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Unstable electrostatic beam modes in free-electron-laser systems

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The electrostatic stability of the free-electron laser is studied for a configuration in which a relativistic electron beam propagates through combined helical wiggler and axial guide fields. Instability is found for certain specific parameter regimes which, in the beam frame, is shown to be purely growing and to require the presence of both the wiggler and axial guide fields. The electrostatic stability is also studied for a confuguration which consists of a linearly polarized wiggler and an axial guide field, for which analogous results are found.

The stability properties of a free-electron-laser (FEL) configuration in which a relativistic electron beam propagates through a combined helical wiggler and axial guide field was investigated by many authors.[1-9] It was pointed out by Freund *et al.*[9] that, in addition to the coherent radiation process, the electrostatic beam modes are intrinsically unstable for a specific class of operating parameters. It is our purpose here to expand upon the discussion in Ref. 1 and to discuss the underlying physical mechanism behind such an instability. To this end, we choose to analyze an idealized model which consists of a cold relativistic fluid described by

$$\frac{\partial}{\partial t}n + \vec{\nabla}\cdot(n\vec{v})=0 \ , \tag{1}$$

$$\frac{d}{dt}\vec{v} = -\frac{e}{\gamma m}\left[\left(\overline{\overline{\mathrm{I}}}-\frac{1}{c^2}\vec{v}\vec{v}\right)\cdot\delta\vec{E}+\frac{1}{c}\vec{v}\times\vec{B}\right] \ , \tag{2}$$

$$\frac{d}{dt}\gamma = -\frac{e}{mc^2}\vec{v}\cdot\delta\vec{E} \ , \tag{3}$$

where n and $\vec{v}$ describe the electron density and velocity, respectively, $\gamma\equiv(1-v^2/c^2)^{-1/2}$,

$$B\equiv B_0\hat{e}_z + B_w(\hat{e}_x\cos k_wz+\hat{e}_y\sin k_wz)$$

is the static magnetic field, δE is the electrostatic field (which is assumed to constitute a small perturbation), and $d/dt\equiv\partial/\partial t+\vec{v}\cdot\vec{\nabla}$ is a convective derivative. The equilibrium state to zeroth order in δE is assumed to be homogeneous (i.e., $\nabla n_0=0$), and is characterized by a velocity[10,11] $\vec{v}_0=v_w\hat{e}_1+v_{\parallel}\hat{e}_3$, where $v_{\parallel}$ is a constant, $v_w\equiv\Omega_wv_{\parallel}/(\Omega_0-k_wv_{\parallel})$, $\Omega_{0,w}\equiv|eB_{0,w}/\gamma mc|$, and

$$\hat{e}_1\equiv\hat{e}_x\cos k_wz+\hat{e}_y\sin k_wz \ ,$$

$$\hat{e}_2\equiv-\hat{e}_x\sin k_wz+\hat{e}_y\cos k_wz \ ,$$

and

$$\hat{e}_3\equiv\hat{e}_z$$

define an orthogonal coordinate frame rotating with the wiggler field. Observe that conservation of energy imposes the requirement that $v_w^2+v_{\parallel}^2=(1-\gamma_0^{-2})c^2$.

Under the assumption of plane-wave solutions of the form $\delta E=\delta\hat{E}\exp(ikz-i\omega t)$, Eqs. (2) and (3) can be reduced to the form

$$-i(\omega-kv_{\parallel})\delta\hat{v}_1+(\Omega_0-k_wv_{\parallel})\delta\hat{v}_2=\frac{e}{\gamma_0m}\beta_w\frac{v_{\parallel}^2}{c^2}\delta\hat{E} \ , \tag{4}$$

$$(\Omega_0-k_wv_{\parallel})\delta\hat{v}_1+i(\omega-kv_{\parallel})\delta\hat{v}_2+\Omega_0\beta_w\delta\hat{v}_3 = -\frac{ie}{\gamma_0m}\beta_w\frac{v_{\parallel}^2}{c^2}\frac{k_wv_{\parallel}}{\omega-kv_{\parallel}}\delta\hat{E} \ , \tag{5}$$

$$\Omega_w\delta\hat{v}_2+i(\omega-kv_{\parallel})\delta\hat{v}_3=\frac{e}{\gamma_0\gamma_{\parallel}^2m}\delta\hat{E} \ , \tag{6}$$

to first order in the electric field, where $\beta_w\equiv v_w/v_{\parallel}$ and $\gamma_{\parallel}\equiv(1-v_{\parallel}^2/c^2)^{-1/2}$. The stability properties, therefore, are determined by Eqs. (4)–(6) in conjunction with the following combination of the continuity equations and Poisson's equation,

$$\frac{e}{m}\delta\hat{E}=\frac{i\omega_b^2}{\omega-kv_{\parallel}}\delta\hat{v}_3 \ , \tag{7}$$

where $\omega_b\equiv(4\pi e^2n_0/m)^{1/2}$ is the beam plasma frequency.

It is clear from the z component of the momentum transfer Eq. (6) that in the absence of a wiggler field there is no coupling between the axial and transverse components of the velocity, and the dispersion relation reduces to the well-known positive and negative energy beam modes $\omega=kv_{\parallel}\pm\omega_b/\gamma_0^{1/2}\gamma_{\parallel}$. However, the parallel-transverse coupling in the presence of the static fields can alter the dielectric properties of the medium. Elimination of $\delta\hat{v}_1$ from Eqs. (4) and (5) shows that

$$[(\omega-kv_{\parallel})^2-(\Omega_0-k_wv_{\parallel})^2]\delta\hat{v}_2 = \Omega_0\beta_w\left[i(\omega-kv_{\parallel})\delta\hat{v}_3-\frac{e}{\gamma_0m}\frac{v_{\parallel}^2}{c^2}\delta\hat{E}\right] \ , \tag{8}$$

and $\delta\hat{v}_2$ is nonzero only if both the axial guide and wiggler fields are present. Thus, the modification to the dispersion properties of the electrostatic beam modes which is of interest here is possible only through the combination of both magnetic fields. The instability can be readily demonstrated by combination of Eqs. (6)–(8) to obtain the following dispersion equation:

$$(\omega-kv_{\parallel})^2=\frac{\omega_b^2}{\gamma_0\gamma_{\parallel}^2}\left[1-\frac{\gamma_{\parallel}^2\beta_w^2\Omega_0(\Omega_0-k_wv_{\parallel})}{[(1+\beta_w^2)\Omega_0-k_wv_{\parallel}](\Omega_0-k_wv_{\parallel})-(\omega-kv_{\parallel})^2}\right] \ . \tag{9}$$

The regime considered in Ref. 1, and which is most relevant to current FEL experiments, is that in which $|\omega - kv_{\parallel}| << |\Omega_0 - k_w v_{\parallel}|$. As a result, the dispersion equation is of the form

$$(\omega - kv_{\parallel})^2 = \frac{\omega_b^2}{\gamma_0 \gamma_{\parallel}^2}\Phi \ , \tag{10}$$

where

$$\Phi \equiv 1 - \frac{\gamma_{\parallel}^2 \beta w^2 \Omega_0}{(1+\beta_w^2)\Omega_0 - k_w v_{\parallel}} \ . \tag{11}$$

It is evident that instability results when $\Phi < 0$. Furthermore, the instability is purely growing (i.e., the real part of the frequency is zero) in the beam frame. Additional information on the parameters necessary for instability is given in Ref. 1. Finally, solution of the complete dispersion equation [Eq. (9)] does not qualitatively affect this conclusion.

In order to understand the underlying physics we consider motion in the absence of an axial guide field. It is clear from (6) that the modification of the dielectric properties results from the presence of a $\delta v_2 \hat{e}_2 \times \vec{B}_w$ force in the momentum-transfer equation. Hence, the essential point is to determine a source for an oscillatory velocity in the direction of $\hat{e}_2$. The possible sources for such a motion are evident from the two-component of the momentum-transfer equation [Eq. (2)] and includes a $\vec{\nabla} \times \vec{B}$ force

$$F_{\vec{\nabla}\times\vec{B}} = -\frac{e}{\gamma_0 mc}(\delta\vec{v}\times\vec{B}_w)\cdot\hat{e}_2 = -\Omega_w \delta v_3 \ , \tag{12}$$

convection (note that $\nabla \hat{e}_1 = k_w \hat{e}_2$) due to the centripetal force arising from the rotation (or gradient) of the wiggler field,

$$F_{\rm cent} = -(\delta\vec{v}\cdot\vec{\nabla}\,\vec{v}_0)\cdot\hat{e}_2 = -k_w v_w \delta v_3 \ , \tag{13}$$

as well as a relativistic contribution which arises from the variation in the total energy. When no axial field is present, $v_w = -\Omega_w/k_w$ and the convection exactly balances the $\vec{\nabla}\times\vec{B}$ force with the result that no net velocity in the $\hat{e}_2$ direction occurs. The relativistic contribution is the sole remaining source, but it can be shown to drive oscillatory motion only in the $\hat{e}_1$ direction. However, the axial guide field tends to increase the transverse velocity (i.e., v_w), and results in enhanced convection as well as a net source which drives an oscillation in the $\hat{e}_2$ direction. As mentioned previously, the finite δv_2 causes a $\delta\vec{v}\times\vec{B}_w$ force in the axial direction which affects partial bunching and modifies the dispersive properties of the medium. As long as $k_w v_{\parallel} > \Omega_0$ the convection acts to oppose the $\vec{\nabla}\times\vec{B}$ force (12), in part, and causes an effective enhancement in the plasma frequency (10). In contrast, when $k_w v_{\parallel} < \Omega_0$ the direction of the $\hat{e}_1$ component of the zeroth-order transverse velocity is reversed (i.e., $v_w > 0$), and convection tends to enhance the effect of the $\vec{\nabla}\times\vec{B}$ force. It is in this regime that instabliity is found.

The actual motion in the case in which instability occurs may be summarized as follows. The electric field drives a fluctuation in the axial velocity which, in turn, causes a net velocity fluctuation in the $\hat{e}_2$ direction by the combined action of the Lorentz force and convention. This velocity then feeds back upon the axial velocity via the Lorentz force ($\delta v_2 \hat{e}_2 \times \vec{B}_w$). The feedback provides the dominant contribution to the axial velocity when

$$(1 - \gamma_{\parallel}^2 v_w^2/c^2)\Omega_0 < k_w v_{\parallel} < \Omega_0 \ , \tag{14}$$

(i.e., $\Phi < 0$) and the net effect of the electric field is to drive axial velocity fluctuations counter to that produced by the "direct" action of the electric field. The combined action of the axial guide and wiggler fields results in a phase shift in the axial motion which causes electron bunching to occur in such a way that the electric field is enhanced. Thus, although this is a nonrelativistic effect, the system acts as though the electrons had a negative mass.

It is also of interest to determine whether an analogous instability exists for a configuration in which the static magnetic fields consist of a linearly polarized wiggler in combination with an axial guide field. In this case we represent the magnetic field in the form $\vec{B} = B_0 \hat{e}_z + B_w \sin k_w z \hat{e}_y$. The equilibrium orbits in this field geometry are

$$v_x = \alpha v_{\parallel} \cos k_w z \ ,$$

$$v_y = \alpha \Omega_0 (k_w v_{\parallel})^{-1} v_{\parallel} \sin k_w z \ ,$$

and

$$v_z = v_{\parallel} \ ,$$

where $\alpha \equiv \Omega_w k_w v_{\parallel}/(\Omega_0^2 - k_w^2 v_{\parallel}^2)$ and oscillatory terms in $2k_w z$ (and higher) have been neglected. Conservation of energy, therefore, imposes the constraint $(1+\beta_\perp^2)v_{\parallel}^2 = (1-\gamma_0^{-2})c^2$, where $\beta_\perp^2 \equiv \frac{1}{2}\alpha^2(1+\Omega_0^2/k_w^2 v_{\parallel}^2)$. Perturbation analysis of Eqs. (1)–(3) about this equilibrium state to first order in δE, and combination of the result with Eq. (7), therefore, yields the following dispersion equation:

$$(\omega - kv_{\parallel})^2 = \frac{\omega_b^2}{\gamma_0\gamma_{\parallel}^2}\left[1 - \frac{\gamma_{\parallel}^2 \beta_\perp^2 \Omega_0^2 (\Omega_0^2 + 3k_w^2 v_{\parallel}^2)}{(1+\beta_\perp^2)\Omega_0^4 - k_w^2 v_{\parallel}^2 (k_w^2 v_{\parallel}^2 - 3\beta_\perp^2 \Omega_0^2)}\right] \ , \tag{15}$$

in the limit in which $|\omega - kv_{\parallel}| << |\Omega_0 - k_w v_{\parallel}|$. This is analogous to the dispersion equation for the helical wiggler field (10), and instability is found when

$$\Omega_0^4 - \beta_\perp^2(\gamma_{\parallel}^2 - 1)\Omega_0^2(\Omega_0^2 + 3k_w^2 v_{\parallel}^2) < k_w^4 v_{\parallel}^4 < \Omega_0^4 \ . \tag{16}$$

As in the case of the helical wiggler, the instability is purely growing in the beam frame, and arises from the same physical mechanism.

The central question raised by this analysis is how the instability will affect the performance of the FEL. On the basis of a linearized theory it has been shown that the growth rates for the amplification of radiation are large (and exceed those found in the limit as $B_0 \to 0$), and the bandwidth is enhanced for the range of parameters leading to the electrostatic beam instability. However, since it might be expected that the electrostatic instability will lead to degradation of beam quality in the nonlinear regime, the effects of this instability on the saturation of the FEL are of prime importance. This question has been addressed by means of a particle simulation of a cold beam in an FEL amplifier,[12] and it was found that (for the parameters considered), the saturation efficiency is greatest when the electrostatic instability is present. It should be remarked that this conclusion is reinforced by experimental results[13,14] in which maximum power was observed for parameters corresponding to the electrostatic instability. Thus, while the question of the effects of the electrostatic beam instability on the FEL has not been conclusively answered (i.e., a more complete parameter study of the nonlinear saturation efficiency is required, as in a knowledge of the effects of a finite velocity spread), it should not be concluded that these effects are necessarily deleterious.

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