Equilibrium fluctuations in fluid layers: Effects of elastic solid boundaries

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The hydrodynamic theory of the dynamics of equilibrium fluctuations in one-component fluid layers confined by two identical elastic solid boundaries is presented. The dynamic structure factor for a fluid layer and the interfacial modes are analyzed under assumptions of continuity of stresses (no interfacial stress) and velocities ("stick" boundary condition) across the fluid-solid interfaces. There are four interfacial modes in such a system with dispersion relations which depend on the component of the wave vector parallel to the interfaces, $k_{\parallel}$. In the limit of $\bar{\rho}/\rho_s \to 0$, where $\bar{\rho}$ and ρ_s are the mean densities of the fluid and solid, respectively, two of these modes are just the well-known Rayleigh waves and the remaining two are the interfacial fluid modes found previously in the case of rigid solid walls [see D. Gutkowicz-Krusin and I. Procaccia, Phys. Rev. Lett. 48, 417 (1982); Phys. Rev. A 27, 2585 (1983)]. For finite small values of $\bar{\rho}/\rho_s$ the attenuation of the interfacial fluid modes is due to dissipative interfacial transport and varies as $k_{\parallel}^{3/2}\bar{\rho}/\rho_s$, provided that $c < c_t$, where c is the speed of sound in the fluid and c_t is the transverse sound speed in the solid. If, however, $c > c_t$, the energy of the interfacial fluid mode is radiated into the solid and the resulting attenuation along the interface varies as $k_{\parallel}(\bar{\rho}/\rho_s)^2$, for small $k_{\parallel}$. Since the speed of the interfacial fluid modes is less than c, the corresponding peaks in the dynamic structure factor are better separated from the unbounded fluid Brillouin peaks and thus easier to observe experimentally than would have been the case with rigid solid walls.

I. INTRODUCTION

Recently[1,2] it has been shown that the spectrum of equilibrium fluctuations (i.e., the dynamic structure factor) for a fluid layer bounded by rigid solid walls depends strongly on the energy and tangential momentum transport across the fluid-solid interfaces. In particular, new interfacial modes have been found; these modes are essentially acoustic modes and are due to the additional dissipation caused by the solid walls. Thus, the interfacial modes should be a sensitive probe of interfacial transport over a wide frequency range in the hydrodynamic regime.

The dynamic structure factor is the Fourier transform of the density autocorrelation function

$$S(\vec{k},\vec{k}',\omega)=\frac{1}{(2\pi)^7}\int_{-\infty}^{\infty}dt\int d^3r\int d^3r'\exp[-i(\omega t+\vec{k}\cdot\vec{r}+\vec{k}'\cdot\vec{r}')]\langle\delta\rho(\vec{r},t)\delta\rho(\vec{r}'0)\rangle\ , \tag{1.1}$$

where brackets denote the equilibrium average. This quantity is of great experimental interest since it can be studied, for example, by light scattering.[3] In the present paper, the dynamic structure factor and the interfacial modes are analyzed for a fluid layer bounded by isotropic elastic solid walls. The boundary conditions on the velocity field in the fluid layer are derived in Sec. II by assuming continuity of the stress, as well as velocity across the interface ("stick" boundary conditions), and neglecting viscous dissipation in and thermal expansion of the solid walls. Together with the previously obtained boundary condition on the fluid temperature,[2] one has a complete set of boundary conditions on the fluctuating hydrodynamic variables in the fluid layer. In Sec. III, the general expression for the dynamic structure factor given previously[2] is evaluated for these generalized boundary conditions.

The effects of acoustic excitations in solid walls on the interfacial modes and on the spectrum of equilibrium fluctuations in the fluid are discussed in Sec. IV. In the limit of small mass density ratio between the fluid and the solid, two types of interfacial modes have been found. One of them is the dissipative fluid mode found previously and the other is the well-known Rayleigh wave[4] which exists at the elastic solid-vacuum interface. For finite values of the fluid-solid density ratio the interfacial fluid modes become modified; not only are there shifts in their speeds, but their attenuation varies as $k_{\parallel}^{3/2}$ (or $\omega^{3/2}$) rather than $k_{\parallel}^2$ (or ω^2), where $k_{\parallel}$ is the wave vector parallel to the interfaces, provided that the sound speed in the fluid c is less than the transverse sound speed in the solid c_t. In this case the amplitudes of the interfacial modes decrease exponentially with the distance from the interface. If, however, $c > c_t$, the decrease is oscillatory and some of the energy of the interfacial modes is radiated into the solid walls. These so-called "leaky waves"[5] have an attenuation coefficient which, in the limit of small $k_{\parallel}$, is independent of dissipative processes and varies as $k_{\parallel}(\bar{\rho}/\rho_s)^2$.

The interfacial fluid modes are of greatest interest in the case $c < c_t$. Then, their speeds and attenuation depend on

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the dissipative interfacial transport and the experimental study of $S(\vec{k},\omega)$ would enable one to determine the nature of the tangential momentum transport (the nature of interfacial stresses) as well as of the energy transport across the fluid-solid interfaces over a wide frequency range in the hydrodynamic regime. Results are summarized in Sec. V.

II. DYNAMICS OF FLUCTUATIONS IN ELASTIC SOLID BOUNDARIES

Consider a system which consists of a fluid layer of infinite extent in the xy plane for $z\in[-L/2,L/2]$ and of elastic solid boundaries for $z\in(-\infty,-L/2)$ and $z\in(L/2,\infty)$. The initial value problem of hydrodynamic fluctuations can be solved most easily by taking the Fourier transform of equations of motion in the xy plane, and taking the Laplace transform of these equations in time. Then a fluctuating hydrodynamic variable δA, in either fluid or solid, is transformed to

$$\tilde{A}(k_{\|},z;s)=\frac{1}{(2\pi)^2}\int_0^\infty dt\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\,\exp[-st-i(k_xx+k_yy)]\delta A(\vec{r},t)\,, \tag{2.1}$$

where

$$k_{\|}^2=k_x^2+k_y^2\,.$$

The spectrum of equilibrium fluctuations in the fluid layer can be determined using the general formula from Ref. 2, provided one has a set of appropriate boundary conditions on the transformed hydrodynamic variables in the fluid.

It has been shown in Ref. 2 that the assumption of continuity of the heat and entropy fluxes across the fluid-rigid solid interface leads to the boundary conditions on the fluid temperature:

$$\left.\frac{d\tilde{T}}{dz}\right|_{z=\pm L/2}\pm\delta_T\tilde{T}(\pm L/2)=0\,, \tag{2.2}$$

where

$$\delta_T(k_{\|},s)=\frac{\rho_sC_{ps}}{\bar{\rho}C_p}\frac{\kappa_s}{\kappa}\lambda_4\,, \tag{2.3}$$

$$\lambda_4=(k_{\|}^2+s/\kappa_s)^{1/2}\,, \tag{2.4}$$

$\bar{\rho}$ is the mean mass density, C_p is the specific heat at constant pressure; $\kappa=\lambda/(\rho C_p)$ is the thermal diffusivity, and subscript s refers to the solid. The boundary condition Eq. (2.2) remains valid for the fluid-elastic solid interface provided that viscous dissipation in and thermal expansion of the elastic solid boundaries are negligible. This assumption is used throughout this paper.

It is usually assumed that the normal and tangential momentum fluxes are continuous across the interface; i.e., that the normal and tangential stresses are continuous. Thus, no interfacial stress is associated with the fluid-solid interface. In order to complete the set of boundary conditions, it is necessary to specify the velocities (or elastic displacements) at the interface. In the absence of interfacial mass transport, the normal component of the velocity must be continuous across the interface. In addition, it is well established experimentally that in the limit of vanishing frequency, the tangential components of the velocity are also continuous, leading to the so-called stick boundary conditions. In the present paper, the consequences of this assumption for the nature of interfacial modes and for the dynamic structure factor are studied in detail in the hydrodynamic frequency regime.

The dynamics of the fluctuations in local displacements in the solid, $\delta\vec{u}_s$, is governed by the equation[4]

$$\frac{\partial^2\delta\vec{u}_s}{\partial t^2}=c_t^2\nabla^2\delta\vec{u}_s+(c_l^2-c_t^2)\vec{\nabla}(\vec{\nabla}\cdot\delta\vec{u}_s)\,, \tag{2.5}$$

where c_l and c_t are the longitudinal and transverse sound speeds, respectively. After taking the appropriate Laplace and Fourier transforms, the equations of motion can be simplified by noting that the static correlation functions between the fluctuations in fluid density and fluctuations in solid displacements must vanish in equilibrium. Therefore the initial values of these displacements, as well as of their time derivatives, can be set to zero without loss of generality. Let

$$\phi_s\equiv\frac{\partial\delta u_{sx}}{\partial x}+\frac{\partial\delta u_{sy}}{\partial y}\,. \tag{2.6}$$

Then, the transformed variables $\tilde{u}_{sz}$ and $\tilde{\phi}_s$ satisfy the equations

$$s^2\tilde{u}_{sz}=c_l^2\frac{d^2\tilde{u}_{sz}}{dz^2}-c_t^2k_{\|}^2\tilde{u}_{sz}+(c_l^2-c_t^2)\frac{d\tilde{\phi}_s}{dz} \tag{2.7}$$

and

$$s^2\tilde{\phi}_s=c_t^2\frac{d^2\tilde{\phi}_s}{dz^2}-c_l^2k_{\|}^2\tilde{\phi}_s-(c_l^2-c_t^2)k_{\|}^2\frac{d\tilde{u}_{sz}}{dz}\,. \tag{2.8}$$

The solutions of Eqs. (2.7) and (2.8) must remain finite as $z\to\pm\infty$. Therefore, if

$$\lambda_5\equiv(k_{\|}^2+s^2/c_t^2)^{1/2} \tag{2.9}$$

and

$$\lambda_6\equiv(k_{\|}^2+s^2/c_l^2)^{1/2}\,, \tag{2.10}$$

the solutions are

$$\tilde{u}_{sz}=\begin{cases}a_1\exp[-\lambda_5(z-L/2)]+a_2\lambda_6\exp[-\lambda_6(z-L/2)], & L/2\le z\le\infty\\ b_1\exp[\lambda_5(z+L/2)]-b_2\lambda_6\exp[\lambda_6(z+L/2)], & -\infty\le z\le -L/2\end{cases} \tag{2.11}$$

and

$$\tilde{\phi}_s = \begin{cases} a_1\lambda_5 \exp[-\lambda_5(z-L/2)] + a_2 k_{\|}^2 \exp[-\lambda_6(z-L/2)], & L/2 \le z \le \infty \\ -b_1\lambda_5 \exp[\lambda_5(z+L/2)] + b_2 k_{\|}^2 \exp[\lambda_6(z+L/2)], & -\infty \le z \le -L/2 . \end{cases} \quad (2.12)$$

The components of the fluctuating part of the stress tensor in an isotropic elastic solid are

$$\sigma_{szz} = \rho_s \left[c_l^2 \frac{\partial \delta u_{sz}}{\partial z} + (c_l^2 - 2c_t^2)\phi_s \right] , \quad (2.13a)$$

$$\sigma_{szx_i} = \rho_s c_t^2 \left[\frac{\partial \delta u_{sz}}{\partial x_i} + \frac{\partial \delta u_{si}}{\partial z} \right], \quad x_i = x,y . \quad (2.13b)$$

The corresponding components of the stress tensor for a Newtonian fluid are

$$\sigma_{zz} = -\delta p + \bar{\rho} \left[\Gamma_v \frac{\partial u_z}{\partial z} + (\Gamma_v - 2\nu)\phi \right] , \quad (2.14a)$$

$$\sigma_{zx_i} = \bar{\rho}\nu \left[\frac{\partial u_z}{\partial x_i} + \frac{\partial u_i}{\partial z} \right], \quad x_i = x,y , \quad (2.14b)$$

where δp is the fluctuating pressure, $\vec{u}$ is the velocity field and ϕ is defined in Eq. (2.6) but without the subscript, ν and ζ are the kinematic shear and bulk viscosities, and $\Gamma_v = \frac{4}{3}\nu + \zeta$ is the longitudinal viscosity. From the continuity of normal stresses, after taking the appropriate Laplace and Fourier transforms, one obtains

$$\rho_s \left[c_l^2 \frac{d\tilde{u}_{sz}}{dz}\bigg|_{z=z_0} + (c_l^2 - 2c_t^2)\tilde{\phi}_s(z_0) \right] = \frac{\bar{\rho}}{\gamma s}(c^2 + \gamma\Gamma_v s) \left[\frac{d\tilde{u}_z}{dz}\bigg|_{z=z_0} + \left[1 - \frac{2\gamma\nu s}{c^2 + \gamma\Gamma_v s} \right] \tilde{\phi}(z_0) \right]$$

$$- \frac{\alpha}{\gamma}\bar{\rho}c^2\tilde{T}(z_0) - \frac{c^2}{\gamma s}\delta\rho(z_0) , \quad (2.15)$$

and, from the continuity of tangential stresses one gets

$$\rho_s c_t^2 \left[\frac{d\tilde{\phi}_s}{dz}\bigg|_{z=z_0} - k_{\|}^2 \tilde{u}_{sz}(z_0) \right]$$

$$= \bar{\rho}\nu \left[\frac{d\tilde{\phi}}{dz}\bigg|_{z=z_0} - k_{\|}^2 \tilde{u}_z(z_0) \right] , \quad (2.16)$$

where $z_0 = \pm L/2$, α is the thermal expansion coefficient, $\gamma = C_p/C_v$, c is the adiabatic sound speed in the fluid, and $\delta\rho(z_0)$ is the initial value of the density fluctuation at the interface. Note that the particular form of the boundary condition in Eq. (2.15) is obtained by using the assumption of local thermal equilibrium to express the fluctuations in pressure in terms of those in temperature and density; the latter can be related to the divergence of the velocity field through the continuity equation. Finally, the continuity of velocities across the interface implies

$$s\tilde{u}_{sz}(z_0) = \tilde{u}_z(z_0) \quad (2.17)$$

and

$$s\tilde{\phi}_{sz}(z_0) = \tilde{\phi}(z_0) . \quad (2.18)$$

Using Eqs. (2.11) and (2.12) to eliminate the interfacial values of fluctuations in solid displacements from Eqs. (2.15)–(2.18), one obtains the boundary conditions on the velocity field in the fluid layer:

$$\frac{d\tilde{u}_z}{dz}\bigg|_{z=\pm L/2} \mp \epsilon_{uu}\tilde{u}_z(\pm L/2) + (1+\epsilon_{u\phi})\tilde{\phi}(\pm L/2)$$

$$-\epsilon_{uT}\tilde{T}(\pm L/2) = \frac{c^2}{c^2+\gamma\Gamma_v s}\frac{\delta\rho(\pm L/2)}{\bar{\rho}} \quad (2.19)$$

and

$$\frac{d\tilde{\phi}}{dz}\bigg|_{z=\pm L/2} \mp \epsilon_{\phi\phi}\tilde{\phi}(\pm L/2) + k_{\|}^2(1+\epsilon_{\phi u})\tilde{u}_z(\pm L/2) = 0 , \quad (2.20)$$

where

$$\epsilon_{\phi\phi} \equiv \frac{\rho_s}{\bar{\rho}}\frac{s}{\nu}\frac{\lambda_6}{k_{\|}^2 - \lambda_5\lambda_6} , \quad (2.21a)$$

$$\epsilon_{\phi u} \equiv \frac{\rho_s}{\bar{\rho}}\frac{c_t^2}{s\nu}\left[1 + \frac{\lambda_5^2 - \lambda_5\lambda_6}{k_{\|}^2 - \lambda_5\lambda_6} \right] - 2 , \quad (2.21b)$$

$$\epsilon_{u\phi} = \frac{\gamma\nu s}{c^2+\gamma\Gamma_v s}\epsilon_{\phi u} , \quad (2.21c)$$

$$\epsilon_{uu} = \frac{\gamma\nu s}{c^2+\gamma\Gamma_v s}\frac{\lambda_5}{\lambda_6}\epsilon_{\phi\phi} , \quad (2.21d)$$

$$\epsilon_{uT} = \frac{\alpha c^2 s}{c^2+\gamma\Gamma_v s} . \quad (2.21e)$$

Equations (2.2), (2.19), and (2.20) constitute a complete set of boundary conditions on the hydrodynamic variables in a fluid layer confined by elastic solid walls.

III. DYNAMIC STRUCTURE FACTOR FOR A FLUID LAYER BOUNDED BY ELASTIC SOLID WALLS

The general solutions for the hydrodynamic variables, as well as for the dynamic structure factor in a fluid layer for any boundary conditions are given in Ref. 2. For boundary conditions given by Eqs. (2.2), (2.19), and (2.20) the coefficients C_i, necessary to specify the solutions of interest, are determined from the equation:

$$\underline{G}\vec{C}+\vec{g}=0\ , \tag{3.1}$$

where

$$\underline{G}=\begin{bmatrix} -A_1(P_1+\delta_T) & A_1(S_1+\delta_T) & -A_2(P_2+\delta_T) & A_2(S_2+\delta_T) & 0 & 0 \\ A_1(P_1+\delta_T) & A_1(S_1+\delta_T) & A_2(P_2+\delta_T) & A_2(S_2+\delta_T) & 0 & 0 \\ B_1(P) & -B_1(S) & B_2(P) & -B_2(S) & B_3(S) & -B_3(P) \\ B_1(P) & B_1(S) & B_2(P) & B_2(S) & -B_3(S) & -B_3(P) \\ D_1(P) & -D_1(S) & D_2(P) & -D_2(S) & D_3(S) & -D_3(P) \\ D_1(P) & D_1(S) & D_2(P) & D_2(S) & -D_3(S) & -D_3(P) \end{bmatrix}, \tag{3.2}$$

where

$$R_i \equiv \tanh(\tfrac{1}{2}\lambda_i L)\ ,\quad i=1,2,3$$

$$P_i \equiv \lambda_i R_i\ ,\quad i=1,2$$

$$P_3 \equiv R_3/\lambda_3\ ,$$

$$S_i \equiv \lambda_i/R_i\ ,\quad i=1,2$$

$$S_3 \equiv 1/\lambda_3 R_3\ ,$$

and

$$\lambda_i \equiv (k_{\parallel}^2 - x_i)^{1/2}\ . \tag{3.3}$$

In addition,

$$A_i \equiv -\frac{(\gamma-1)}{\alpha}\frac{x_i}{s+\gamma\kappa x_i}\ ,\quad i=1,2 \tag{3.4}$$

and, for $F=P,S$,

$$B_i(F) \equiv \epsilon_{u\phi}k_{\parallel}^2 - \epsilon_{uT}A_i + x_i + \epsilon_{uu}F_i\ ,\quad i=1,2 \tag{3.5a}$$

$$B_3(F) = \epsilon_{u\phi} + \epsilon_{uu}F_3\ , \tag{3.5b}$$

$$D_i(F) = k_{\parallel}^2(\epsilon_{\phi\phi} + \epsilon_{\phi u}F_i)\ ,\quad i=1,2 \tag{3.6a}$$

$$D_3(F) = \epsilon_{\phi\phi} + (x_3 + \epsilon_{\phi u}k_{\parallel}^2)F_3\ , \tag{3.6b}$$

where, from Ref. 2,

$$x_{1,2} = -\frac{s}{2\kappa(c^2+\gamma\Gamma_v s)}\{c^2+s(\Gamma_v+\gamma\kappa)\mp[(c^2+s\Gamma_v+s\gamma\kappa)^2-4\kappa s(c^2+\gamma\Gamma_v s)]^{1/2}\} \tag{3.7a}$$

and

$$x_3 = -\frac{s}{\nu}\ . \tag{3.7b}$$

The vector $\vec{g}$ depends on the initial density fluctuation and is

$$\vec{g}^{\,T} \equiv (0,g_1,0,g_2,0,g_3)\ , \tag{3.8a}$$

where

$$g_1 = -A_0A_1A_2\int_{-L/2}^{L/2} dz\, \delta\rho(z)(\lambda_2 x_1\{\lambda_1\cosh[\lambda_1(L/2-z)]+\delta_T\sinh[\lambda_1(L/2-z)]\} - \lambda_1 x_2\{\lambda_2\cosh[\lambda_2(L/2-z)]+\delta_T\sinh[\lambda_2(L/2-z)]\})\ , \tag{3.8b}$$

$$g_2=-A_0\int_{-L/2}^{L/2}dz\,\delta\rho(z)(A_2\lambda_2x_1\{(x_1+\epsilon_{u\phi}k_{\parallel}^2-\epsilon_{uT}A_1)\sinh[\lambda_1(L/2-z)]+\epsilon_{uu}\lambda_1\cosh[\lambda_1(L/2-z)]\}$$
$$-A_1\lambda_1x_2\{(x_2+\epsilon_{u\phi}k_{\parallel}^2-\epsilon_{uT}A_2)\sinh[\lambda_2(L/2-z)]+\epsilon_{uu}\lambda_2\cosh[\lambda_2(L/2-z)]\})\ , \tag{3.8c}$$

$$g_3=-A_0k_{\parallel}^2\int_{-L/2}^{L/2}dz\,\delta\rho(z)(A_2\lambda_2x_1\{\epsilon_{\phi u}\lambda_1\cosh[\lambda_1(L/2-z)]+\epsilon_{\phi\phi}\sinh[\lambda_1(L/2-z)]\}$$
$$-A_1\lambda_1x_2\{\epsilon_{\phi u}\lambda_2\cosh[\lambda_2(L/2-z)]+\epsilon_{\phi\phi}\sinh[\lambda_2(L/2-z)]\})\ , \tag{3.8d}$$

and

$$A_0\equiv\frac{c^2}{c^2+\gamma\Gamma_v s}\,\frac{1}{\rho_0\lambda_1\lambda_2(A_2x_1-A_1x_2)}\ . \tag{3.9}$$

Therefore, using the general formula from Ref. 2, the L-dependent contribution to the dynamic structure factor is

$$S_L(\vec{k},\vec{k}',s)=\frac{2(-1)^{n+n'}c^2}{(A_2x_1-A_1x_2)(c^2+\gamma\Gamma_v s)}\left[\frac{A_2x_1Y_1(k_\perp,k_\perp')}{k^2-x_1}-\frac{A_1x_2Y_2(k_\perp,k_\perp')}{k^2-x_2}\right]\ , \tag{3.10}$$

where

$$Y_1=\frac{P_1x_1}{(k')^2-x_1}+\frac{P_1W_1}{T(\tilde{U})}+\frac{k_\perp k_\perp' W_2}{T(\tilde{V})}\ , \tag{3.11}$$

$$Y_2=\frac{P_2x_2}{(k')^2-x_2}+\frac{P_2W_3}{T(\tilde{U})}+\frac{k_\perp k_\perp' W_4}{T(\tilde{V})}\ , \tag{3.12}$$

$$W_1=\frac{P_1x_1}{(k')^2-x_1}[A_2(P_2+\delta_T)-A_1\tilde{U}_2]-\frac{A_1P_2x_2}{(k')^2-x_2}(\delta_T+P_2-\tilde{U}_2)\ , \tag{3.13}$$

$$W_2=\frac{x_1}{(k')^2-x_1}[A_2(S_2+\delta_T)-A_1\tilde{V}_2]-\frac{A_1x_2}{(k')^2-x_2}(\delta_T+S_2-\tilde{V}_2)\ , \tag{3.14}$$

$$W_3=\frac{A_2P_1x_1}{(k')^2-x_1}(\delta_T+P_1-\tilde{U}_1)+\frac{P_2x_2}{(k')^2-x_2}[A_2\tilde{U}_1-A_1(P_1+\delta_T)]\ , \tag{3.15}$$

$$W_4=\frac{A_2x_1}{(k')^2-x_1}(\delta_T+S_1-\tilde{V}_1)+\frac{x_2}{(k')^2-x_2}[A_2\tilde{V}_1-A_1(S_1+\delta_T)]\ , \tag{3.16}$$

where

$$\tilde{U}_i=\frac{B_i(P)D_3(P)-B_3(P)D_i(P)}{\epsilon_{uu}D_3(P)-\epsilon_{\phi u}k_{\parallel}^2B_3(P)}\ , \tag{3.17}$$

$$\tilde{V}_i=\frac{B_i(S)D_3(S)-B_3(S)D_i(S)}{\epsilon_{uu}D_3(S)-\epsilon_{\phi u}k_{\parallel}^2B_3(S)}\ , \tag{3.18}$$

$$T(\tilde{U})=A_1(P_1+\delta_T)\tilde{U}_2-A_2(P_2+\delta_T)\tilde{U}_1\ , \tag{3.19}$$

$$T(\tilde{V})=A_1(S_1+\delta_T)\tilde{V}_2-A_2(S_2+\delta_T)\tilde{V}_1\ . \tag{3.20}$$

It is easy to check that the matrix $S_L(\vec{k},\vec{k}',s)$ is symmetric.

In the limit of rigid solid walls (i.e., $c_t,c_l\to\infty$, $c_t/c_l=\text{const}$) $\epsilon_{\phi\phi}$, $\epsilon_{\phi u}\propto c_t^2$, it follows from Eqs. (2.21), (3.17), and (3.18) that $\tilde{U}_i\to P_i-k_{\parallel}^2P_3$ and $\tilde{V}_i\to S_i-k_{\parallel}^2S_3$. Therefore, in this limit one obtains the expression for the dynamic structure factor for the case of stick boundary conditions on the fluid velocity, previously derived in Ref. 2. The same results can be obtained even for soft solid boundaries such that $\rho_s/\bar{\rho}\to\infty$, in which case $\epsilon_{\phi\phi}$ and $\epsilon_{\phi u}$ are proportional to $\rho_s/\bar{\rho}$.

The expression for the L-dependent contribution to the dynamic structure factor can be simplified considerably in the limit of vanishing coefficient of thermal expansion of the fluid α. Since $(\gamma-1)\propto\alpha^2$, this is the limit of $\gamma\to1$. An example is water at 4 °C and atmospheric pressure. In this limit, from Eq. (3.7a),

$$x_1=-\frac{s^2}{c^2+\Gamma_v s}\ ,$$

$$x_2=-\frac{s}{\kappa}\ ,$$

and, since $A_1\propto\alpha\to0$ and $A_2\propto\alpha^{-1}\to\infty$,

$$S_L(k,k',s)=\frac{2(-1)^{n+n'}c^2}{c^2+\Gamma_v s}\,\frac{x_1}{(k^2-x_1)[(k')^2-x_1]}\times\left[P_1-\frac{P_1^2}{\tilde{U}_1}-\frac{k_\perp k_\perp'}{\tilde{V}_1}\right]\ . \tag{3.21}$$

IV. ANALYTIC STRUCTURE OF $S(\vec{k},\vec{k}',\omega)$

The normal hydrodynamic modes of the fluid layer are found from the poles of the dynamic structure factor. The analytic structure of $S_\infty(\vec{k},\omega)$ is well known[4,5]: in the limit of small k the three poles of $S_\infty(\vec{k},\omega)$ lead to the dispersion relation

$$\omega=i\kappa k^2 \tag{4.1}$$

for the diffusive heat mode and

$$\omega=\pm ck+i\Gamma k^2 \tag{4.2}$$

for the two propagating sound modes where

$$\Gamma=\tfrac{1}{2}[\Gamma_v+(\gamma-1)\kappa] \tag{4.3}$$

is the attenuation coefficient for the bulk sound modes. In the small k limit $S_\infty(\vec{k},\omega)$ is the sum of three Lorentzian peaks:

$$\frac{S_\infty(\vec{k},\omega)}{S(\vec{k})}=\frac{1}{\pi\gamma}\left[\frac{(\gamma-1)\kappa k^2}{(\kappa k^2)^2+\omega^2}+\frac{1}{2}\frac{\Gamma k^2}{(\Gamma k^2)^2+(\omega-ck)^2}+\frac{1}{2}\frac{\Gamma k^2}{(\Gamma k^2)^2+(\omega+ck)^2}\right]. \tag{4.4}$$

More interesting is the analytic structure of $S_L(\vec{k},\vec{k}',\omega)$. The nature of the sound and heat modes in the fluid layer, and their dispersion relations and amplitudes, are the same for rigid and elastic solid boundaries, provided that stick boundary condition is used in the former case, and has been fully discussed in Ref. 2. Only a short summary is given here.

The dispersion relation for the propagating bulk sound modes is determined from

$$K^2=x_1\ ,\quad K=k,k'\ , \tag{4.5}$$

and is, in the limit of small K,

$$\omega=\pm cK+i\Gamma K^2\ . \tag{4.6}$$

The amplitudes of these modes vanish unless $k=k'$. The contribution from $S_L(\vec{k},\vec{k},\omega)$ cancels that of $S_\infty(\vec{k},\omega)$ so that in sufficiently thin fluid layers, such that

$$k_\perp L\left[\frac{\Gamma k}{c}\right]^2\left[\frac{k}{k\perp}\right]^2 \ll 1 \tag{4.7}$$

is satisfied, the Brillouin peaks disappear.

The dispersion relation for the diffusive heat mode is found from

$$K^2=x_2\ ,\quad K=k,k' \tag{4.8}$$

and is

$$\omega=i\kappa K^2\ . \tag{4.9}$$

These modes also contribute only to the diagonal part of the dynamic structure factor and the contribution from $S_L(\vec{k},\vec{k},\omega)$ cancels the Rayleigh peak from $S_\infty(\vec{k},\omega)$ provided L is sufficiently small so that

$$\tfrac{1}{4}k_\perp L\frac{k^4}{k_\perp^2}\frac{(\gamma-1)\kappa\,|\,\Gamma_v-\kappa\,|}{c^2}\ll 1\ . \tag{4.10}$$

There is also an infinite number of poles of $S_L(\vec{k},\vec{k}',\omega)$ found from the conditions

$$P_1=\infty \tag{4.11}$$

and

$$P_2=\infty\ . \tag{4.12}$$

The solutions of Eq. (4.11) are the waveguide sound modes with dispersion relation

$$\omega=\pm cK(m)+i\Gamma K^2(m)\ ,\qquad K^2(m)\equiv k_{\parallel}^2+\frac{(2m+1)^2\pi^2}{L^2}\ . \tag{4.13}$$

Their amplitudes vanish for sufficiently small values of L, given by condition (4.7) with $k_\perp$ replaced by $(2m+1)\pi/L$. The solutions of Eq. (4.12) are the waveguide heat modes with dispersion relation

$$\omega=i\kappa K^2(m)\ ; \tag{4.14}$$

their amplitudes vanish for small values of L satisfying the condition (4.10) with $k_\perp$ replaced by $(2m+1)\pi/L$.

Therefore, for sufficiently small values of L the amplitudes of the bulk sound and heat modes vanish and the total intensity of density fluctuations is in the interfacial modes whose dispersion relation is given by the solutions of

$$T(\tilde{U})=0 \quad \text{or} \quad T(\tilde{V})=0\ . \tag{4.15}$$

In general, the dispersion relation for the interfacial modes depends on the thickness L of the fluid layer and on $k_{\parallel}$, the component of wave vector parallel to the interfaces. Analytic results for the dispersion relation can be found in the limit of L sufficiently large so that

$$\mathrm{Re}(\tfrac{1}{2}\lambda_i L)\gg 1\ ,\quad i=1,2,3 \tag{4.16}$$

and sufficiently small $k_{\parallel}$ so that

$$k_{\parallel}D/c\ll 1\ , \tag{4.17}$$

where D is any transport coefficient. The assumption (4.16) leads to

$$P_i\approx S_i\approx\lambda_i\ ,\quad i=1,2\ ,\qquad P_3\approx S_3\approx 1/\lambda_3\ , \tag{4.18}$$

and, since $\tilde{U}_i\approx\tilde{V}_i$, to

$$T(\tilde{U})\approx T(\tilde{V})\ . \tag{4.19}$$

The dispersion relation for the interfacial modes in the presence of elastic solid boundaries is rather complicated; therefore, it is useful to consider first the simpler case with $\gamma=1$. Then, the L-dependent contribution to the dynamic structure factor is given by Eq. (3.21) and the dispersion relation for the interfacial modes is determined from the condition

$$\tilde{U}_1\approx\tilde{V}_1=0 \tag{4.20}$$

or

$$(\lambda_1\lambda_3-k_{\parallel}^2)[\epsilon_{\phi\phi}^2(\lambda_5/\lambda_6)-\epsilon_{\phi u}^2 k_{\parallel}^2]+x_3\{\epsilon_{\phi\phi}[(\lambda_3+\lambda_1\lambda_5)/\lambda_6]+2\epsilon_{\phi u}k_{\parallel}^2\}+x_3^2=0\ . \tag{4.21}$$

Making explicit the dependence on the density ratio $\bar{\rho}/\rho_s$, Eq. (4.21) is equivalent to

$$U_0F_0+\frac{\bar{\rho}}{\rho_s}F_1+\left[\frac{\bar{\rho}}{\rho_s}\right]^2F_2=0\ , \tag{4.22}$$

where

$$U_0=\lambda_1\lambda_3-k_{\parallel}^2\ , \tag{4.23a}$$

$$F_0=(\lambda_5^2+k_{\parallel}^2)^2-4k_{\parallel}^2\lambda_5\lambda_6\ , \tag{4.23b}$$

$$F_1=\frac{s^4}{c_t^4}(\lambda_1\lambda_5+\lambda_3\lambda_6)+\frac{2k_{\parallel}^2 s}{c_t^2}(s-2\nu U_0)\left[\frac{s^2}{c_t^2}+2k_{\parallel}^2-2\lambda_5\lambda_6\right]\ , \tag{4.23c}$$

$$F_2=\frac{s^2}{c_t^4}(\lambda_5\lambda_6-k_{\parallel}^2)(s^2+4k_{\parallel}^2\nu s-4k_{\parallel}^2\nu^2U_0)\ . \tag{4.23d}$$

In the limit $\bar{\rho}/\rho_s\to 0$ there are four solutions of Eq. (4.21). The dispersion relation for two of the modes is determined from

$$U_0(\omega_0)=0 \tag{4.24}$$

and is

$$\omega_0=\pm ck_{\parallel}+\tfrac{1}{2}i(\Gamma_v+\nu)k_{\parallel}^2\ ; \tag{4.25}$$

this is just the dispersion relation for the interfacial fluid modes found previously in the case of the rigid solid walls with stick boundary conditions on the velocity field at the interfaces.[2] Since in the present case $\gamma=1$, the bulk sound attenuation coefficient $\Gamma=\Gamma_v/2$, while the additional attenuation due to shear created by the boundaries $\Gamma'=\nu/2$.

The dispersion relation for the remaining two modes can be found from

$$F_0(\omega_{R0})=0\ , \tag{4.26}$$

which leads to the Rayleigh waves propagating along the elastic solid-vacuum interfaces; i.e.,

$$\omega_{R0}=\pm c_R k_{\parallel}\ , \tag{4.27}$$

where the speed of the Rayleigh wave c_R is determined from the equation

$$X^3-8X^2+8[3-2(c_t/c_l)^2]X-16[1-(c_t/c_l)^2]=0\ , \tag{4.28}$$

where

$$X\equiv(c_R/c_t)^2\ ;$$

depending on the value of the Poisson ratio for the solid walls[4]

$$0.874c_t\le c_R\le 0.955c_t\ .$$

Since the dissipation in the elastic solid boundaries has been neglected, the Rayleigh waves are not attenuated to zeroth order in $\bar{\rho}/\rho_s$. For $\gamma=1$, the dissipation is due to the transport of tangential momentum across the fluid-solid interfaces.

To first order in $\bar{\rho}/\rho_s$, the interfacial fluid modes become modified due to coupling to the acoustic modes of the solid. The dispersion relation for the interfacial modes may be written

$$\omega=\omega_0+ik_{\parallel}^{3/2}\psi[(1\pm i)(\tfrac{1}{2}\nu c)^{1/2}+\psi_1\nu k_{\parallel}^{1/2}]\left[\frac{\bar{\rho}}{\rho_s}\right]+O\left[\left[\frac{\bar{\rho}}{\rho_s}\right]^2\right]\ , \tag{4.29}$$

where ω_0 is given in Eq. (4.25),

$$\psi=\frac{\lambda_{60}(\lambda_{50}^2-1)^2}{4\lambda_{50}\lambda_{60}-(\lambda_{50}^2+1)^2}\ , \tag{4.30a}$$

$$\psi_1=2\frac{1-2\lambda_{50}\lambda_{60}+\lambda_{50}^2}{\lambda_{60}(\lambda_{50}^2-1)}\ , \tag{4.30b}$$

$$\lambda_{50}\equiv(1-c^2/c_t^2)^{1/2}\ , \tag{4.30c}$$

and

$$\lambda_{60}\equiv(1-c^2/c_l^2)^{1/2}\ . \tag{4.30d}$$

If $c_t>c$, λ_{50} is real and so are ψ and ψ_1. Hence, for small values of $k_{\parallel}$, the speed of the interfacial fluid modes is

$$c_f=c\left[1-\psi\left[\frac{\nu k_{\parallel}}{2c}\right]^{1/2}\frac{\bar{\rho}}{\rho_s}+O\left[\left[\frac{\bar{\rho}}{\rho_s}\right]^2\right]\right]\ , \tag{4.31}$$

i.e., it depends on the wave vector $k_{\parallel}$, while the attenuation is

$$\mathrm{Im}(\omega)=\psi(\tfrac{1}{2}\nu c)^{1/2}\frac{\bar{\rho}}{\rho_s}k_{\parallel}^{3/2}+\left[\tfrac{1}{2}(\Gamma_v+\nu)+\psi\psi_1\nu\frac{\bar{\rho}}{\rho_s}\right]k_{\parallel}^2+O\left[\left[\frac{\bar{\rho}}{\rho_s}\right]^2\right]\ . \tag{4.32}$$

Therefore, in general, in the limit of sufficiently small $k_{\parallel}$ and small $\bar{\rho}/\rho_s$, the attenuation of the interfacial modes varies as $k_{\parallel}^{3/2}$ (or $\omega^{3/2}$) rather than $k_{\parallel}^2$ as is usually the case. In the limit $c_t,c_l\gg c$, the above expressions simplify to

$$c_f\approx c\left[1-\frac{1}{2}\frac{(c/c_t)^2}{1-(c_t/c_l)^2}\left[\frac{\nu k_{\parallel}}{2c}\right]^{1/2}\frac{\bar{\rho}}{\rho_s}+\cdots\right] \tag{4.33}$$

while the attenuation of the interfacial fluid modes becomes

$$\mathrm{Im}(\omega)\approx\frac{1}{2}\frac{(c/c_t)^2}{1-(c_t/c_l)^2}(\tfrac{1}{2}\nu c)^{1/2}k_{\parallel}^{3/2}\frac{\bar{\rho}}{\rho_s}+\left[\tfrac{1}{2}(\Gamma_v+\nu)-\nu\frac{(c/c_l)^2}{1-(c_t/c_l)^2}\frac{\bar{\rho}}{\rho_s}\right]k_{\parallel}^2+\cdots\ . \tag{4.34}$$

The leading contribution to the dispersion relation which depends on the density ratio $\bar{\rho}/\rho_s$ and which is of order $k_{\parallel}$ is

$$\Delta\omega = \mp c k_{\|} \psi^2 \left[\frac{\bar{\rho}}{\rho_s}\right]^2 + O(k_{\|}^{3/2}) . \tag{4.35}$$

Hence, for $c_t < c$, as is the case, for example, for a water layer at room temperature confined by gold walls, λ_{50} is imaginary so that ψ is complex. Therefore, the contribution to the dispersion relation of order $(\bar{\rho}/\rho_s)^2$ may be the dominant contribution to the attenuation coefficient in the limit of small $k_{\|}$; this contribution is

$$2\,\mathrm{Re}(\psi)\mathrm{Im}(\psi) \left[\frac{\bar{\rho}}{\rho_s}\right]^2 c k_{\|} + O(k_{\|}^{3/2}) \tag{4.36}$$

and thus varies as $k_{\|}$ (or ω) for small frequencies. Note that this attenuation is not due to the dissipative processes at the interface or in the fluid but rather to the radiation of energy into the solid walls; the interfacial modes are then called leaky waves.[5]

The dispersion relation for the Rayleigh waves also changes for finite values of $\bar{\rho}/\rho_s$. Let

$$\omega_R = \pm c_R k_{\|} + i\omega_{R1} \left[\frac{\bar{\rho}}{\rho_s}\right] + O\left[\left[\frac{\bar{\rho}}{\rho_s}\right]^2\right] , \tag{4.37}$$

and

$$\lambda_{1R} \equiv [1-(c_R/c)^2]^{1/2} , \tag{4.38a}$$

$$\lambda_{5R} \equiv [1-(c_R/c_t)^2]^{1/2} . \tag{4.38b}$$

Then

$$\omega_R = c_{R1} k_{\|} \psi_R \left[\pm i + (1\pm i)\left[\frac{\nu k_{\|}}{2c_R}\right]^{1/2} \psi_{R1}\right] , \tag{4.39a}$$

where

$$\psi_R = \frac{\lambda_{5R}(\lambda_{5R}^2+1)^2(\lambda_{5R}^4-1)^2}{4\lambda_{1R}[2\lambda_{5R}^2(\lambda_{5R}^2-3)(\lambda_{5R}^2+1)^3+(\lambda_{5R}^2+1)^4+16\lambda_{5R}^4]} , \tag{4.39b}$$

$$\psi_{R1} = \frac{1}{\lambda_{1R}} + \frac{4\lambda_{5R}(\lambda_{1R}\lambda_{5R}-\lambda_{5R}^2-1)}{(\lambda_{5R}^2+1)^2} . \tag{4.39c}$$

Since, typically, $c_R > c$, λ_{1R} is imaginary and so is ψ_R. In such cases, therefore, it is the Rayleigh wave which is a leaky wave[5] (radiating energy into the fluid) with an attenuation coefficient which varies as $k_{\|}$, while the attenuation due to dissipative processes at the interfaces and in the fluid through the interfacial momentum transport varies as $k_{\|}^{3/2}$. However, when $c_R < c$ the attenuation of the Rayleigh waves is due purely to dissipation and varies as $k_{\|}^{3/2}$. Finally, when $c_t > c_R > c$ the dominant interfacial modes are the fluid modes with the attenuation coefficients $\propto k_{\|}^{3/2}\bar{\rho}/\rho_s$ whereas the attenuation of the Rayleigh waves is $\propto k_{\|}\bar{\rho}/\rho_s$ and thus much higher for small values of $k_{\|}$. On the other hand, if $c > c_t > c_R$, the attenuation of the interfacial fluid modes has contributions which vary as $k_{\|}^{3/2}\bar{\rho}/\rho_s$ and $k_{\|}(\bar{\rho}/\rho_s)^2$, while the attenuation of the Rayleigh waves $\propto k_{\|}^{3/2}\bar{\rho}/\rho_s$. Hence, the latter are the dominant interfacial modes for small $k_{\|}$. An interesting case arises if $c_R < c < c_t$; in this case the attenuation due to radiation is unimportant and both the interfacial fluid modes and the Rayleigh waves are attenuated only due to dissipative processes with attenuation $\propto k_{\|}^{3/2}\bar{\rho}/\rho_s$. Hence, in this case, there should be two distinct new peaks in the diagonal part of the dynamic structure factor for $\omega > 0$. The separation of the two peaks, and thus their observability, depends, however, on the various transport and thermodynamic coefficients of the two media.

To first order in $\bar{\rho}/\rho_s$, the contribution of the interfacial fluid modes to the diagonal part of the dynamic structure factor is

$$\frac{\Delta S^{\pm}(\vec{k},\omega)}{S(\vec{k})} = \frac{2k_{\|}}{\pi L}\mathrm{Re}\left[\frac{\left[\frac{\nu k_{\|}}{2c}\right]^{1/2}(1\mp i)+\psi\frac{\bar{\rho}}{\rho_s}}{\left[k_\perp^2 \mp i\frac{\nu}{c}k_{\|}^3\right]\left[i\omega \mp ick_{\|} + \frac{1}{2}(\Gamma_v+\nu)k_{\|}^2 + (1\pm i)(\frac{1}{2}\nu c)^{1/2}k_{\|}^{3/2}\psi\frac{\bar{\rho}}{\rho_s}\right]}\right] . \tag{4.40}$$

If $c_t > c$, then ψ is real and, since typically $k_\perp^2 \gg \nu k_{\|}^3/c$, Eq. (4.40) reduces to

$$\frac{\Delta S^{\pm}(\vec{k},\omega)}{S(\vec{k})} = \frac{2}{\pi k_{\|} L}\left[\frac{k_{\|}}{k_\perp}\right]^2\left[\left[\frac{\nu k_{\|}}{2c}\right]^{1/2}[\tfrac{1}{2}(\Gamma_v+\nu)k_{\|}^2 \mp (\omega \mp ck_{\|})] + \tfrac{1}{2}(\Gamma_v+\nu)\psi\frac{\bar{\rho}}{\rho_s}k_{\|}^2\right]$$
$$\times\left[\left[\tfrac{1}{2}(\Gamma_v+\nu)k_{\|}^2 + (\tfrac{1}{2}\nu c)^{1/2}\psi\frac{\bar{\rho}}{\rho_s}k_{\|}^{3/2}\right]^2 + \left[\omega \mp ck_{\|} \pm (\tfrac{1}{2}\nu c)^{1/2}\psi\frac{\bar{\rho}}{\rho_s}k_{\|}^{3/2}\right]^2\right]^{-1} . \tag{4.41}$$

Hence, the interfacial fluid modes contribute non-Lorentzian peaks to the dynamic structure factor. While the amplitudes of these peaks are not significantly different in the limit of small $\bar{\rho}/\rho_s$ for the two cases of rigid and elastic solid walls, the decrease in the speed of propagation of the interfacial modes in the latter case leads to the increased separation of the corresponding peaks from the unbounded fluid Brillouin peaks. Therefore, the interfacial fluid modes should be easier to study experimentally, for example, by light scattering, than was suggested previously.[1,2]

The contribution of the Rayleigh waves to the diagonal part of the dynamic structure factor for small values of $\bar{\rho}/\rho_s$ is

$$\frac{\Delta S_R^{\pm}(\vec{k},\omega)}{S(\vec{k})}=\frac{2}{\pi L k_{\parallel}}\left[\frac{\bar{\rho}}{\rho_s}\right]\frac{k_{\parallel}^2(c_R/c)^2}{k_{\perp}^2+[1-(c_R/c)^2]k_{\parallel}^2}\times\mathrm{Re}\left[\frac{\psi_R/\lambda_{1R}}{i\left[\omega\mp c_R k_{\parallel}\pm c_R k_{\parallel}\psi_R[1+(\nu k_{\parallel}/2c_R)^{1/2}\psi_{R1}]\frac{\bar{\rho}}{\rho_s}\right]+(\tfrac{1}{2}\nu c_R)^{1/2}\psi_R\psi_{R1}\frac{\bar{\rho}}{\rho_s}k_{\parallel}^{3/2}}\right]. \tag{4.42}$$

For $c_R<c$, as is the case for a water layer confined by gold walls, λ_{1R} is real and so are ψ_R and ψ_{R1}. Then Eq. (4.42) reduces to

$$\frac{\Delta S_R^{\pm}(\vec{k},\omega)}{S(\vec{k})}\approx\frac{2}{\pi k_{\parallel}L}\left[\frac{\bar{\rho}}{\rho_s}\right]^2\frac{k_{\parallel}^2(c_R/c)^2}{k_{\perp}^2+k_{\parallel}^2\lambda_{1R}^2}\frac{(\tfrac{1}{2}\nu c_R)^{1/2}}{\lambda_{1R}}\times\left[\frac{\psi_R^2\psi_{R1}k_{\parallel}^{3/2}}{(\tfrac{1}{2}\nu c_R)\psi_R^2\psi_{R1}^2\left[\frac{\bar{\rho}}{\rho_s}\right]^2k_{\parallel}^3+\left[\omega\mp c_R k_{\parallel}\left[1-\psi_R\frac{\bar{\rho}}{\rho_s}\right]\pm(\tfrac{1}{2}\nu c_R)^{1/2}\psi_R\psi_{R1}\frac{\bar{\rho}}{\rho_s}k_{\parallel}^{3/2}\right]^2}\right]. \tag{4.43}$$

Thus, the Rayleigh waves contribute to the dynamic structure factor Lorentzian peaks whose amplitudes are small for small values of $\bar{\rho}/\rho_s$.

The situation is not qualitatively different for $\gamma\neq 1$. Again, consider the dispersion relation in the limit of small values of $\bar{\rho}/\rho_s$. Since this typically implies metallic boundaries, it is consistent to assume that the thermal diffusivity of the solid is much higher than that of the fluid. In the limit of perfectly conducting solid boundaries, the dispersion relation is determined from the condition

$$A_1(B_2D_3-B_3D_2)=A_2(B_1D_3-B_3D_1) \tag{4.44}$$

or from Eq. (4.22), but with λ_1 replaced by Λ, where

$$\Lambda=[k_{\parallel}^2+\lambda_1\lambda_2+\gamma s^2/(c^2+\gamma\Gamma_v s)]/(\lambda_1+\lambda_2)\ . \tag{4.45}$$

In the limit of $\bar{\rho}/\rho_s\to 0$, the dispersion relation for the interfacial fluid modes is

$$\omega_0=\pm c k_{\parallel}+i(\Gamma+\Gamma')k_{\parallel}^2\ , \tag{4.46a}$$

where

$$\Gamma=\tfrac{1}{2}[\Gamma_v+(\gamma-1)\kappa]\ , \tag{4.46b}$$

$$\Gamma'=\tfrac{1}{2}[\sqrt{\nu}+(\gamma-1)\sqrt{\kappa}]^2\ . \tag{4.46c}$$

To first order in $(\bar{\rho}/\rho_s)$ the dispersion relation is

$$\omega=\pm c k_{\parallel}\left[1-\psi\left[\frac{\Gamma' k_{\parallel}}{c}\right]^{1/2}\frac{\bar{\rho}}{\rho_s}\right]+i\left[(\Gamma' c)^{1/2}\psi\frac{\bar{\rho}}{\rho_s}k_{\parallel}^{3/2}+\left[\Gamma+\Gamma'+(2\Gamma'\nu)^{1/2}\psi\psi_1\frac{\bar{\rho}}{\rho_s}\right]k_{\parallel}^2\right], \tag{4.47}$$

where ψ and ψ_1 are defined in Eqs. (4.30). Thus, since for $\gamma>1$ the sound in the fluid is not isothermal, the attenuation of the interfacial modes increases due to heat conduction from the fluid to the solid boundaries. In addition, the effective speed of these modes decreases.

For $\gamma>1$ the dispersion relation for the Rayleigh waves is still given by Eq. (4.37), but $\psi_{R1}(\gamma=1)$ defined in Eq. (4.39b) is now modified to

$$\psi_{R1}(\gamma)=\psi_{R1}(1)+(\gamma-1)\left[\frac{\kappa}{\nu}\right]^{1/2}\frac{(c_R/c)^2}{\lambda_{1R}}\ . \tag{4.48}$$

While the analytic expressions for the dispersion relation for either the interfacial fluid modes or the Rayleigh waves were obtained in the limit of small density ratio $\bar{\rho}/\rho_s$ and high thermal conductivity of the solid boundaries, the exact expression for the dynamic structure factor, Eq. (3.10), can be easily evaluated for any values of the parameters. In Fig. 1, the dynamic structure factor is shown for a layer of water at 4 °C bounded by stainless-steel walls for different values of L in the vicinity of the Brillouin peak of the unbounded fluid. The interfacial fluid mode is clearly seen for $\omega\approx 7.095\times 10^8\ \mathrm{sec}^{-1}$; while its amplitude increases with decreasing layer thickness, the amplitudes of the Brillouin and the waveguide modes decrease. The interfacial peak is well separated from the Brillouin peak since the speed of the interfacial modes decreases due to the acoustic coupling to the boundaries. In Fig. 2, the dynamic structure factor shown is the same as in Fig. 1 but for walls made of Pyrex, which has a higher value of $\bar{\rho}/\rho_s$ than does stainless steel. It is seen that the interfacial peak, centered at $\omega\approx 7.075\times 10^8\ \mathrm{sec}^{-1}$ has similar amplitude and remains well separated from the Brillouin peak. In addition, an unusual decrease in the Brillouin peak due to Pyrex boundaries is seen here. The dynamic structure factor for the water layer bounded by

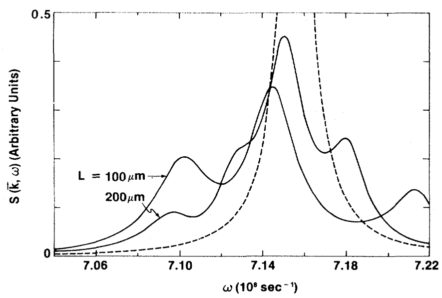

FIG. 1. Dynamic structure factor $S(\vec{k},\omega)$ for a water layer bounded by stainless-steel walls for two values of the layer thickness L. Parameters for water at 277 K are $\gamma=1.0$, $\rho=1.0$ g/cm^3, $c=1.42\times10^5$ cm/sec, $\nu=1.57\times10^{-2}$ cm^2/sec, $\zeta=4.82\times10^{-2}$ cm^2/sec. Parameters for stainless steel are $\rho_s=7.9$ g/cm^3, $c_t=3.1\times10^5$ cm/sec, $c_l=5.79\times10^5$ cm/sec. Components of the wave vector are $k_{\|}=5\times10^3$ cm^{-1}, $k_{\perp}=2\pi\times10^2$ cm^{-1}. The Brillouin peak of the unbounded fluid normalized to unity at maximum (dashed curve) is shown for comparison. The peak corresponding to the interfacial fluid mode is centered at $\omega\approx7.095\times10^8$ sec^{-1}.

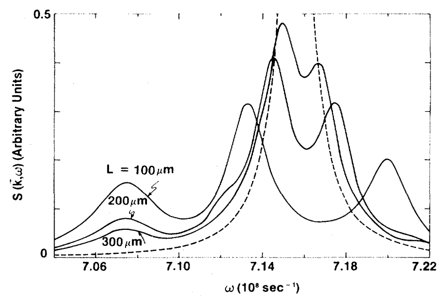

FIG. 2. Dynamic structure factor $S(\vec{k},\omega)$ for a water layer bounded by Pyrex walls for three values of the layer thickness L. The water parameters and the wave vector are as in Fig. 1. Parameters for Pyrex are $\rho_s=2.32$ g/cm^3, $c_t=3.28\times10^5$ cm/sec, $c_t=5.64\times10^5$ cm/sec. The Brillouin peak of the unbounded fluid normalized to unity at maximum (dashed curve) is shown for comparison. The peak corresponding to the interfacial fluid mode is centered at $\omega\approx7.075\times10^8$ sec^{-1}.

platinum walls looks very similar to that for Pyrex walls. However, in the case of gold walls ($c_t<c$), the interfacial fluid peak disappears but a very small peak due to the Rayleigh wave appears at $\omega\approx k_{\|}c_R$.

V. SUMMARY AND CONCLUSIONS

The exact analytic expression for the dynamic structure factor for a fluid layer confined by elastic solid boundaries has been given, under assumptions of continuity of stresses and velocities, as well as entropy and heat fluxes, across the fluid-solid interfaces. In addition, it has been assumed that viscous dissipation in and thermal expansion of the solid boundaries are negligible. In the limit of small ratio of the mass densities of the fluid and solid, $\bar{\rho}/\rho_s$, and small values of $k_{\|}$, analytic expressions for the dispersion relations of various interfacial modes, as well as their contributions to the dynamic structure factor are given. In this limit, four interfacial modes are found. In the limit of $\bar{\rho}/\rho_s\to0$ two of these modes are just the Rayleigh waves and two are the interfacial fluid modes. For small finite values of $\bar{\rho}/\rho_s$, these modes become coupled and the nature of the dominant interfacial modes depends on the values of c, c_R, and c_t; i.e., the adiabatic speed of sound in the fluid, the speed of the Rayleigh wave, and the speed of transverse sound in the solid, respectively. For $c<c_R<c_t$ the dominant modes are the interfacial fluid modes with attenuation $\propto k_{\|}^{3/2}\bar{\rho}/\rho_s$ for small $k_{\|}$ due to dissipative processes whereas the attenuation of the Rayleigh waves varies as $k_{\|}\bar{\rho}/\rho_s$. For $c_R<c<c_t$ both the interfacial fluid modes and the Rayleigh waves have attenuation $\propto k_{\|}^{3/2}\bar{\rho}/\rho_s$. If, however, $c<c_R<c_t$, the interfacial fluid modes are strongly damped with attenuation $\propto k_{\|}(\bar{\rho}/\rho_s)^2$; in this case the Rayleigh waves are the dominant surface modes. Because the coupling between the acoustic modes in the solid and fluid results in a decrease of the speed of the interfacial modes, the corresponding peaks in the dynamic structure factor should be easier to study experimentally than would have been the case with rigid solid boundaries. Such study would enable one to determine the nature of the tangential momentum and energy transport across the fluid-solid interfaces over a wide frequency range in the hydrodynamic regime, and thus dependence of such transport on the nature of interfaces.

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