Thermal magnetic fluctuations and anomalous electron diffusion in a mirror-confined plasma

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The electron test particle cross-field diffusion due to thermally excited magnetostatic modes with ergodic field lines is investigated. Estimate shows that in mirror-confined plasmas, the electron transport (and hence the electron thermal conduction) caused by the magnetostatic mode exceeds the convective as well as the classical transport.

One of the most central points in the understanding of the transport process in magnetically confined plasmas is the anomalous test particle diffusion and electron thermal conductivity. Earlier simulations by Okuda and Dawson[1] have shown that the systems with closed magnetic-field lines exhibit enhanced transport compared to the systems with open lines of force, i.e., systems in which the lines of force fill the volume ergodically. Later on they showed[2] that mirroring can significantly enhance the plasma transport in an open magnetic-field-lines system due to the formation of electrostatic convective cells[3] which will, otherwise, die out quickly and produce little plasma transport. Chu, Chu, and Ohkawa[4] have shown that, other than the electrostatic convective cell, a zero-frequency magnetostatic mode can also exist in a magnetized plasma and its spatially fluctuating magnetic fields provide paths of escape for electrons which would otherwise be anchored to the external magnetic field. Similar to the convective cell, the long-wavelength magnetostatic mode can persist for a long time. For the closed field-line system the electron diffusion (and hence the electron thermal conduction) caused by the magnetostatic modes exceeds the classical value even in an equilibrium plasma. On the other hand, if the mode becomes turbulent, anomalous diffusion due to the resulting stochastic magnetic field can occur. There are several mechanisms which can give rise to enhanced levels of magnetic fluctuations in a plasma.[5] Consequently, increased electron diffusion and electron heat loss is expected to occur.[6]

In this Brief Report we study the effect of mirroring on cross-field plasma transport due to the magnetostatic mode. We have performed an approximate calculation of the test particle diffusion and electron thermal conductivity to be expected in this case, using the fluid model and assuming that only the zero-frequency magnetostatic mode contributes. We find that the electron diffusion rate due to magnetostatic mode scales in this case as $T^{7/2}B_0^{-2}$ compared to the classical scaling $nT^{-1/2}B_0^{-2}$ (where n is the background plasma density, T is the electron temperature, and B_0 is the external magnetic field). This diffusion rate also competes with the convective transport rate[1] which scales as $n^{-1}T^{3/2}$ or $T^{3/2}B_0^{-2}$ depending on the strength of the magnetic field.

The dynamics of the magnetostatic mode in a magnetized plasma is governed by the electron momentum equation in the z direction,

$$\partial_t v_z + \vec{v}\cdot\vec{\nabla} v_z = -\frac{e}{m}E_z + \mu\nabla^2 v_z - \nu v_z \ , \tag{1}$$

where v_z can be obtained from the wave equation for the perturbation A_z,

$$\nabla^2 A_z = \frac{4\pi e n}{c} v_z \ . \tag{2}$$

Here, n is the electron density, ν is the electron collision frequency, and μ is the collective shear viscosity. The corresponding perturbed field quantities are

$$\vec{B}_1 = \vec{\nabla} A_z \times \hat{z} \tag{3}$$

and

$$E_z = -\frac{1}{c}\partial_t A_z \ . \tag{4}$$

These equations will give the linear dispersion relation for the magnetostatic mode as

$$\omega = -\frac{i(\nu+\mu k^2)}{1+\omega_{pe}^2/k^2c^2} \ . \tag{5}$$

The corresponding electron test particle diffusion rate for the above-mentioned case can be written as

$$D = \sum_k \frac{\langle B_1^2(k)\rangle}{B_0^2} v_{\mathrm{the}}^2 \tau(k) \ , \tag{6}$$

where $v_{\mathrm{the}} = (T/m)^{1/2}$ is the electron thermal velocity (which comes after averaging over the Maxwellian equilibrium distribution) and $\tau(k)$ is the lifetime (time for correlations to die out) of the field fluctuations of mode k.

Now for the three-dimensional plasma in thermal equilibrium, we can write the power spectra for fluctuating magnetic field by using the fluctuationdissipation theorem[7]

$$\frac{\langle B_1^2(k)\rangle}{8\pi}L_{\parallel}L_1L_2=\frac{T}{2}\frac{\omega_{pe}^2}{c^2k^2+\omega_{pe}^2}\ , \tag{7}$$

where L_1 and L_2 are the linear dimensions perpendicular to the flux tubes and $L_{\parallel}$ is the one parallel to the flux tubes. If we assume that only k's which contribute to diffusion are those perpendicular to $\vec{B}_0$, then D_m is given by

$$D_m=\frac{1}{L_{\parallel}L_1L_2}\sum_{k_2,k_3}\frac{4\pi T\omega_{pe}^2}{\omega_{pe}^2+c^2k^2}\frac{v_{\text{the}}^2\tau(k)}{B_0^2}\ . \tag{8}$$

To calculate now the diffusion rate in the presence of mirroring, it is reasonable to assume that $\tau(k)$ is the time required for a particle to scatter into the loss cone and be lost from the mirrors. Using the prediction of Kamimura and Dawson[2] for $\tau(k)$, i.e.,

$$\tau(k)=\tau_c\ln(B_{\max}/B_{\min})\ , \tag{9}$$

which shows that the loss term varies logarithmically with the mirror ratio and is proportional to the collision rate τ_c, we can estimate the diffusion rate as

$$D_m=\frac{T^2}{m}\frac{\tau_c}{B_0^2\lambda^2L_{\parallel}}\ln\left[\frac{B_{\max}}{B_{\min}}\right]\ln\left[\frac{1+\lambda_D^2k_{\max}^2}{1+\lambda_D^2k_{\min}^2}\right]\ , \tag{10}$$

where $\lambda=c/\omega_{pe}$ is the skin depth.

Since τ_c scales as $n^{-1}T^{3/2}$, we find that $D_m\sim T^{7/2}B_0^{-2}$ as compared to the classical scaling of $nT^{-1/2}B_0^{-2}$. We may add here that the magnetic mirroring also enhances the transport due to the convective cell and the diffusion rate D_c is given by[2]

$$D_c=\frac{c^2T\tau_c}{B_0^2\lambda_D^2L_{\parallel}\epsilon}\ln\left[\frac{B_{\max}}{B_{\min}}\right]\ln\left[\frac{1+\lambda_D^2k_{\max}^2}{1+\lambda_D^2k_{\min}^2}\right]\ , \tag{11}$$

where the dielectric constant

$$\epsilon=1+\frac{\omega_{pe}^2}{\Omega_e^2}+\frac{\omega_{pi}^2}{\Omega_i^2}$$

and λ_D is the Debye length.

Depending on the magnetic-field strength, the diffusion rate D_c scales as $n^{-1}T^{3/2}$ or $T^{3/2}B_0^{-2}$. The three diffusion rates, therefore, scale as

$$D_c{:}D_m{:}D_\nu=\begin{cases}n^{-1}T^{3/2}{:}\,T^{7/2}B_0^{-2}{:}\,nT^{-1/2}B_0^{-2}, & \text{for } \dfrac{\omega_{pi}^2}{\Omega_i^2}\gg 1\\[2ex] T^{3/2}B_0^{-2}{:}\,T^{7/2}B_0^{-2}{:}\,nT^{-1/2}B_0^{-2}, & \text{for } \dfrac{\omega_{pi}^2}{\Omega_i^2}\ll 1\ .\end{cases}$$

Thus at high temperatures the electron diffusion due to the magnetostatic mode can dominate over the convective as well as collisional diffusion. As the diffusion due to the magnetostatic mode is not ambipolar, it will, therefore, give rise only to enhanced electron shear viscosity and electron heat conductivity but not the particle diffusion. Comparing D_m with D_c we have

$$\frac{D_m}{D_c}=\left[\frac{T}{mc^2}\right]^2\left[1+\frac{\omega_{pe}^2}{\Omega_e^2}+\frac{\omega_{pi}^2}{\Omega_i^2}\right]\times\ln\left[\frac{1+\lambda^2k_{\max}^2}{1+\lambda^2k_{\min}^2}\right]\Big/\ln\left[\frac{1+\lambda_D^2k_{\max}^2}{1+\lambda_D^2k_{\min}^2}\right]\ .$$

For a superstrong magnetic field ($\Omega_i^2\gg\omega_{pi}^2$), the contribution from convective cells dominates over that of magnetostatic modes by a factor $(T/mc^2)^2$. However, for high-β plasmas (e.g., mirror-confined plasmas), the contribution from magnetostatic modes would be larger than that of convective cells. For the thermonuclear condition ($n=10^{14}$ cm^{-3}, $T=10$ keV, $L_{\parallel}=10^3$ cm), the ratio D_m/D_c is 17.9 for $\beta=1$ and 89.5 for $\beta=5$. Remember that our estimate for the cross-field diffusion is made for thermal-equilibrium plasma. Of course the diffusion can be enhanced by many orders of magnitude if the magnetic fluctuations exceed the thermal level and could result in enhanced electron heat conductivity and rapid spreading of current.[8]

In conclusion, we have briefly shown that the magnetic mirroring enhances the electron cross-field test particle diffusion due to the thermally excited magnetostatic mode and the rate of diffusion becomes much larger than the convective as well as the classical diffusion. However, one should be careful in applying this result to a real machine where other effects such as magnetic curvature and inhomogeneities are also present. Such effects may drastically alter the picture. For example, it was pointed out by Nozaki[9] that in an inhomogeneous plasma the magnetostatic mode becomes a finite real frequency mode which may inhibit the electron cross-field transport substantially. The investigtion of such effects will be attempted elsewhere.

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