Nonlinear coupling of drift waves, convective cells, and magnetic drift modes in finite-β plasmas

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A set of three coupled nonlinear equations describing electrostatic drift waves, drift Alfvén waves, convective cells, and magnetic drift modes has been derived for finite ion temperature. The reduction to simplified nonlinear model equations in different β regimes and the consequences for transport properties are discussed.

INTRODUCTION

The importance of low-frequency vortex modes for the anomalous transport in magnetic confinement devices is a question that has recently attracted considerable attention.[1–20] One of the main points of interest is the influence of magnetic shear which is generally supposed to almost eliminate the diffusion due to convective cells. There are, however, indications that an appreciable part of this diffusion remains also in the presence of magnetic shear[6] since small cells may still overlap and since the ponderomotive force creates driven convective cells with $k_{\|}=0$ also for linear modes with finite $k_{\|}$. It was also recently pointed out[5] that for a reasonable ordering for a toroidal plasma the electric potential obeys the two-dimensional convective cell equation if β (the ratio of plasma and magnetic field pressure) is larger than δ^2/q^2 where q is the safety factor and δ is the inverse aspect ratio. In the present paper we generalize the treatment of Ref. 5 to include also the magnetic drift mode, which in the presence of shear becomes the tearing mode, and a more explicit nonlinear equation for drift Alfvén waves. We also include a curvature drift and finite ion temperature and discuss the influence of β, the finite-Larmor-radius (FLR) parameter $k^2\rho^2$, and different orderings for the electromagnetic potential.

BASIC EQUATIONS

We are here going to assume that β is so small that a representation of the electric field in terms of two potentials is adequate. This means that we should have $\beta<\delta$ where δ is the inverse aspect ratio.[18] We may then use the gauge condition

$$\vec{A}=-\psi\hat{z}$$

leading to the representations

$$\vec{B}=\hat{z}\times\vec{\nabla}\psi+B_0\hat{z},\quad \vec{E}=-\vec{\nabla}\phi+\frac{1}{c}\frac{\partial\psi}{\partial t}\hat{z}\,, \tag{1}$$

where ψ plays the role of a poloidal flux function. We write the perpendicular particle velocities as

$$\vec{v}_{\perp j}=-\frac{1}{B_0}(c\vec{\nabla}\phi+v_{zj}\vec{\nabla}_{\perp}\psi)\times\hat{z}-\frac{c}{B_0\Omega_{c_j}}\frac{d}{dt}\vec{\nabla}_{\perp}\phi+\frac{c}{q_jB_0n_j}(\hat{e}_{\|}\times\vec{\nabla}p_j)+\vec{v}_{gj}\,, \tag{2}$$

where

$$\vec{v}_{gj}=-\frac{g_j}{\Omega_{c_j}}\hat{y}$$

is included only for ions. The ion temperature effects can to lowest order be included by including the ion diamagnetic drift (both background and perturbed parts) in the convective part of the time derivative $\partial/\partial t+\vec{v}\cdot\vec{\nabla}$. This procedure is in agreement with results obtained by orbit averaging.[18,21] A corresponding result for the parallel electron motion is that the diamagnetic drift should not appear in the convective part of the time derivative. This is seen by comparison with the drift kinetic equation. From Ampères law we now obtain the parallel current

$$j_z=\frac{c}{4\pi}\Delta_{\perp}\psi\,. \tag{3}$$

Neglecting the parallel ion motion we obtain, from (3),

$$v_{ez}=-\frac{c}{4\pi e n_0}\left[1-\frac{\delta n_e}{n_0}\right]\Delta_{\perp}\psi\,, \tag{4}$$

where δn_e is the perturbation in electron density and we expanded for $\delta n_e\ll n_0$.

Using now the low-frequency approximation

$$\vec{\nabla}\cdot\vec{j}=0 \tag{5}$$

we find, in combination with (2) and (3), neglecting $v_{\|}$ for ions,

$$\frac{d}{dt}\Delta_{\perp}\phi-\frac{v_A^2}{c}\frac{\partial}{\partial z}\Delta_{\perp}\psi+\frac{B_0^2}{m_ic}v_g\frac{\partial}{\partial y}\left[\frac{\delta n_i}{n_0}\right]=\frac{v_A^2}{B_0c}(\hat{z}\times\vec{\nabla}_{\perp}\psi)\cdot\vec{\nabla}\left[\left[1-\frac{\delta n_e}{n_0}\right]\Delta_{\perp}\psi\right]. \tag{6}$$

The equation of motion for electrons along z may be written using (4) and $\delta p=\gamma T\delta n$,

$$\left[1-\frac{c^2}{\omega_{pe}^2}\Delta_\perp\right]\frac{\partial\psi}{\partial t}-c\frac{\partial\phi}{\partial z}+\gamma_e\frac{cT_e}{en_0}\frac{\partial\delta n_e}{\partial z}+v_{de}\frac{\partial\psi}{\partial y}=\frac{c^2}{\omega_{pe}^2}(\vec{v}_E\cdot\vec{\nabla})\Delta_\perp\psi-(\vec{v}_E+\vec{v}_{de})\cdot\vec{\nabla}\psi+\frac{c^2v_0}{\omega_{pe}^2B_0}(\hat{z}\times\vec{\nabla}\psi)\cdot\vec{\nabla}(\Delta_\perp\psi), \quad (7)$$

where $\vec{v}_E$ is the $\vec{E}\times\vec{B}$ drift, $\vec{v}_{de}$ is the perturbed diamagnetic drift, ω_{pe} is the electron plasma frequency, and v_0 is the background parallel electron velocity. The electron continuity equation, finally, may be written

$$\frac{\partial}{\partial t}\left[\frac{\delta n_e}{n_0}\right]+\frac{e}{T_e}v_{de}\frac{\partial\phi}{\partial y}-\frac{e}{T_e}\frac{v_A^2}{c}\rho^2\Delta\frac{\partial\psi}{\partial z}=\rho^2\frac{v_A^2}{cB_0}(\hat{z}\times\vec{\nabla}\psi)\cdot\vec{\nabla}\left[1-\frac{\delta n_e}{n_0}\right]\Delta\psi-\frac{c}{B_0}(\hat{z}\times\vec{\nabla}\phi)\cdot\vec{\nabla}\frac{\delta n_e}{n_0}\ . \quad (8)$$

Equations (6)—(8), in combination with an assumption of quasineutrality, form a closed set of equations describing nonlinear low-frequency modes in inhomogeneous plasma. They may also allow for the presence of shear if an x- (radially) dependent background part of ψ [related to v_0 through (4)] is included. Assuming ψ_0 (but not $\Delta_\perp\psi_0$) to be zero at the rational surface and restricting the analysis to a region close to the rational surface we may treat ψ as a perturbation in Eqs. (6)—(8) when $\Delta_\perp\ln\psi_0 \ll \Delta_\perp\ln\psi$.

LINEAR DISPERSION RELATION

For reference it is convenient to write down the linear dispersion relation corresponding to the system (6)—(8). Neglecting, for simplicity, the current associated with ψ_0 we obtain, assuming $v_g < v_d$,

$$\left[1-\frac{\omega}{\omega_{*e}-\omega}\frac{m_e}{m_i\beta}k_\perp^2\rho^2\right]\left[\omega(\omega-\omega_{*i})+\kappa g\frac{k_y^2}{k_\perp^2}\right]-k_{\parallel}^2v_A^2\frac{\gamma_e\omega_{*e}-\omega}{\omega_{*e}-\omega}=k_\perp^2\rho^2k_{\parallel}^2v_A^2\gamma_e\frac{\omega_{*i}-\omega}{\omega_{*e}-\omega}\ , \quad (9)$$

where $\rho=c_s/\Omega_{c_i}$, $c_s=(T_e/m_i)^{1/2}$, and $\kappa=-d/dx\ \ln n_0$.

Unless $\omega\approx\omega_{*e}$ the expression in the first set of parentheses is normally close to unity and (9) then describes drift Alfvén waves for $\gamma_e=1$ (thermalized electrons). If we include shear, $k_{\parallel}$ becomes x dependent and (9) is replaced by the eigenvalue equation of collisionless drift tearing modes. In the limit $k_{\parallel}=0$ there are three solutions of (9). Putting the quantity first enclosed within the first parentheses equal to zero we obtain the magnetic drift mode (electromagnetic)

$$\omega=\frac{\omega_{*e}}{1+(m_e/m_i\beta)k_\perp^2\rho^2}=\frac{\omega_{*e}}{1+k_\perp^2c^2/\omega_{pe}^2}\ . \quad (10)$$

For a homogeneous plasma this mode turns into the magnetostatic mode[3] considered to be the magnetic equivalent of the convective cell mode. The quantity in the second set of parentheses gives the two electrostatic interchange modes. For vanishing gravity (curvature) they turn into the ion drift mode and the convective-cell mode, $\omega=0$.

SIMPLIFIED NONLINAR EQUATIONS IN DIFFERENT β REGIMES

We will here make the assumption suggested above and drop the cubic terms in the right-hand sides of (6)—(8). We then normalize time and space by $\Omega_{c_i}^{-1}$ and $\rho=c_s/\Omega_{c_i}$, respectively, and obtain, using quasineutrality, the system

$$\left[\frac{\partial}{\partial t}+v_{di}\frac{\partial}{\partial y}\right]\Delta_\perp\hat{\phi}-\frac{v_A^2}{c}\Delta_\perp\frac{\partial\hat{\psi}}{\partial z}+v_g\frac{\partial}{\partial y}\left[\frac{\delta n}{n_0}\right]=\frac{v_A^2}{c^2}(\hat{z}\times\vec{\nabla}\hat{\psi})\cdot\vec{\nabla}(\Delta_\perp\hat{\psi})-\left[\hat{z}\times\hat{\nabla}\left[\hat{\phi}+\gamma_i\frac{\delta n}{n}\right]\right]\cdot\vec{\nabla}(\Delta_\perp\hat{\phi})\ , \quad (11a)$$

$$\left[1-\frac{m_e}{m_i\beta}\Delta_\perp\right]\frac{\partial\hat{\psi}}{\partial t}-c\frac{\partial\hat{\phi}}{\partial z}+\gamma_ec\frac{\partial}{\partial z}\left[\frac{\delta n_e}{n_0}\right]+v_{de}\frac{\partial\hat{\psi}}{\partial y}=(\hat{z}\times\vec{\nabla}\hat{\phi})\cdot\vec{\nabla}\left[\frac{m_e}{m_i\beta}\Delta_\perp-1\right]\hat{\psi}-\gamma_e\left[\hat{z}\times\vec{\nabla}\frac{\delta n}{n}\right]\cdot\vec{\nabla}\hat{\psi}$$
$$-\frac{v_0}{c}\frac{m_e}{m_i\beta}(\hat{z}\times\vec{\nabla}\hat{\psi})\cdot\vec{\nabla}(\Delta_\perp\hat{\psi})\ , \quad (11b)$$

$$\frac{\partial}{\partial t}\left[\frac{\delta n}{n}\right]+v_{de}\frac{\partial\hat{\phi}}{\partial y}-\frac{v_A^2}{c}\Delta_\perp\frac{\partial\hat{\psi}}{\partial z}=\frac{v_A^2}{c^2}(\hat{z}\times\vec{\nabla}\hat{\psi})\cdot\vec{\nabla}(\Delta_\perp\hat{\psi})-(\hat{z}\times\vec{\nabla}\phi)\cdot\vec{\nabla}\frac{\delta n}{n}\ , \quad (11c)$$

where $\hat{\phi}=e\phi/T_e$, $\hat{\psi}=e\psi/T_e$, and we included a factor T_i/T_e in γ_i.

We now introduce a small parameter ϵ of order $\delta n/n$. We then adopt the ordering (compare Ref. 5)

$$\hat{\phi}\sim\frac{\delta n}{n}\sim\frac{\rho}{a}\sim\frac{\omega}{\Omega_{c_i}}\sim\epsilon,\ v_g\sim\delta v_d\ ,$$

$$k_{\parallel}=\rho\frac{k_yx}{L_s}\sim\frac{\rho}{L_s}\sim\frac{\rho}{Rq}\sim\epsilon\frac{\delta}{q}\ ,$$

where L_s is the shear length. The ordering of ∇ (corresponding to $k\rho$) will be left open but is usually assumed to be of order 1. The ordering of $\hat{\psi}$ is a crucial point. For an electromagnetic mode it is natural to assume that the electrostatic and electromagnetic contributions to $E_{\parallel}$ are comparable. This leads to $\hat{\psi}\sim(k_{\parallel}c/\omega)\hat{\phi}$, or

$$\text{(i):}\ \hat{\psi}\sim(\delta/q)c\hat{\phi}=\epsilon c\delta/q\ .$$

For low β, i.e., $\beta \ll \delta^2/q^2$, it turns out that another ordering of $\hat{\psi}$ is necessary for the terms in (11a) to balance. Thus, putting the first two terms equal, we obtain the ordering

(ii): $\hat{\psi} \sim (\beta c q/\delta)\hat{\phi}$.

For the electromagnetic case we use the ordering (i) for $\hat{\psi}$. This leads to the orderings of the terms in (ii):

$$\left[\frac{\partial}{\partial t}+v_{di}\frac{\partial}{\partial y}\right]\Delta_\perp\hat{\phi} \sim O(\epsilon^2\Delta_\perp), \quad \frac{v_A^2}{c}\Delta_\perp\frac{\partial}{\partial z}\psi \sim O\left[\frac{\delta^2}{Bq^2}\Delta_\perp\epsilon^2\right], \quad v_q\frac{\partial}{\partial y}\left[\frac{\delta n}{n_0}\right] \sim O(\delta\epsilon^2) , \tag{12a}$$

$$\frac{v_A^2}{c^2}(\hat{z}\times\vec{\nabla}\hat{\psi})\cdot\vec{\nabla}(\Delta_\perp\psi) \sim O\left[\frac{\delta^2}{\beta q^2}\epsilon^2\Delta_\perp^2\right], \quad \hat{z}\times\vec{\nabla}\left[\phi+\gamma_i\frac{\delta n}{n}\right]\cdot\vec{\nabla}(\Delta_\perp\hat{\phi}) \sim O(\epsilon^2\Delta_\perp^2) ;$$

$$\left[1-\frac{m_e}{m_i\beta}\Delta_\perp\right]\frac{\partial}{\partial t}\hat{\psi} \sim O\left[\epsilon^2\frac{\delta}{q}c\left[1-\frac{m_e}{m_i\beta}\Delta_\perp\right]\right], \quad c\frac{\partial}{\partial z}\hat{\phi} \sim O\left[\epsilon^2\frac{\delta}{q}c\right], \quad \gamma_e c\frac{\partial}{\partial z}\left[\frac{\partial n_e}{n_0}\right] \sim O\left[\epsilon^2\frac{\delta}{q}c\right] ,$$

$$v_{de}\frac{\partial}{\partial y}\hat{\psi} \sim O\left[\epsilon^2\frac{\delta}{q}c\right], \quad (\hat{z}\times\vec{\nabla}\hat{\phi})\cdot\vec{\nabla}\left[\frac{m_e}{m_i\beta}\Delta_\perp-1\right]\hat{\psi} \sim O\left[\epsilon^2\frac{\delta}{q}c\Delta_\perp\left[\frac{m_e}{m_i\beta}\Delta_\perp-1\right]\right] , \tag{12b}$$

$$\gamma_e\left[\hat{z}\times\vec{\nabla}\frac{\delta n}{n}\right]\cdot\vec{\nabla}\hat{\psi} \sim O\left[\epsilon^2\frac{\delta}{q}c\Delta_\perp\right], \quad \frac{v_0}{c}\frac{m_e}{m_i\beta}(\hat{z}\times\vec{\nabla}\hat{\psi})\cdot\vec{\nabla}\hat{\psi}\cdot\vec{\nabla}(\Delta_\perp\hat{\psi}) \sim O\left[v_0\frac{m_e}{m_i\beta}\Delta_\perp^2\epsilon^2\frac{\delta^2}{q^2}c\right] ;$$

$$\frac{\partial}{\partial t}\left[\frac{\delta n}{n}\right] \sim O(\epsilon^2), \quad v_{de}\frac{\partial}{\partial y}\hat{\phi} \sim O(\epsilon^2), \quad v_A^2\Delta_\perp\frac{\partial}{\partial z}\hat{\psi} \sim O\left[\epsilon^2\frac{\delta^2}{\beta q^2}\Delta_\perp\right] , \tag{12c}$$

$$\frac{v_A^2}{c^2}(\hat{z}\times\vec{\nabla}\hat{\psi})\cdot\vec{\nabla}(\Delta_\perp\hat{\psi}) \sim O\left[\epsilon^2\frac{\delta^2}{\beta q^2}\Delta_\perp^2\right], \quad (\hat{z}\times\vec{\nabla}\hat{\phi})\cdot\vec{\nabla}\left[\frac{\partial n}{n}\right] \sim O(\epsilon^2\Delta_\perp) .$$

As a first general remark we point out that the nonlinear terms are comparable to the linear terms when $\Delta_\perp \sim 1$ and the weak turbulence limit requires $\Delta_\perp \ll 1$ in agreement with the electrostatic case.[7,8] Furthermore, we immediately observe two characteristic β values which come out of the ordering in (12). The lowest value

$$\beta \sim (m_e/m_i)\Delta_\perp$$

is associated with parallel electron inertia. These terms are important from the point of view that they correspond to a deviation from ideal magnetohydrodynamics leading to magnetic field lines which are not frozen in. This is an important feature of tearing modes. We will, however, here usually assume

$$\beta > (m_e/m_i)\Delta_\perp ,$$

thus neglecting these terms. The second typical β is $\beta = \delta^2/q^2$. This β is typical of electromagnetic modes and as we shall see the system becomes electrostatic in both the limits $\beta \ll \delta^2/q^2$ and $\beta \gg \delta^2/q^2$. Starting with the low β case we should use the ordering (ii) of $\hat{\psi}$. Then multiplying (11b) by $\delta/q\epsilon^2 c$, the terms, for

$$\beta \gg (m_e/m_i)\Delta ,$$

compare as

$$\frac{\delta}{q\epsilon^2}\left[1-\frac{m_e}{m_i\beta}\Delta_\perp\right]\frac{\partial}{\partial t}\hat{\psi} \sim O(\beta) ,$$

$$\frac{\delta}{q\epsilon^2}\frac{\partial}{\partial z}\hat{\phi} \sim O\left[\frac{\delta^2}{q^2}\right], \quad \frac{\gamma_0\delta}{q\epsilon^2}\frac{\partial}{\partial z}\left[\frac{\delta n_e}{n_0}\right] \sim O\left[\frac{\delta^2}{q^2}\right] ,$$

$$\frac{v_{de}}{q\epsilon^2 c}\frac{\partial}{\partial y}\hat{\psi} \sim O(\beta) ,$$

$$\frac{\delta}{q\epsilon^2 c}(\hat{z}\times\vec{\nabla}\hat{\phi})\cdot\vec{\nabla}\left[\frac{m_e}{m_i\beta}\Delta_\perp-1\right]\hat{\psi} \sim O(\Delta\beta) , \tag{13}$$

$$\frac{\gamma_e\delta}{q\epsilon^2 c}\left[\hat{z}\times\vec{\nabla}\frac{\delta n}{n}\right]\cdot\vec{\nabla}\hat{\psi} \sim O(\Delta\beta) ,$$

$$\frac{v_0\delta}{q\epsilon^2 c^2}\frac{m_e}{m_i\beta}(\hat{z}\times\vec{\nabla}\hat{\psi})\cdot\vec{\nabla}\hat{\psi}\cdot\vec{\nabla}(\Delta_\perp\hat{\psi}) \sim O(\Delta\beta) .$$

Thus, in the limit $\beta \ll \delta^2/q^2$ only the second and third terms remain in (11) and we obtain the Boltzmann distribution ($\gamma_e = 1$)

$$\frac{\delta n}{n} = \hat{\phi} . \tag{14}$$

Now, subtracting (11c) from (11b) and using (14) we obtain the equation

$$\frac{\partial}{\partial t}(\Delta_\perp - 1)\hat{\phi} + (v_{di}\Delta_\perp + v_g - v_d)\frac{\partial \hat{\phi}}{\partial y} = [(1+\gamma_i)\vec{\nabla}\hat{\phi}\times\hat{z}]\cdot\vec{\nabla}(\Delta_\perp \phi) . \quad (15)$$

Equation (15) is the nonlinear equation for electrostatic drift waves, and is the generalization of the Hasegawa-Mima equation[7] for finite ion temperature (γ_i contains T_i/T_e) and curvature v_g. We notice here that due to the Boltzmann distribution (14) the curvature only introduces a frequency shift and does not cause interchange mode solutions. As is well known, Eq. (15) leads to cascading towards smaller and larger $k_\perp$.[8] In the limit of weak nonlinearity ($\Delta_\perp \ll 1$) the pump wave is the wave with the largest frequency. Equation (15) has two constants of motion, total energy and total enstrophy (squared vorticity) and describes turbulence with many features in common with Navier-Stokes turbulence. Another interesting property is that it may lead to zonal flows have an inhibiting influence on the transport.[8]

The reason why Eq. (15) does not contain interchange mode solutions is that the charge separation due to the perpendicular motion is shielded by the free-electron motion along B_0 as described by (14). When β increases the induction force increases and impedes the free-electron motion. This leads to electromagnetic interchange modes.

When $\beta \sim \delta^2/q^2$ the system (11) describes nonlinear drift Alfvén waves or tearing modes. In order to obtain a simplified equation for drift Alfvén waves we shall assume that $\Delta_\perp < 1$ (weakly nonlinear) so that we can drop nonlinear terms of order δ. Then differentiating (11a) with respect to time and substituting (11b) and (11c), we obtain, for $\gamma_e = 1$,

$$\left[\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t} + v_{di}\frac{\partial}{\partial y}\right] - v_A^2\frac{\partial^2}{\partial z^2}\right]\Delta\hat{\phi} + \kappa g\frac{\partial^2\hat{\phi}}{\partial y^2} + v_A^2\frac{\partial}{\partial z}\left[\frac{\partial}{\partial z}\frac{\delta n}{n_0} + \frac{v_{de}}{c}\frac{\partial\hat{\psi}}{\partial y}\right]$$
$$= \frac{v_A^2}{c^2}\frac{\partial}{\partial t}(\hat{z}\times\vec{\nabla}_\perp\hat{\psi})\cdot\vec{\nabla}\Delta_\perp\hat{\psi} + \frac{\partial}{\partial t}\left[\vec{\nabla}\left[\hat{\phi} + \gamma_i\frac{\delta n}{n}\right]\times\hat{z}\right]\cdot\vec{\nabla}(\Delta_\perp\hat{\phi}) + \frac{v_A^2}{c}\Delta_\perp\frac{\partial}{\partial z}\left[(\vec{\nabla}\hat{\phi}\times\hat{z})\cdot\vec{\nabla}\hat{\psi} + \left[\vec{\nabla}\frac{\delta n}{n}\times\hat{z}\right]\cdot\vec{\nabla}\hat{\psi}\right], \quad (16)$$

where $\kappa = -d/dx \ln n_0$ and $\kappa g = -v_g v_{di}$.

In order to eliminate the last linear term it is convenient to use the ion continuity equation

$$\left[\frac{\partial}{\partial t} + v_g\frac{\partial}{\partial y}\right]\frac{\delta n}{n_0} + v_{de}\frac{\partial\hat{\phi}}{\partial y} - \Delta_\perp\left[\frac{\partial}{\partial t} + v_{di}\frac{\partial}{\partial y}\right]\hat{\phi} = (\vec{\nabla}\hat{\phi}\times\hat{z})\cdot\vec{\nabla}\frac{\delta n}{n_0} + (\hat{z}\times\vec{\nabla}\hat{\phi})\cdot\vec{\nabla}(\Delta_\perp\hat{\phi}) . \quad (17)$$

Now, operating with $\partial/\partial z$ on (17) and with $(v_{de}/c)\partial/\partial y$ on (11b) and adding the two equations we obtain, dropping the linear terms of order δ,

$$\left[\frac{\partial}{\partial t} + v_{de}\frac{\partial}{\partial y}\right]\left[\frac{\partial}{\partial z}\frac{\delta n_i}{n} + \frac{v_{de}}{c}\frac{\partial\hat{\psi}}{\partial y}\right] = \frac{\partial}{\partial z}(\vec{\nabla}\hat{\phi}\times\hat{z})\cdot\vec{\nabla}\frac{\delta n}{n} + \frac{v_{de}}{c}\frac{\partial}{\partial y}\left[\vec{\nabla}\hat{\phi}\times\hat{z} + \vec{\nabla}\frac{\delta n}{n}\times\hat{z}\right]\cdot\vec{\nabla}\hat{\psi} . \quad (18)$$

We notice that the main linear part of the expression

$$\frac{\partial}{\partial z}\frac{\delta n}{n} + \frac{v_{de}}{c}\frac{\partial\hat{\psi}}{\partial y}$$

vanishes. Now, operating with $\partial/\partial t + v_{de}\,\partial/\partial y$ on (16) and substituting (18) we find that the nonlinear terms of (18) cancel the last nonlinear part of (16) to leading order. This follows from the fact that, due to (16) and (11b), we have $\partial\hat{\psi}/\partial t \approx c\,\partial\hat{\phi}/\partial z$ (compare also the ordering assumed for $\hat{\psi}$). Then, inverting the operator $\partial/\partial t + v_{de}\,\partial/\partial y$, we obtain the equation

$$\left[\frac{\partial^2}{\partial t^2} + v_{di}\frac{\partial^2}{\partial t\,\partial y} - v_A^2\frac{\partial^2}{\partial z^2}\right]\Delta_\perp\hat{\phi} + \kappa g\frac{\partial^2\hat{\phi}}{\partial y^2} = \frac{v_A^2}{c^2}\frac{\partial}{\partial t}(\hat{z}\times\vec{\nabla}_\perp\hat{\psi})\cdot\vec{\nabla}(\Delta_\perp\hat{\psi}) + \frac{\partial}{\partial t}\left[\vec{\nabla}\left[\hat{\phi} + \gamma_i\frac{\delta n}{n}\right]\times\hat{z}\right]\cdot\vec{\nabla}(\Delta_\perp\hat{\psi}) \quad (19)$$

which is our reduced nonlinear equation for drift Alfvén waves. Since we assumed $\Delta < 1$ we can substitute linear expressions for $\hat{\psi}$ and δn in terms of $\hat{\phi}$ into the nonlinear terms. The equation thus obtained is identical to the model equation derived in Ref. 11 if the nonlinear term containing δn is dropped. This is equivalent to assuming $T_i < T_e$.

Equation (19) with $T_i \sim T_e$ can also be obtained from the nonlinear equation for ballooning modes derived in Ref. 18 in toroidal coordinates if the shearless limit is taken. In Ref. 11 it was found that (19) can give the same type of cascade towards smaller and larger $k_\perp$ as is typical of electrostatic drift waves.[7] Explosively unstable solutions and possibilities for up conversion were also found. Another important electromagnetic mode is obtained for $k_\parallel \leq \epsilon^2\delta/q^2$. We obtain, from (11b),

$$\left[1 - \frac{m_e}{m_i\beta}\Delta_\perp\right]\frac{\partial\hat{\psi}}{\partial t} + v_{de}\frac{\partial\hat{\psi}}{\partial y} = (\hat{z}\times\vec{\nabla}\hat{\phi})\cdot\vec{\nabla}\left[\frac{m_e}{m_i\beta}\Delta_\perp - 1\right]\hat{\psi} - \gamma_e\left[\hat{z}\times\vec{\nabla}\frac{\delta n}{n}\right]\cdot\vec{\nabla}\hat{\psi} - \frac{v_0}{c}\frac{m_e}{\beta m_i}(\hat{z}\times\vec{\nabla}\hat{\psi})\cdot\vec{\nabla}(\Delta_\perp\hat{\psi}) . \quad (20)$$

This is the equation for the magnetic drift mode,[4] indicating possibilities for coupling to drift waves, convective cells, and drift Alfvén waves. For the interaction between magnetic drift modes with $k_{\|}\approx 0$, however, $\hat{\phi}$ and $\delta n/n$ are small and in fact generated by v_0 (ψ_0) as found in Ref. 19. In this case

$$\hat{\phi}\sim\frac{v_0}{c}\frac{\kappa\Omega_{ci}}{\omega k}\hat{\psi}$$

and no nonlinearity remains for $v_0=0$. As was pointed out in Ref. 16, for a homogeneous case Eq. (20) turns into the Hasegawa-Mima equation[7] for $\hat{\phi}=\delta n/n=0$ but $v_0\neq 0$. For this approximation to be correct for an inhomogeneous plasma with the above relation for $\hat{\phi}$ and $\omega\sim\omega_{*e}$ we must require that $k^2\rho^2\gg 1$. In the homogeneous case the magnetic drift mode turns into the magnetostatic mode which can cause signficant electron thermal conductivity due to perturbations of the magnetic flux surfaces.[3] The coupled set of equations (20) and (11a) was also recently investigated from the nonlinear stability point of view in the homogeneous approximation with $\beta\gg(m_e/m_i)\Delta_\perp$ and $v_g=0$. It was found that nonlinearly growing solutions exist in a background of random phase waves.[22]

We now turn to the large β region, i.e., $\beta\gg\delta^2/q^2$. We then observe from (12a) that the electromagnetic terms vanish from (11a) and we obtain the two-dimensional system

$$\left[\frac{\partial}{\partial t}+v_{di}\frac{\partial}{\partial y}\right]\Delta_\perp\hat{\phi}+v_g\frac{\partial}{\partial y}\left[\frac{\delta n}{n_0}\right]=-\left[\hat{z}\times\vec{\nabla}\left[\phi+\gamma_i\frac{\delta n}{n}\right]\right]\cdot\vec{\nabla}(\Delta_\perp\hat{\phi})\,, \tag{21a}$$

$$\frac{\partial}{\partial t}\left[\frac{\delta n}{n}\right]+v_{de}\frac{\partial\hat{\phi}}{\partial y}=(\vec{\nabla}\hat{\phi}\times\hat{z})\cdot\vec{\nabla}\frac{\delta n}{n}\,. \tag{21b}$$

This is a generalization of the system derived in Ref. 10 for finite ion temperature. It was found to lead to spectral cascade rules of the same type as for the Hasegawa-Mima equation but also to possibilities for up conversion. In the limit $v_g\rightarrow 0$ the equations couple only through the nonlinear terms, and (21a) is the nonlinear equation for the ion drift branch. In order to obtain the convective-cell mode it is necessary to differentiate (21a) with respect to time and to substitute (21b). This leads to the equation

$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}+v_{di}\frac{\partial}{\partial y}\right]\Delta\hat{\phi}-\kappa g\frac{\partial^2\hat{\phi}}{\partial y^2}=-\frac{\partial}{\partial t}\left[\hat{z}\times\vec{\nabla}\left[\hat{\phi}+\gamma_i\frac{\delta n}{n}\right]\right]\cdot\vec{\nabla}(\Delta_\perp\hat{\phi})-v_g\frac{\partial}{\partial y}(\vec{\nabla}\hat{\phi}\times\hat{z})\cdot\vec{\nabla}\frac{\delta n}{n}\,. \tag{22}$$

The reason why it is not possible to obtain the convective cell mode in the limit $v_g\rightarrow 0$ in (21) is the singularity in $\delta n/n$ when $\omega\rightarrow 0$ as seen from (21b). As was pointed out in Ref. 5, the two-dimensional character in the limit $\beta\gg\delta^2/q^2$ is very important since it can give rise to substantial convective cell transport in a system with magnetic shear. The presence of curvature (v_g) may, however, lead to a strong reduction of the transport. In this case the higher-order process derived in Ref. 11 and studied in Ref. 17 becomes important. Owing to the finite $k_{\|}$ allowed for here, it is, however, not clear if the effective curvature experienced by this mode is favorable; and if this is not the case, the usual convective cell diffusion remains strong.

CONCLUSIONS

We have seen how a comparatively general nonlinear system (11) reduces to well-known nonlinear equations in different β regimes and for different orderings of $k_{\|}$. In particular, the eigenfrequency of the convective cell (interchange mode) and the magnetic drift modes are important. When $\beta>\delta^2/q^2$ the influence of shear on the convective cell mode disappears. The influence of shear ($k_{\|}v_a$) and curvature (κg) are comparable when $\beta=\delta/q^2$. Thus, for $\beta>\delta/q^2$ the question of the eigenfrequency is mainly determined by the question of the effective average curvature experienced by the mode. This can only be determined by the mode structure and requires the solution of the poloidal eigenvalue problem. The question of the transport thus boils down to the question of the stability of ballooning modes. For the magnetic drift mode we notice that the ordering $\Delta>1$, $\omega\sim\epsilon\Delta$ would eliminate the linear eigenfrequency, leading to a situation where the dynamics is dominated by the nonlinear terms, and this may lead to enhanced transport. Another interesting property of the system (11) is that it contains possibilities for nonlinearly unstable solutions. These may occur for modes driven by the combined influence of curvature and pressure and described by (19), which is the slab limit[11] of a model equation for ballooning modes[18] and for tearing modes[22] described by (11) for $v_g=0$ but finite ψ_0. The background part ψ_0 is associated with a background current in the toroidal direction and with magnetic shear. The interesting result for a homogeneous plasma obtained in Ref. 16 that the magnetostatic mode in the presence of current obeys the Hasegawa-Mima equation has turned out to apply for an inhomogeneous plasma only in the strongly nonlinear region $k^2\rho^2>1$.

Another interesting mode which is closely related to the modes studied here is the curvature-driven trapped electron mode.[23] In the low β limit this mode turns into the electrostatic ubiquitous mode,[24] while for large β it is hard to distinguish from the ballooning mode. Here an electrostatic mode is also obtained in the limit when all electrons are trapped.

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