PHYSICAL REVIEW A VOLUME 28, NUMBER 3 SEPTEMBER 1983

# Surface polaritons in nonlinear media

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(Received 20 August 1982)

It is shown that the general dispersion relation of electromagnetic surface waves propagating at the interface of certain types of nonlinear media can be obtained exactly without first solving for the field profiles across the boundary.

Surface wave propagation is of interest in many branches of physics. Recently, there have been many attempts to consider nonlinear effects of surface waves on solid[1–4] as well as plasma[5–7] boundaries. Because of the complexity of the problem, usually drastic simplifying assumptions have to be made.

Agranovich *et al.*[1] have derived the dispersion relation for nonlinear surface wave propagation at the interface of an isotropic linear medium and a nonlinear medium having a diagonal dielectric tensor with the components $\epsilon_{xx}=\epsilon_{yy}=\epsilon_0(\omega)+\alpha|E_{\parallel}|^2$, $\epsilon_{zz}=\epsilon(\omega)$, where $\vec{E}_{\parallel}$ is the wave electric field parallel to the surface, and $\omega$ is the wave frequency. As was pointed out in the paper, the results have restricted applications even for uniaxial solids, since the form of $\epsilon_{ij}$ is rather idealized.

In this paper, we generalize the problem considered in Ref. 1 to the case in which both media are nonlinear, having dielectric tensors of the more general form $\epsilon_{ij}=\delta_{ij}\epsilon_{ij}(\omega,|E_{\parallel}|^2)$. That is, the component $\epsilon_{zz}$ is also nonlinear. A novel method is presented with which the nonlinear dispersion relation for the electromagnetic surface waves (polaritons) can be obtained directly from the boundary conditions without first having to solve for the field profiles. Examples for which complete analytical solutions can be obtained are given.

We consider surface wave propagation on the interface $z=0$ between two nonlinear dielectric media. If both media are isotropic in the $(x,y)$ plane, without loss of generality, we can write, for the surface waves,

$$E_{x,z}=\mathcal{E}_{x,z}(z)\exp(-i\omega t+ikx)+\text{c.c.} ,$$

$$B_y=\mathcal{B}_y(z)\exp(-i\omega t+ikx)+\text{c.c.} ,$$

$$E_y=B_x=B_z=0 .$$

The relevant Maxwell's equations are

$$\frac{d}{dz}\mathcal{B}_y=i\frac{\omega}{c}\epsilon_{xx}\mathcal{E}_x , \tag{1}$$

$$k\mathcal{B}_y=-\frac{\omega}{c}\epsilon_{zz}\mathcal{E}_z , \tag{2}$$

$$\frac{d}{dz}\mathcal{E}_x-ik\mathcal{E}_z=i\frac{\omega}{c}\mathcal{B}_y . \tag{3}$$

The boundary conditions are

$$[\mathcal{E}_x]=0 , \tag{4}$$

$$[\mathcal{B}_y]=0 , \tag{5}$$

where the square brackets denote a jump in the value of the argument across the boundary. The components $\epsilon_{xx}$ and $\epsilon_{zz}$ of the dielectric tensor are functions of $\mathcal{E}_x^2$.

Eliminating $\mathcal{E}_z$ and $\mathcal{B}_y$, we obtain the differential equation

$$\frac{d}{dz}\left[A(\mathcal{E}_x^2)\frac{d}{dz}\mathcal{E}_x\right]=\epsilon_{xx}\mathcal{E}_x , \tag{6}$$

where

$$A(\mathcal{E}_x^2)=\epsilon_{zz}/(k^2-\omega^2\epsilon_{zz}/c^2) .$$

Equation (6) has to be applied separately in the two adjoining media. It can be integrated once if the factor $A(\mathcal{E}_x^2)\,d\mathcal{E}_x/dz$ is multiplied. The result, after some manipulation, is

$$A^2(\mathcal{E}_x^2)\left[\frac{d}{dz}\mathcal{E}_x\right]^2=\int^{\mathcal{E}_x^2}\epsilon_{xx}(\xi)A(\xi)\,d\xi+C , \tag{7}$$

where the constant $C$ is to be determined by the condition

$$\mathcal{E}_x=d\mathcal{E}_x/dz=0 \text{ for } |z|\to\infty .$$

Equation (7) is in the form of a quadrature and can therefore, in principle, be integrated for any given $\epsilon_{xx}$ and $\epsilon_{zz}$. However, in the following, we show that the dispersion relation can be obtained without solving (7).

Using Eqs. (1)–(3), one can write the magnetic field in the form

$$\mathcal{B}_y=i\frac{\omega}{c}A(\mathcal{E}_x^2)\frac{d}{dz}\mathcal{E}_x . \tag{8}$$

Comparing (7) and (8), we note that the boundary condition (5) can be applied without solving explicitly for $\mathcal{E}_x$. Thus one gets

$$\left[C+\int^{\mathcal{E}_x^2(0)}\epsilon_{xx}(\xi)A(\xi)\,d\xi\right]=0 , \tag{9}$$

 

where the square-bracket notation has been used.

Denoting the two media by the superscripts I ($z>0$) and II ($z<0$), and using (4), we obtain

$$C^{\mathrm{I}}+\int^{\mathcal{E}_x^2(0)}\epsilon_{xx}^{\mathrm{I}}(\xi)A^{\mathrm{I}}(\xi)\,d\xi = C^{\mathrm{II}}+\int^{\mathcal{E}_x^2(0)}\epsilon_{xx}^{\mathrm{II}}(\xi)A^{\mathrm{II}}(\xi)\,d\xi \ , \quad (10)$$

where $\mathcal{E}_x(0)=\mathcal{E}_x^{\mathrm{I}}(0)=\mathcal{E}_x^{\mathrm{II}}(0)$ is the value of the electric field $\mathcal{E}_x$ at the boundary, and is a measure of the nonlinearity.

Equation (10) is the general nonlinear dispersion relation of the surface polaritons. It relates the electric field amplitude at the surface to the frequency $\omega$ and the wave vector $k$.

As an example, let us consider the problem investigated in Ref. 1. Here, medium I is linear, so that $\epsilon_{xx}^{\mathrm{I}}=\epsilon_{zz}^{\mathrm{I}}=\epsilon^{\mathrm{I}}=$constant. Medium II is given by $\epsilon_{xx}^{\mathrm{II}}=\epsilon_0+\alpha\mathcal{E}_x^2$ and $\epsilon_{zz}^{\mathrm{II}}=\epsilon=$constant. Application of (10) leads immediately to the dispersion relation

$$\epsilon^{\mathrm{I}^2}/\tilde{\kappa}^2=\epsilon[\epsilon_0+\tfrac{1}{2}\alpha\mathcal{E}_x^2(0)]/\kappa^2 \ , \quad (11)$$

where $\tilde{\kappa}^2=k^2-\omega^2\epsilon^{\mathrm{I}}/c^2$ and $\kappa^2=k^2-\omega^2\epsilon/c^2$. Equation (11) is equivalent to the dispersion relation in Ref. 1.

We now consider the case in which both media are nonlinear, given by dielectric tensor components of the form $\epsilon_{xx}=\epsilon_1+\alpha\mathcal{E}_x^2$, $\epsilon_{zz}=\epsilon_2+\beta\mathcal{E}_x^2$. That is, both media are nonlinear with respect to every axis. In order to obtain analytical results, we shall consider the small amplitude limit. Thus the function $A(\mathcal{E}_x^2)$ can be approximated by $a+b\mathcal{E}_x^2$, where $a=\epsilon_2/(k^2-\omega^2\epsilon_2/c^2)$ and $b=\beta(k^2-\omega^2\epsilon_2/c^2)^2$. Application of (10) leads to the dispersion relation

$$[a\epsilon_1]+\tfrac{1}{2}[b\epsilon_1+\alpha a]\mathcal{E}_x^2(0)=0 \ , \quad (12)$$

where we have again used the square-bracket notation.

The profile for $\mathcal{E}_x$ for this problem can be obtained by integrating (7). We obtain for each medium

$$\mathcal{E}_x=\left[\frac{-2\epsilon_1 a}{\alpha a-3\epsilon_1 b}\right]^{1/2}\operatorname{sech}\left[\left[\frac{\epsilon_1}{a}\right]^{1/2}(z-z_0)\right] \ . \quad (13)$$

Thus it is necessary that $\epsilon_1/a>0$ and $(\alpha a-3\epsilon_1 b)<0$ for localized solutions to exist. These conditions can also be obtained directly from (7) by requiring its right-hand side to be positive definite.

For fixed surface value $\mathcal{E}_x=\mathcal{E}_x(0)$, the constants of integration $z_0^{\mathrm{I}}$ and $z_0^{\mathrm{II}}$ are to be determined by the boundary condition (4). Thus

$$z_0^j=-\left[\frac{a^j}{\epsilon_1^j}\right]^{1/2}\operatorname{sech}^{-1}\left[\left[\frac{\alpha^j-3\epsilon_1^j b^j}{-2\epsilon_1^j a^j}\right]^{1/2}\mathcal{E}_x(0)\right] \ , \quad (14)$$

where $j=$I, II.

One can also verify by applying (8) to (13) that the boundary condition (5) yields a dispersion relation identical to (12).

To conclude, we have derived the general dispersion relation of nonlinear surface polaritons for media whose dielectric tensor is diagonal and independent of the electric field component perpendicular to the surface. Thus our results are particularly applicable to surface plasma waves, which usually satisfy $E_x \gg E_z$, especially near the cutoff frequencies.[5–7]

We have not included in our investigation a possible nonlinear modulation of the field in the direction of propagation. Such a modulation might result in the localization of the waves also parallel to the surface. This problem is still under investigation.

## ACKNOWLEDGMENTS

This research was supported by the Sonderforschungsbereich 162 Plasmaphysik Bochum/Jülich.

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