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Vortex and magnetostatic-mode turbulence produced by a Buchsbaum mode

H. U. Rahman* and P. K. Shukla
Institut für Theoretische Physik, Ruhr-Universität Bochum, 4630 Bochum, Federal Republic of Germany

S. G. Tagare
Department of Mathematics and Computer/Information Sciences, University of Hyderabad, Hyderabad, India
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It is found that the modulational instability of a finite-amplitude Buchsbaum mode can give rise to enhanced vortex (convection cells) and magnetostatic modes in a D-T fusion plasma. The relevance of our investigation to the anomalous particle and heat losses in a Tokamak device is briefly pointed out.

I. INTRODUCTION

It is well established that a plasma, consisting of electrons and two kinds of ions, supports a Buchsbaum or ion-ion hybrid wave.[1,2] The latter propagates almost perpendicular to the external magnetic field, and the wave frequency ω, which is much smaller than the electron gyrofrequency Ω_e, lies between the gyrofrequency of the two species of ions. Since a typical fusion plasma consists of two ion species (e.g., D and T), Buchsbaum resonances are frequently encountered in a tokamak device. The possibility of ion heating by normal resonance absorption of ion-ion hybrid waves has been documented in the past.[3–5].

Although, in a practical situation, one has to worry about the mechanism of introducing ion-ion hybrid waves from an external source, there now exist numerous ways by which this goal can be accomplished. For example, large-amplitude Buchsbaum modes can originate during the lower-hybrid wave heating,[6,7] or in the presence of field-aligned currents[8] in a plasma with impurities. Taking the external wave to be at the Buchsbaum frequency, Satya *et al.*[9] and Bujarbarua *et al.*[10] investigated parametric instabilities involving the low-frequency decay product as the drift wave, ion-acoustic wave, or the "cold" ion-Bernstein mode. Such nonlinear effects can lead to significant plasma heating or enhanced particle losses depending on the excitation of particular low-frequency oscillations in plasmas.

In this paper, we investigate a new kind of parametric interaction of the Buchsbaum mode. In particular, we consider nonlinear interaction of the latter with zero-frequency vortex,[11] and magnetostatic[12] modes.

This study is motivated by the fact that the zero-frequency modes[11,12] can cause cross-field particle diffusion even in an equilibrium plasma. The transport properties of the plasma are expected to be enhanced once the energy density of the dampened modes far exceeds the thermal level. Here, we present a novel mechanism, i.e., the modulational instability, which could be responsible for the generation of enhanced near-zero-frequency turbulence in the presence of the Buchsbaum mode.

Section II contains basic equations and appropriate particle velocities in different frequency regimes. We decompose all the field quantities in two parts, namely, a high-frequency Buchsbaum mode and low-frequency vortex or magnetostatic modes. A coupled set of equations describing their nonlinear interaction is derived. The linear theory of modes under consideration is reviewed in Sec. III. Here, for completeness, we also briefly present the expressions for the diffusion coefficients associated with the zero-frequency modes. In Sec. IV we carry out a normal mode analysis on the basic equations. Nonlinear dispersion relations are derived in Sec. V. Expressions for the growth rate are obtained analytically. For illustrative purposes, we point out the relevance of our work to a tokamak device. Finally, Sec. VI contains a brief summary and problems which have to be solved in the future.

II. BASIC EQUATIONS

Consider a cold magnetized plasma consisting of electrons (mass m_e, charge $-e$), and two species of ions (mass m_a,m_b, charge Z_ae,Z_be) embedded in a uniform magnetic field $\vec{B}_0 = B_0\hat{z}$. Nonlinear interactions of Buchsbaum modes with convective cells[11] and the magnetostatic modes[12,13] are governed by

$$\partial_t n_e + \vec{\nabla}\cdot n_e \vec{v}_e = 0 , \tag{1}$$

$$(\partial_t - \mu_e \nabla^2 + \nu_e + \vec{v}_e\cdot\vec{\nabla})\vec{v}_e = -\frac{e}{m_e}\left[\vec{E} + \frac{1}{c}\vec{v}_e \times (\vec{B}_0 + \vec{b})\right] - \frac{T_e}{m_e n_e}\nabla n_e , \tag{2}$$

$$\partial_t n_\alpha + \vec{\nabla}\cdot n_\alpha \vec{v}_\alpha = 0 , \tag{3}$$

$$(\partial_t - \mu_\alpha \nabla^2 + \nu_\alpha + \vec{v}_\alpha\cdot\vec{\nabla})\vec{v}_\alpha = \frac{Z_\alpha e}{m_\alpha}\left[\vec{E} + \frac{1}{c}\vec{v}_\alpha \times (\vec{B}_0 + \vec{b})\right] , \tag{4}$$

$$\vec{\nabla}\cdot\vec{E} \equiv 4\pi e(Z_a n_a + Z_b n_b - n_e) , \tag{5}$$

$$\vec{b} = \vec{\nabla}\times\vec{A} , \quad \vec{E} = -\frac{1}{c}\partial_t \vec{A} - \nabla\Phi , \tag{6}$$

$$\vec{\nabla}\times\vec{b} = \frac{4\pi}{c}\vec{J} . \tag{7}$$

In Eqs. (1)—(7), $\alpha = a,b$ for each ion species, $\vec{J}$ is the total plasma current, $\vec{b}$ is the wave magnetic field, Φ and $\vec{A}$ are the scalar and vector potentials, respectively, $n,\vec{v}$ are the density and fluid velocity, and the subscripts e,α denote corresponding quantities associated with electrons or ion species. Furthermore, ν and μ are the collision frequency and the gryoviscosity, respectively.

In the following, we first derive the wave equation for the ion-hybrid waves, taking into account the interaction with zero-frequency electrostatic convective cells. Near the Buchsbaum resonances, the waves are nearly electrostatic and are governed by the two-fluid and Poisson equations. Also, for simplicity, we assume ions to be singly charged, i.e., $Z_a = 1 = Z_b$.

Decomposing the field quantities into their high- and low-frequency components, we have

$$\begin{aligned} n_\alpha &= N_\alpha + n_\alpha^h + n_\alpha^l , \\ n_e &= N_e + n_e^h + n_e^l , \\ \vec{v}_j &= \vec{v}_j^h + \vec{v}_j^l , \\ \vec{E} &= -\nabla\phi - \nabla\psi , \end{aligned} \tag{8}$$

where $j = e,\alpha$, $N_e = N_a + N_b$, N_e and N_α are the unperturbed densities, and the superscripts h and l and potentials ϕ and ψ denote the high- and low-frequency components, respectively.

For the Buchsbaum mode, we have

$$\omega_0 \sim \Omega_\alpha \equiv \frac{eB_0}{m_\alpha c} \ll \Omega_e \left[\Omega_e = \left|\frac{eB_0}{m_e c}\right|\right] .$$

Thus, the motion of the ions is very sensitive to the external magnetic field. The magnetized electrons execute $\vec{E}\times\vec{B}_0$ motion, and also suffer a polarization drift, as well as a drift along $B_0\hat{z}$. Applying these facts in the basic equations, we get the expressions for the velocities corresponding to Buchsbaum modes:

$$\vec{v}_{e\perp}^h = -\frac{c}{B_0}\vec{\nabla}_\perp\phi\times\hat{z} + \frac{c}{B_0\Omega_e}\partial_t\vec{\nabla}_\perp\phi - \frac{c^2}{B_0^2\Omega_e}[(\vec{\nabla}_\perp\psi\times\hat{z}\cdot\vec{\nabla})\vec{\nabla}_\perp\phi + (\vec{\nabla}_\perp\phi\times\hat{z}\cdot\vec{\nabla}_\perp)\vec{\nabla}_\perp\psi] , \tag{9}$$

$$\partial_t^2 v_{e\parallel}^h = \frac{e}{m_e}\nabla_\parallel\partial_t\phi + \frac{ec}{m_e B_0}(\vec{\nabla}_\perp\psi\times\hat{z}\cdot\vec{\nabla}_\perp)\nabla_\parallel\phi , \tag{10}$$

where we have used $\vec{v}_{e\perp}^l = -c\vec{\nabla}_\perp\psi\times\hat{z}/B_0$, the details of which shall follow later.

Similarly, for two kinds of ions, we find

$$\begin{aligned}(\partial_t^2 + \Omega_\alpha^2)\vec{v}_{\alpha\perp}^h = {} & -\frac{e\Omega_\alpha}{m_\alpha}\vec{\nabla}_\perp\phi\times\hat{z} - \frac{e}{m_\alpha}\partial_t\vec{\nabla}_\perp\phi + \frac{ec}{m_\alpha B_0}(\vec{\nabla}_\perp\psi\times\hat{z}\cdot\vec{\nabla}_\perp)\vec{\nabla}_\perp\phi \\ & -\frac{c}{B_0}[(\partial_t\vec{v}_{\alpha\perp}^h\cdot\vec{\nabla}_\perp)\vec{\nabla}_\perp\psi\times\hat{z} - 2(\vec{\nabla}_\perp\psi\times\hat{z}\cdot\vec{\nabla}_\perp)\partial_t\vec{v}_{\alpha\perp}^h + \Omega_\alpha(\vec{v}_{\alpha\perp}^h\cdot\vec{\nabla}_\perp)\vec{\nabla}_\perp\psi]\end{aligned} \tag{11}$$

and

$$\partial_t^2 v_{\alpha\parallel}^h = -\frac{e}{m_\alpha}\nabla_\parallel\partial_t\phi - \frac{ec}{m_\alpha B_0}(\vec{\nabla}_\perp\psi\times\hat{z}\cdot\vec{\nabla}_\perp)\nabla_\parallel\phi . \tag{12}$$

The equations of continuity are

$$\partial_t^2 n_e^h + \vec{\nabla}\cdot\partial_t\vec{v}_e^h + \frac{c}{B_0}\vec{\nabla}_\perp\psi\times\hat{z}\cdot\vec{\nabla}_\perp(\vec{\nabla}_\perp\cdot\vec{v}_e^h) = 0 \tag{13}$$

and

$$\partial_t^2 n_\alpha^h+\vec\nabla\cdot\partial_t\vec v_\alpha^h+\frac{c}{B_0}\vec\nabla_\perp\psi\times\hat z\cdot\vec\nabla_\perp(\vec\nabla_\perp\cdot\vec v_\alpha^h)=0\ , \tag{14}$$

where now the densities are normalized by the average plasma density N_e. Substituting Eqs. (13) and (14) into Poisson's equation, and using Eqs. (9)—(12), we obtain

$$[(1+\omega_{pe}^2/\Omega_e^2)\nabla_\perp^2\partial_t^2+(\partial_t^2+\omega_{pe}^2)\nabla_{||}^2]\partial_t\phi-\sum_{\alpha=a,b}4\pi e N_\alpha\partial_t^2(\vec\nabla\cdot\vec v_\alpha^h)$$
$$=-\frac{c}{B_0}\omega_{pe}^2(\vec\nabla_\perp\psi\times\hat z\cdot\vec\nabla_\perp)\nabla_{||}^2\phi+\sum_{\alpha=a,b}\frac{\omega_{p\alpha}^2}{\Omega_\alpha}(\vec\nabla_\perp\psi\times\hat z\cdot\vec\nabla_\perp)\partial_t(\vec\nabla_\perp\cdot\vec v_{\alpha\perp}^h)\ . \tag{15}$$

Equations (11) and (15) constitute the wave equation for ion-ion hybrid waves.

For electrostatic convective cells, we have $\omega \ll \Omega_\alpha$ and $\nabla_{||}=0$. Thus, the relevant velocities are given by

$$\vec v_{e\perp}^l=-\frac{c}{B_0}\nabla_\perp\psi\times\hat z+\frac{c}{B_0\Omega_e}(\partial_t-\mu_e\nabla^2)\nabla_\perp\psi-\frac{c^2}{B_0^2\Omega_e}\langle(\vec\nabla_\perp\phi\times\hat z\cdot\vec\nabla_\perp)\vec\nabla_\perp\phi\rangle\ , \tag{16}$$

$$\vec v_{\alpha\perp}^l=-\frac{c}{B_0}\vec\nabla_\perp\psi\times\hat z-\frac{c}{B_0\Omega_\alpha}(\partial_t-\mu_\alpha\nabla^2)\vec\nabla_\perp\psi-\frac{1}{\Omega_\alpha}\langle\vec v_{\alpha\perp}^h\cdot\vec\nabla_\perp\vec v_{\alpha\perp}^h\times\hat z\rangle\ , \tag{17}$$

where $\mu_\alpha=0.3\nu_\alpha\rho_\alpha^2$, $\rho_\alpha=v_{t\alpha}/\Omega_\alpha$, $v_{t\alpha}=(T_\alpha/m_\alpha)^{1/2}$ is the thermal velocity of species α. In Eqs. (16) and (17), we have included the contributions of the slowly varying ponderomotive force driven velocities which arise from the beating of the two Buchsbaum modes. The angular brackets denote averaging over one high-frequency wave period. Furthermore, self-interaction nonlinearities are not included here. On using the equations

$$\partial_t n_e^l+N_e\vec\nabla_\perp\cdot\vec v_{e\perp}^l+\vec\nabla_\perp\cdot\langle n_e^h\vec v_e^h\rangle=0\ , \tag{18}$$

$$\partial_t n_\alpha^l+N_\alpha\vec\nabla_\perp\cdot\vec v_{\alpha\perp}^l+\vec\nabla_\perp\cdot\langle n_\alpha^h\vec v_\alpha^h\rangle=0\ , \tag{19}$$

$$\nabla^2\psi=4\pi e(n_e^l-n_a^l-n_b^l)\ , \tag{20}$$

together with (16) and (17), we readily obtain the equation for convective cells

$$\left[1+\omega_{pe}^2/\Omega_e^2+\sum_{\alpha=a,b}\frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\right]\partial_t\nabla_\perp^2\psi-\sum_{\alpha=a,b}\frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\mu_\alpha\nabla_\perp^4\psi$$
$$=\frac{c}{B_0}\frac{\omega_{pe}^2}{\Omega_e^2}\vec\nabla_\perp\cdot\langle(\vec\nabla_\perp\phi\times\hat z\cdot\vec\nabla_\perp)\vec\nabla_\perp\phi\rangle+\frac{\omega_{pe}^2}{\Omega_e^2}\langle\vec\nabla_\perp\phi\times\hat z\cdot\vec\nabla_\perp n_e^h/N_e\rangle$$
$$+\vec\nabla_\perp\cdot\sum_{\alpha=a,b}\omega_{p\alpha}^2\frac{m_\alpha}{e}\left\langle\frac{n_\alpha^h}{N_\alpha}\vec v_{\alpha\perp}^h-\frac{1}{\Omega_\alpha}(\vec v_{\alpha\perp}^h\cdot\vec\nabla_\perp\vec v_{\alpha\perp}^h\times\hat z)\right\rangle\ , \tag{21}$$

where the lowest-order linear as well as nonlinear terms are retained.

Next, we obtain the relevant equations describing nonlinear interaction of magnetostatic modes with the Buchsbaum mode. For the magnetostatic modes,[12,13] ions do not play any role and the electron motion is primarily along the external magnetic field. We thus have the following equations which govern the dynamics of driven magnetostatic modes:

$$\partial_t v_{e||}^m+(\vec v_e^h\cdot\vec\nabla)v_{e||}^h=-\frac{e}{m_e}E_{||}^m-\nu_e v_{e||}^m+\mu_e\nabla^2 v_{e||}^m\ ,\quad E_{||}^m=-\frac{1}{c}\partial_t A_{||}\ ,\quad\text{and } B_\perp^m=\vec\nabla_\perp A_{||}\times\hat z\ . \tag{22}$$

Here, the superscript m denotes field quantities associated with the magnetostatic modes.

Using the relations

$$v_{e||}^m=\frac{c}{4\pi N_e}\nabla^2 A_{||}\ , \tag{23}$$

which follow from (6) and (7),

$$\vec v_{e\perp}^h=-\frac{c}{B_0}\vec\nabla\phi\times\hat z+\frac{c}{B_0\Omega_e}\partial_t\vec\nabla_\perp\phi\ , \tag{24}$$

and

$$\partial_t v_{e\parallel}^h = \frac{e}{m_e}\nabla_\parallel \phi \ , \tag{25}$$

we find from (22) the wave equation for the magnetostatic mode in the presence of Buchsbaum modes:

$$(\lambda^2\nabla_\perp^2 - 1)\partial_t A_\parallel + \lambda^2(\nu_e - \mu_e \nabla_\perp^2)\nabla_\perp^2 A_\parallel = \frac{c}{\Omega_e}\left\langle \left[(\vec{\nabla}_\perp \phi \times \hat{z}\cdot\vec{\nabla}_\perp) v_{e\parallel}^h - \frac{1}{\Omega_e}\partial_t \vec{\nabla}_\perp \phi \cdot \vec{\nabla}_\perp v_{e\parallel}^h \right] \right\rangle , \tag{26}$$

where $\lambda = c/\omega_{pe}$ is the inertial length, and $\omega_{pe} = (4\pi e^2 N_e/m_e)^{1/2}$ is the electron plasma frequency.

The high-frequency velocities are affected by the magnetic field $B_\perp^m$ of the magnetostatic modes. Straightforward algebra yields a dynamical equation for the Buchsbaum mode taking into account its interaction with the magnetostatic mode. We find

$$[(1+\omega_{pe}^2/\Omega_e^2)\nabla_\perp^2\partial_t^2 + (\partial_t^2 + \omega_{pe}^2)\nabla_\parallel^2]\phi - 4\pi e N_\alpha \partial_t(\vec{\nabla}_\perp \cdot \vec{v}_{\alpha\perp}^h) = \sum_\alpha \frac{\omega_{p\alpha}^2}{c}\nabla_\parallel(\vec{v}_{\alpha\perp}^h \cdot \vec{\nabla}_\perp A_\parallel) - \frac{\omega_{pe}^2}{B_0}\vec{\nabla}_\perp(\nabla_\parallel \phi)\cdot \vec{\nabla}_\perp A_\parallel \times \hat{z}$$
$$- \frac{\omega_{pe}^2}{B_0}\nabla_\parallel[\vec{\nabla}_\perp \phi \times \hat{z}\cdot \vec{\nabla}_\perp(\lambda^2\nabla^2 A_\parallel - A_\parallel)] , \tag{27}$$

where

$$(\partial_t^2 + \Omega_\alpha^2)\partial_t \vec{v}_{\alpha\perp}^h = \frac{e\Omega_\alpha}{m_\alpha}\vec{\nabla}_\perp \partial_t \phi \times \hat{z} - \frac{e}{m_\alpha}\partial_t^2 \vec{\nabla}_\perp \phi - \frac{e}{m_\alpha}\frac{\Omega_\alpha}{B_0}(\Omega_\alpha \nabla_\parallel \phi \vec{\nabla}_\perp A_\parallel \times \hat{z} + \nabla_\parallel \partial_t \phi \vec{\nabla}_\perp A_\parallel) , \tag{28}$$

and $\vec{v}_{e\perp}^m = 0$ has been used.

Equations (25)—(28) describe the nonlinear interaction of Buchsbaum modes with the magnetostatic modes.

III. LINEAR MODES

In the absence of nonlinear interactions, we have the Buchsbaum mode, convection cells, and magnetostatic modes as normal modes of the plasma. First, we consider the Buchsbaum mode. Fourier transforming (11) and (15), we obtain

$$k^2 = -\frac{\omega_{pe}^2}{\Omega_e^2}k^2 + \frac{\omega_{pe}^2}{\omega^2}k_\parallel^2 + \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2 - \Omega_\alpha^2}k_\perp^2 + \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}k_\parallel^2 , \tag{29}$$

where $k^2 = k_\perp^2 + k_\parallel^2$ and $\omega_{p\alpha}^2 = 4\pi N_\alpha e^2/m_\alpha$. For perpendicular propagation, i.e., $k_\parallel = 0$, and

$$\omega_{pe}^2 \gg \Omega_e^2 \gg \omega^2 ,$$

we find

$$\frac{\omega_{pa}^2}{\omega^2 - \Omega_a^2} + \frac{\omega_{pb}^2}{\omega^2 - \Omega_b^2} = \frac{\omega_{pe}^2}{\Omega_e^2} . \tag{30}$$

When $\omega \sim \Omega_a, \Omega_b$, Eq. (30) has the solution

$$\omega^2 = \omega_B^2 = \frac{\omega_{pe}^2 \Omega_b^2 + \omega_{pb}^2 \Omega_a^2}{\omega_{pe}^2 + \omega_{pb}^2} , \tag{31}$$

where ω_B denotes the Buchsbaum resonance frequency.[1,2]

For a plasma with

$$\frac{m_a}{m_b} \ll \frac{N_b}{N_a} \ll \frac{m_b}{m_a} , \tag{32}$$

ω_B^2 takes the form

$$\omega_B^2 \simeq \omega_{pb}^2 \Omega_a^2/\omega_{pa}^2 . \tag{33}$$

This is also obtained by assuming $\Omega_a^2 \gg \omega^2 \gg \Omega_b^2$ in Eq. (30), and requiring the solution to be consistent with their assumptions.

Second, we consider the convective cells in the presence of two species of ions. Neglecting nonlinear terms in Eq. (21) and Fourier transforming, we obtain

$$\omega = -ik_\perp^2 \sum_\alpha \frac{\mu_\alpha}{1 + \Omega_\alpha^2/\omega_{p\alpha}^2} . \tag{34}$$

We see that the ion gyrofrequency gives rise to a normal mode whose electric field damps out exponentially. The equilibrium electric field energy density in the convective cells is

$$\frac{\langle E^2 \rangle}{8\pi} = \frac{T}{2}\left[1 + \frac{\omega_{pe}^2}{\Omega_e^2} + \frac{\omega_{pa}^2}{\Omega_a^2} + \frac{\omega_{pb}^2}{\Omega_b^2}\right] , \tag{35}$$

where T is the plasma temperature, and we have assumed a Maxwellian distribution of particles. Since both the electrons and ions move with the velocity $\vec{v}_1 = c\vec{E} \times \hat{z}/B_0$, the cross-field diffusion coefficient is given by[11]

$$D=\int_0^\infty \langle \vec{v}_1(t),\vec{v}_1(t+\tau)\rangle d\tau \tag{36a}$$

$$=\frac{cT^{1/2}}{B_0}\left[\left[1+\frac{\omega_{pa}^2}{\Omega_a^2}+\frac{\omega_{pb}^2}{\Omega_b^2}\right]^{-1}\frac{2}{L_{\|}}\times\ln\left[\frac{L_\perp}{2\pi\lambda_e}\right]\right]^{1/2}, \tag{36b}$$

where $L_{\|}$ and $L_\perp$ are the parallel and perpendicular dimensions of the system, and λ_e is the electron Debye length.

Third, the linear dispersion of the zero-frequency magnetostatic mode is obtained from (26). Neglecting nonlinear terms, and Fourier analyzing, one obtains[12]

$$\omega=-\frac{i(\nu_e+\mu_e k^2)}{1+\omega_{pe}^2/c^2k^2}. \tag{37}$$

As discussed earlier, ion motion does not enter in the mode dynamics. The energy of the mode is contained in the wave magnetic field and the particle motion along the external magnetic field. A test particle streaming along $B_0\hat{z}$ with velocity v_0 suffers a perpendicular drift given by

$$\vec{v}_1=v_0\vec{b}/B_0 \tag{38}$$

which describes the particle motion along the perturbation magnetic field $\vec{b}$. Using (36a), (38), and the equilibrium magnetic field energy density[12]

$$\frac{\langle b^2\rangle}{8\pi}=\frac{T}{2}\frac{c^2k^2}{c^2k^2+\omega_{pe}^2}, \tag{39}$$

we find[12]

$$D_m^e=\frac{T}{B_0}\left[\frac{2}{m_eL_{\|}}\ln\left[\frac{L\omega_{pe}}{2\pi c}\right]\right]^{1/2}, \tag{40}$$

where $2\pi cL/\omega_{pe}\ll 1$, and $D_m^i \ll D_m^e$. A hypothetical relation between the electron thermal conductivity K_T and D_m^e is $K_T=nD_m^e$.

In the next section, we discuss the mechanism of parametric interaction which can enhance the level of the zero-frequency modes, as given above.

IV. NONLINEAR DISPERSION RELATIONS

A. Electrostatic convective cells

First, we consider the parametric instability in which the electrostatic convective cells $(\omega,\vec{k})$ and the high-frequency sidebands $(\omega_\pm,\vec{k}_\pm)$ are excited in the presence of a finite amplitude Buchsbaum mode $(\omega_0,\vec{k}_0)$. Accordingly, we split the high-frequency fields into three components, namely, the pump and the upper and lower sidebands. Thus,

$$\begin{bmatrix}\phi^h\\ \vec{v}_\alpha^h\end{bmatrix}=\begin{bmatrix}\phi_0\\ \vec{v}_{\alpha 0}\end{bmatrix}\exp(i\vec{k}_0\cdot\vec{x}-i\omega_0 t)+\text{c.c.}+\begin{bmatrix}\phi_+\\ \vec{v}_{\alpha +}\end{bmatrix}\exp(i\vec{k}_+\cdot\vec{x}-i\omega_+ t)+\begin{bmatrix}\phi_-\\ \vec{v}_{\alpha -}\end{bmatrix}\exp(i\vec{k}_-\cdot\vec{x}-i\omega_- t), \tag{41}$$

where $\omega_\pm=\omega\pm\omega_0$, and $\vec{k}_\pm=\vec{k}\pm\vec{k}_0$.

The low-frequency potential ψ is assumed to have a space-time dependence of the form

$$\psi=\psi\exp(i\vec{k}\cdot\vec{x}-i\omega t). \tag{42}$$

Inserting (41) and (42) into (11), (15), and (21), and matching the phasor, we obtain, after some algebra,

$$A_\pm\phi_\pm=B_\pm\psi\begin{bmatrix}\phi_0\\ \phi_0^*\end{bmatrix}, \tag{43}$$

$$c\psi=D_+\phi_+\phi_0^*+D_-\phi_-\phi_0, \tag{44}$$

where we have defined

$$A_\pm=-i\omega_\pm\left[\left[1+\frac{\omega_{pe}^2}{\Omega_e^2}-\sum_{\alpha=a,b}\frac{\omega_{p\alpha}^2}{\omega_\pm^2-\Omega_\alpha^2}\right]\omega_\pm^2k_\pm^2-\omega_{pe}^2k_{\pm\|}^2\right], \tag{45}$$

$$B_{\pm}=\mp\frac{c}{B_0}\omega_{pe}^2(\vec{k}\times\hat{z}\cdot\vec{k}_0)k_{0\parallel}^2$$
$$+\frac{c}{B_0}\omega_0^2\sum_\alpha\frac{\omega_{p\alpha}^2}{(\omega_0^2-\Omega_\alpha^2)}\left\{(\vec{k}\times\hat{z}\cdot\vec{k}_0)\left[\left[1-\frac{2\omega_0^2\pm\Omega_\alpha^2)}{\omega_0^2-\Omega_\alpha^2}\right](\vec{k}\cdot\vec{k}_{0\perp}\pm k_{0\perp}^2)+\frac{\omega_0^2}{\omega_0^2-\Omega_\alpha^2}\vec{k}\cdot\vec{k}_{0\perp}\pm k_{0\perp}^2\right]\right.$$
$$\left.\mp\frac{i\omega_0\Omega_\alpha}{\omega_0^2-\Omega_\alpha^2}[(\vec{k}\times\hat{z}\cdot\vec{k}_0)^2\pm(\vec{k}\cdot\vec{k}_0)(\vec{k}\cdot\vec{k}_{0\perp}\pm k_{0\perp}^2)]\right\}, \tag{46}$$

$$C=ik^2\left[1+\frac{\omega_{pe}^2}{\Omega_e^2}+\sum_{\alpha=a,b}\frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\right](\omega+i\Gamma_c), \tag{47}$$

$$D_{\pm}=\frac{c}{B_0}\frac{\omega_{pe}^2}{\omega_0^2}(\vec{k}\times\hat{z}\cdot\vec{k}_0)(2\vec{k}\cdot\vec{k}_0\pm k^2)\frac{k_{0\parallel}^2}{k_{0\perp}^2}$$
$$+\frac{c}{B_0}\sum_\alpha\frac{\omega_{p\alpha}^2}{(\omega_0^2-\Omega_\alpha^2)^2}\{2(\vec{k}\times\hat{z}\cdot\vec{k}_0)[\omega_0^2(\vec{k}\cdot\vec{k}_0\pm k_{0\perp}^2)+\Omega_\alpha^2(2\vec{k}\cdot\vec{k}_{0\perp}\mp k_{0\perp}^2\pm k^2)]$$
$$+i\omega_0\Omega_\alpha[k_{0\perp}^2(3\vec{k}\cdot\vec{k}_{0\perp}\pm k_{0\perp}^2\pm 2k^2)\pm 2(\vec{k}\cdot\vec{k}_0\times\hat{z})^2]\}, \tag{48}$$

where

$$\Gamma_c=\left[\sum_\alpha\frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\mu_\alpha k^2\right]\left[1+\frac{\omega_{pe}^2}{\Omega_e^2}+\sum_\alpha\frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\right]^{-1}.$$

Combining (43) and (44), we get the nonlinear dispersion equation

$$C=\left[\frac{B_+D_+}{A_+}+\frac{B_-D_-}{A_-}\right]|\phi_0|^2. \tag{49}$$

This equation describes the modulational instability of the ion-ion hybrid wave involving the long-lived convective cells as the modulating agent.

B. Magnetostatic mode

For the excitation of the magnetostatic mode, again, we split the high-frequency fields into three components. The low-frequency vector potential $A_{\parallel}$ is assumed to have space-time dependence as given in Eq. (42). Here, equations similar to (43) and (44) can be obtained, with the replacement $\psi\rightarrow A_{\parallel}$, and

$$A_{\pm}=\left[1+\frac{\omega_{pe}^2}{\Omega_e^2}-\sum_{\alpha=a,b}\frac{\omega_{p\alpha}^2}{\omega_{\pm}^2-\Omega_\alpha^2}\right]\omega_{\pm}^2k_{\pm\perp}^2-\omega_{pe}^2k_{0\parallel}^2, \tag{50}$$

$$B_{\pm}=\frac{i\omega_{pe}^2k_{0\parallel}}{B_0}(2+\lambda^2k^2)\vec{k}\times\hat{z}\cdot\vec{k}_0-\sum_{\alpha=a,b}\frac{\omega_{p\alpha}^2\Omega_\alpha}{\omega_0^2-\Omega_\alpha^2}\frac{\omega_0k_{0\parallel}}{B_0}(k^2\pm 2\vec{k}\cdot\vec{k}_{0\perp}), \tag{51}$$

$$C=i(1+\lambda^2k_\perp^2)(\omega+i\Gamma_m), \tag{52}$$

$$D_{\pm}=\pm\frac{c^2}{B_0}\frac{k_{0\parallel}\rho_e^2}{\omega_0}(k^2\pm\vec{k}\cdot\vec{k}_{0\perp})(\vec{k}_0\times\hat{z}\cdot\vec{k}), \tag{53}$$

where

$$\Gamma_m=\frac{\lambda^2k_\perp^2(\mu_e k_\perp^2+\nu_e)}{(1+k_\perp^2\lambda^2)},$$

and $\rho_e=v_{te}/\Omega_e$. Equation (49) is then also the dispersion relation describing the coupling of the ion-ion hybrid wave with the magnetostatic mode. In both cases, we have assumed the pump to be weak so that only terms up to order $|\phi_0|^2$ have been considered.

V. STABILITY ANALYSIS

A. Electrostatic convective cells

Because of the complexity of the algebra, it is difficult to analyze the instability in full detail. However, the instability growth rate ($\gamma = \text{Im}\omega$) can be calculated analytically for different limits. For example, if $\vec{k} \perp \vec{k}_0$, then Eq. (49) becomes

$$(\omega + i\Gamma_c)(\omega^2 - \delta\omega^2) = -\frac{c^2}{B_0^2} \frac{\omega k_{0\perp}^2 (\vec{k} \times \hat{z} \cdot \vec{k}_{0\perp}) P^2 |\phi_0|^2}{\omega_{Pa}^2 + \omega_{Pb}^2)\eta k^2} , \quad (54)$$

where

$$P = \frac{[\omega_{Pa}^2 \Omega_a (\omega_0^2 - \Omega_b^2)^2 + \omega_{Pb}^2 \Omega_b (\omega_0^2 - \Omega_a^2)^2]}{(\omega_0^2 - \Omega_a^2)^{3/2} (\omega_0^2 - \Omega_b^2)^{3/2}} ,$$

$$\eta = 1 + \omega_{pe}^2/\Omega_e^2 + \omega_{pa}^2/\Omega_a^2 + \omega_{pb}^2/\Omega_b^2 ,$$

and $\delta\omega$ is the frequency shift. For $|\omega| \gg \Gamma_c$ and $\delta\omega$, the growth rate is

$$\gamma_c = \frac{c}{B_0} \frac{|k_{0\perp}|}{|k|} \left[\frac{|\vec{k} \times \hat{z} \cdot \vec{k}_{0\perp}| P^2 |\phi_0|^2}{\eta(\omega_{Pa}^2 + \omega_{Pb}^2)} \right]^{1/2} . \quad (55)$$

Next, we consider when $\vec{k}$ is nearly parallel to $\vec{k}_0$, and $|\vec{k}| \ll |\vec{k}_0|$. Equation (49) then takes the form

$$(\omega + i\Gamma_c)(\omega^2 - \delta\omega^2) = -\frac{c^2}{B_0^2} \frac{k_{0\perp}^4 |\vec{k} \cdot \vec{k}_{0\perp}| P^2 \omega}{\eta k^2 (\omega_{Pa}^2 + \omega_{Pb}^2)} |\phi_0|^2 , \quad (56)$$

and the corresponding growth rate is

$$\gamma_c = \frac{c}{B_0} k_{0\perp}^2 \left[\frac{|\vec{k} \cdot \vec{k}_{0\perp}| P^2 |\phi_0|^2}{\eta k^2 (\omega_{Pa}^2 + \omega_{Pb}^2)} \right]^{1/2} . \quad (57)$$

We have thus considered two regimes of parameters, and have shown the possibility of convective cell (vortex modes) generation by a large-amplitude ion-ion hybrid wave.

B. Magnetostatic modes

For the magnetostatic mode, the coupling is not so strong and the maximum growth rate can occur only for $k^2 \ll \vec{k} \cdot \vec{k}_0 \ll k_0^2$. We can write the dispersion relation (49) for this case as

$$(\omega + i\Gamma_m)(\omega^2 - \delta\omega^2) = \frac{4ic^2}{B_0^2} \frac{k_{0\parallel}^2 \rho_e^2 (\vec{k} \cdot \vec{k}_{0\perp})^2 (\vec{k}_0 \cdot \vec{k} \times \hat{z}) Q\omega |\phi_0|^2}{\omega_0^3 k_{0\perp}^2 (1 + \lambda^2 k_\perp^2)} , \quad (58)$$

where

$$Q = \frac{\omega_{Pa}^2 \Omega_a (\omega_0^2 - \Omega_b^2) + \omega_{Pb}^2 \Omega_b (\omega_0^2 - \Omega_b^2)}{\omega_{Pa}^2 + \omega_{Pb}^2} .$$

For $\omega \gg \delta\omega$ and Γ_m, the growth rate is given by

$$\gamma_m \approx \frac{c}{B_0} \left[\frac{k_{0\parallel}^2 \rho_e^2 |\vec{k} \cdot \vec{k}_{0\perp}|^2 (\vec{k}_0 \cdot \vec{k} \times \hat{z}) 2Q |\phi_0|^2}{\omega_0^3 k_{0\perp}^2 (1 + \lambda^2 k_\perp^2)} \right]^{1/2} \quad (59)$$

We note that magnetostatic modes obtain a real frequency which is of the order of the growth rate in the presence of an external pump.

As an illustration, we apply the results of the present investigation to a laboratory plasma. Accordingly, we choose typical parameters: $N_e = 10^{12}$ cm^{-3}, $N_a/N_e = 0.5$, $B_0 = 25$ kG, $E_0 = 15$ V/cm, and $T_e = 10^2$ eV. It is found that growth rates (57) and (59) compete with those found earlier.[9,10]

VI. SUMMARY

A plasma composed of two types of ion species, as occurs in tokamak devices, supports a resonance at the so-called Buchsbaum or ion-ion hybrid frequency. At this layer a mode-converted electrostatic Buchsbaum mode can directly interact with the ion, and can cause wave absorption. On the other hand, anomalous absorption takes place due to the parametric interaction process in which a finite-amplitude ion-ion hybrid wave further decays into a daughter wave and a low-frequency ion-acoustic or an ion-Bernstein wave. In an inhomogeneous plasma, the latter is replaced by drift waves or drift-Alfvén waves. In particular, it was found that a Buchsbaum mode with a moderate electric field amplitude ~ 15 V/cm can indeed produce numerous kinds of nonlinear effects.

In this paper, we have discussed a new kind of nonlinear effect which can be the cause of anomalous particle or heat losses in a tokamak device. Specifically, we have considered the nonlinear interaction of zero-frequency vortex and magnetostatic modes in the presence of the ion-ion hybrid wave which arise in the presence of two species of ions. The process of four-wave interaction results in a modulational instability which can enhance the level of low-frequency modes. In a driven system, both the vortex and the magnetostatic modes obtain a real frequency. Therefore, the particle diffusion, always being anomalous in the presence of driven fluctuations, could be enhanced[14] by an order of magnitude. For cases involving a purely growing mode, one expects an even higher diffusion rate.

At the present time, it is not possible to present a self-consistent theory of the parameter interaction, as has been discussed here. However, one[14,15] expects that in a fully developed turbulent state most of the energy of the system could be contained in the low-frequency modes. Thus, if the energy density of the latter is two orders of magnitude higher than the thermal level, then the diffusion coefficient would be increased by one order of magnitude. A definite answer to this question is only possible provided that we know the enhanced electric and magnetic field spectra of convective cells and the magnetostatic modes. This problem has to be investigated separately.

Finally, our investigation should be refined to include such effects as the plasma inhomogeneity, the gravity, and the magnetic shear. In particular, one[11] finds that in the presence of the magnetic shear the two-dimensional convective cells can obtain a real frequency due to a finite $k_{\|}$. On the other hand, the shear can also allow the electrons to move rapidly along $B_0\hat{z}$, thereby destroying the two-dimensional particle motion. In this case, slow electrons may follow a Boltzmann equilibrium ($n_e^l = N_e e\psi/T_e$) and the three-dimensional ion motion yields the ion sound oscillations in a low-β plasma. Furthermore, due to the magnetic shear, the magnetostatic modes can appear[16] only in a region of thickness $(L_s\lambda)^{1/2}$, where $\lambda = c/\omega_{pe}$ in the electron inertial length, and L_s is the shear length in the neighborhood of the $\vec{k}\cdot\vec{B}=0$ surfaces. Since the Buchsbaum modes are also localized in the presence of the magnetic shear, one encounters an eigenvalue problem for nonlinear interaction purposes. However, on the basis of recent analytical works,[17] we can suggest the applicability of our theory to a plasma with magnetic shear. Like drift-convective cells, we anticipate the localization of enhanced modes near discrete magnetic surfaces. In the presence of enhanced fluctuations (e.g., originating due to the parametric processes) the effective mean free path would be much shorter than the classical value. If the effective mean free path or the length of the system is shorter than the shear length, then the shear has no appreciable effect on suppressing the convective-cell-enhanced diffusion.[18] Otherwise, the diffusion is proportional to L_s near the closed surfaces. In conclusion, we mention that for a real system with magnetic shear our theory has to be revised in a systematic manner. This, however, would lead us far beyond the scope of the present investigation.

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*Permanent address: Department of Physics, Quaid-i-Azam University, Islamabad, Pakistan.

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