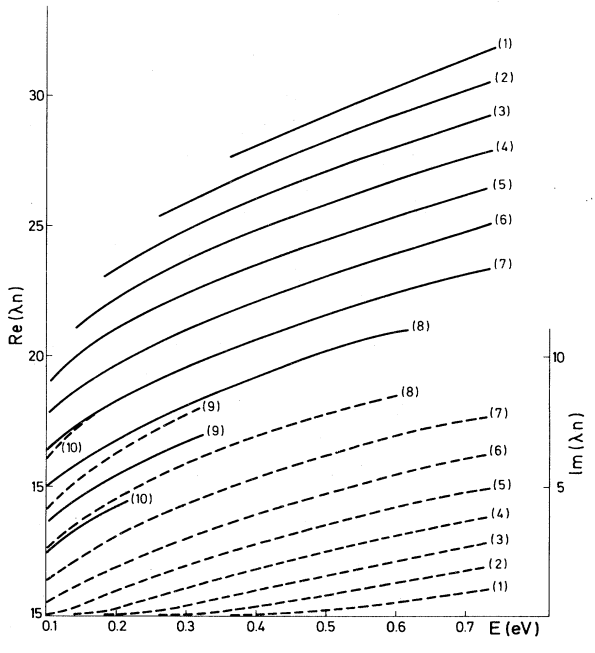

FIG. 2. Energy variation of the real and imaginary parts of the Regge poles for H^+-He. Labels of the poles are indicated by numbers.

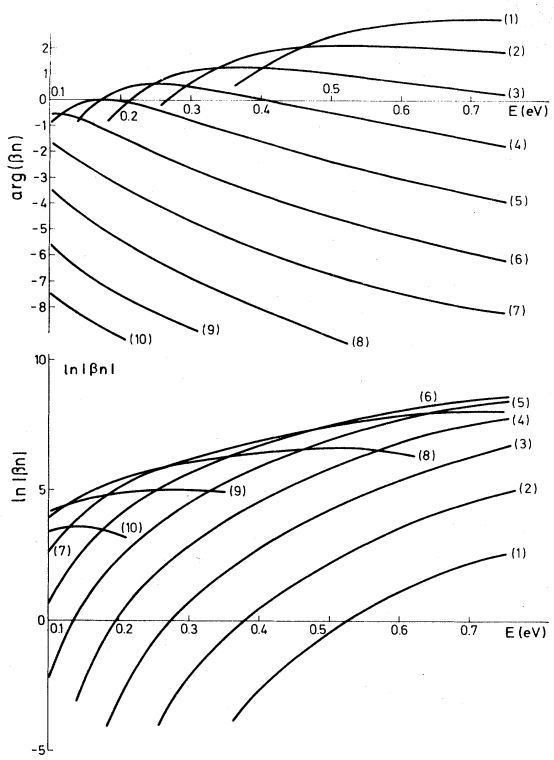

FIG. 3. Energy variation of the logarithm of the residues for H^+-He. Numbers indicate label of the poles.

calculated at approximately 0.1-eV intervals, using an exact method.[17]

Indeed the poles have almost linear dependence with energy, but to achieve better accuracy we have used the interpolation procedure with parabola, through three calculated points of the poles. Numerical tests showed that such a fitting procedure is sufficient to achieve good accuracy. However, even better results were obtained, with the same parabolic fit, if instead of energy we used the square root of energy as an independent variable. In other words, it appears that the pole trajectories are more linear in the square root of energy than in the energy variable. The test for the accuracy of the fitting procedure was that the difference between the extrapolated and calculated value of the real part of the next neighboring poles is less than 0.1.

Almost linear dependence of the poles with energy is not peculiar to this example. This fact has been noted for various potentials and it seems that such a behavior has more to do with the presence of the hard core in the potential than with its functional form. Qualitative arguments confirming this have been discussed on various occasions.[26–28]

The energy dependence of the residues is more complicated. It is an oscillatory function of energy, with increasing amplitudes. However, the logarithm of the residue behaves more regularly with energy (or the square root of energy). In Fig. 3 we show the energy dependence of the logarithm of the residues. The dependence is less linear than in the case of the poles but still the fit with parabola gives sufficient accuracy for the interpolated values of residues. Later we will make a crucial test for the accuracy of the interpolation procedure by calculating the differential cross section for these values and comparing it with the cross section obtained for the calculated poles and residues.

Let us now look at the question of the convergence of the series for f_R in (2.1). Since this point has already been discussed[24] we will only review the essential results. For a given energy the pole which is closest to the real axis we will index by $n=1$ and all the other poles with increasing imaginary part will carry the subsequent index n. The pole $n=1$ may have a very small imaginary part, in which case it can be shown that its contribution in the sum f_R is negligible. Only the poles with imaginary part of the order 0.01, or greater, will noticeably contribute to the cross section, especially if the real part is a half-integer. This point is of a general validity for the poles with a small imaginary part; their contribution in the cross section is greatly enhanced only when their real part is a half-integer. Since this happens only for a certain energy, the cross section undergoes a rapid change in the interval around this point, thus producing a resonance. However, for the poles with a very small imaginary part, this rapid change occurs in a very narrow energy interval, and that is why we can neglect their contribution in the cross section. For this reason some of the trajectories in Fig. 2 do not continue to $E=0.1$ eV, since below the points where they stop the imaginary part of the pole becomes very small according to the criteria given above.

The poles with a large imaginary part do not show such a behavior and their contribution in the scattering amplitude is given by

$$f_R^{(n)} \sim -\frac{\pi i}{k}\left[\frac{2\lambda_n}{\pi \sin(\theta)}\right]^{1/2} \beta_n e^{i\lambda_n\theta - i\pi/4}$$

and the module of $f_R^{(n)}$ is approximately

$$|f_R^{(n)}| \sim \exp[\theta\,\mathrm{Im}(\lambda_n) + \ln|\beta_n|] \ .$$

Therefore, the convergence of the series for f_R will depend on whether or not $\ln|\beta_n|$ as a function of n is always increasing faster than $\theta\,\mathrm{Im}(\lambda_n)$. But what is immediately obvious is that, if the series converges, then the rate of convergence depends on θ. In fact, the smaller θ the poorer the convergence of the series. As we will see later, this fact causes considerable difficulty in the forward scattering space.

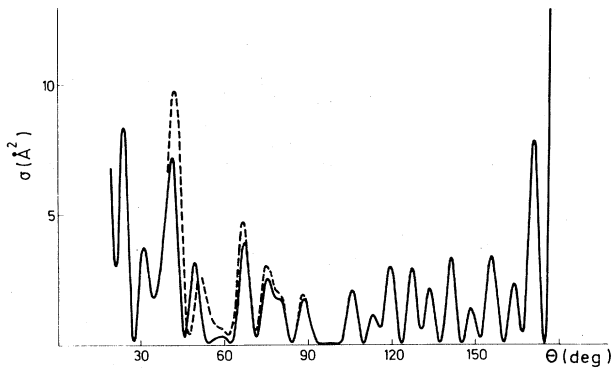

FIG. 4. H^+-He differential cross section for collision energy $E=0.41556$ eV. Solid line represents calculations of the partial-wave series, while the dashed line is obtained from (2.1). For the explanation of why there is deviation between the two curves for small angles, see the text.

A look at Fig. 3 shows that $\ln|\beta_n|$ is, at first, an increasing function of n for all E, but for certain n, changes direction and becomes a decreasing function. The turning point depends on E, but in the interval shown in Fig. 3 this is somewhere around $n=6$—8. Therefore, in our example, the series is convergent since $\mathrm{Im}(\lambda_n)$ is always an increasing function of n. The rule that $\ln|\beta_n|$ reaches a maximum is a general one. It can be qualitatively proved[20] but in many examples it was also shown to be true.[29] Therefore, the convergence of the series for f_R is not typical of our case but has a general validity.

We have shown how to calculate the Regge poles and residues, how to interpolate them, and how individual poles contribute in the scattering amplitude. In addition, we have seen that the series f_R converges. Let us now look at how the Regge-pole representation of the scattering amplitude compares with the partial-wave series and how accurate is the energy interpolation of the poles and residues concerning the differential cross section.

The differential cross section for $E=0.41556$ eV was calculated from the partial-wave series and (2.1). For this energy the poles and residues were calculated numerically from the Schrödinger equation. In Fig. 4 we show results of this calculation.

We notice that the two calculations give almost identical results, except for angles smaller than $\theta\sim 80°$. Below this angle the two calculations increasingly differ one from the other. This is not because we did not include enough poles to achieve convergence in the series f_R, but because of accumulation of numerical errors.

In the sum for f_R the module of each term $f_R^{(n)}$ is n and θ dependent, as discussed earlier. Typically, if θ gets smaller, then $|f_R^{(n)}|$ gets larger, until below a certain angle, when some $|f_R^{(n)}|$ exceed the value of $|f_R|$, i.e., they exceed the contribution of orbiting and resonances in the differential cross section. This usually happens in the diffraction region. Therefore, in the sum f_R some terms will cancel each other out and in this way lose the most significant figures. Since each term $f_R^{(n)}$ is given accurately to only the first few significant figures (the poles λ_n and residues β_n are obtained by numerical integration of the Schrödinger equation and therefore can only be given accurately to a few significant figures), it is obvious that below a certain angle we will not obtain meaningful results. All the significant figures of $f_R^{(n)}$ will cancel out, leaving only the random values. This problem does not exist in the partial-wave summation, because a typical contribution of a single partial wave is smaller than the value of the scattering amplitude, especially in the diffraction region.

That this is indeed the case is confirmed in the calculation of the differential cross section for $E=0.6$ eV, shown in Fig. 5.

For this energy the poles λ_n and residues β_n were obtained by the interpolation procedure described earlier. The nearest calculated values of these quan-

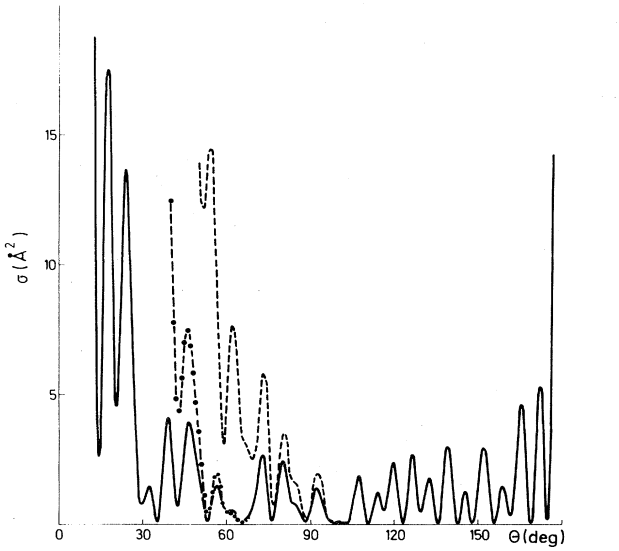

FIG. 5. H^+-He differential cross section for $E=0.6$ eV. Solid line represents the calculation of the partial-wave series, while the dashed line is obtained from (2.1) with the interpolated poles and residues. Dotted-dashed line is obtained from (2.1) but with the calculated poles and residues. See the text for explanation of why the two curves obtained from (2.1) differ so much for small angles.

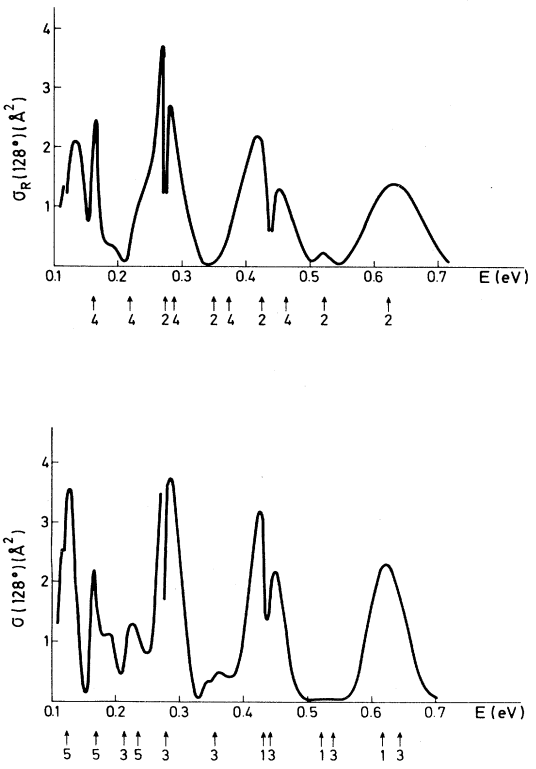

FIG. 6. Energy variation of the differential cross section for $\theta=128°$. Upper curve represents pure contribution of the poles in the differential cross section, while the lower curve represents full differential cross section. We have also indicated the position of the resonances predicted from the energy variation of the poles. Numbers correspond to the labels of the poles: even labels are in the upper curve and odd labels are in the lower curve. Discussion of this is given in the text.

tities were at $E=0.623$ and 0.519 eV. The full line represents the calculation of the partial-wave series, while the broken line represents the calculation with (2.1), but using the interpolated poles and residues. The deviation of the two results for larger angles begins then in Fig. 4 simply because the interpolated values of λ_n and β_n are less accurately given than the corresponding calculated values. However, when we calculated the values of λ_n and β_n for the same energy and used them instead of the interpolated ones, the agreement with the partial-wave series is greatly improved, as shown in Fig. 5 by the dashed-dotted lines. The last result is in accordance with the previous discussion about the stability of the series f_R for the small scattering angle.

The last example also shows that, at least for the angles greater than $\theta>100°$, the series f_R is given quite accurately using the interpolation procedure for λ_n and β_n with parabola. Therefore, with a great deal of confidence, we can analyze the energy dependence of the differential cross section for large angles. We have done this for $\theta=128°$, and results are shown in Fig. 6. The upper part of the figure shows $|f_R|^2$ while the lower part shows $|f_R+f_B|^2$, i.e., the true differential cross section. These two figures are given so that we can compare how much the structure of cross section is influenced by the presence of the f_B amplitude.

As can be noticed, the contribution of f_B does not essentially change the pattern of the energy dependence of the cross section $|f_R|^2$. This is because the radius of the hard core for H^+-He is small, approximately $r\sim0.5$ Å, therefore f_B is also small. The change is only in details such as around $E\sim0.52$ and 0.2 eV. However, both cross sections are quite complex which is due to the large number of resonances. In Fig. 6 the position of the resonances, labeled by the index of the poles, are shown by arrows: even labels in the upper part and odd labels in the lower part of the figure. This was done for convenience.

We immediately notice that a single pole shows up several times as a resonance and this is whenever its real part obtains a half-integer value. The rule is that for increasing energy a single pole produces a wider resonance. For example, the pole $n=4$ produces a series of resonances at $E=0.115$, 0.165, 0.22, and 0.29 eV. The lowest-lying resonance is very narrow and hardly visible, while the next is quite prominent, but the next is buried in the interference with the other resonances. Therefore, even though the last two may have a non-negligible contribution in the cross section, they are overshadowed by the contribution of the neighboring resonances.

Here we encounter the basic problem in the observation of resonances in the H^+-He collision. As can be seen from Fig. 6, several resonances are very often lying close to each other and their contribution may add or cancel, therefore we will not observe a clear picture of a single resonance. There are exceptions, however, such as the very narrow resonances, but they are too narrow for practical observation. The exceptions are also the $n=4$ resonance at energy $E=0.165$ eV, which is isolated from the other, and the lowest-lying $n=1$ resonance. The latter produces a dip in the contribution of the resonances $n=2$ and 3. It is interesting to notice that the next $n=1$ resonance has no affect on the cross section. It has little affect on f_R, but once f_B is included, the effect is washed out. This is another example of how several close-lying resonances can cancel each other out.

The interference between f_R and f_B can extract

out a resonance from the sum f_R, such as the one for $E=0.225$ eV. This peak does not exist in $|f_R|^2$, however, it appears in $|f|^2$. A possible explanation is as follows: the close-lying resonances $n=4$ and 5 ($n=3$ is too narrow to contribute significantly in f_R) give contributions which produce a uniform line (see Fig. 6, upper part). When f_B is added it cancels the contribution of the pole $n=5$, hence, the contribution of the $n=4$ resonance is left and is observed as a peak in $|f|^2$.

The discussion so far shows the whole complexity of the resonance structure of the H^+-He system. It is further complicated by the energy and angle averaging, but in this work we will not discuss this point.

Calculations for different angles have been done but the results are not essentially different from the one which was discussed here. The angle $\theta=128°$ used here is typical of large-angle scattering regions and it is believed that no qualitatively new insight can be obtained by scanning all other angles.

IV. CONCLUSION

In this work we have analyzed the resonance structure of the H^+-He system, which is very pronounced if the energy dependence of the differential cross section is measured for a fixed scattering angle. However, observation of a single resonance is difficult because in most cases several of them are close lying, producing a collective effect in the cross section. In such cases an individual resonance can only be observed if it is very narrow. But in the experiment its contribution is averaged out due to the energy spread of the atomic beam and the finite angular resolution of detectors. There are exceptions, however, noted in the discussion of Sec. III.

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