# Shifts of hydrogen lines from electron collisions in dense plasmas

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The semiclassical impact-parameter method is used to calculate shifts of Lyman and Balmer lines from levels $n \leq 6$ caused by electrons in a dense plasma. Comparison with measurements and estimates of shifts due to ion quadrupole interactions indicate that shifts from distant electron-atom collisions are the major cause of observed shifts of Balmer lines. Blue shifts from ion-atom quadrupole interactions are relatively small for low series members.

## I. INTRODUCTION

Hydrogen lines from stellar atmospheres and dense laboratory plasmas are broader than other lines of similar wavelengths emitted from the same region because hydrogen is subject to linear Stark effect. This effect, in the quasistatic approximation for the slowly varying ion-produced fields, gives symmetrical (Holtsmark) profiles[1] which reflect the statistical distribution of field magnitudes. Electron-produced fields usually vary so rapidly that their broadening effects can be calculated using a suitably generalized impact approximation.[2] To the extent that collisional coupling between levels of different principal quantum numbers can be neglected, the electron broadening is also essentially symmetrical and often comparable to the broadening by ions; indeed, measured profiles are nearly symmetrical and only slightly red shifted.[3–6]

Such shifts are of interest in connection with relativistic red shifts[4] of stellar and solar hydrogen lines and also with plasma shifts of ionized helium lines or lines from other one-electron ions. The latter lines are responding quite similarly to electron- and ion-produced fields, except that for them perturber-radiator correlations are much more important because of the long-range Coulomb interactions. The corresponding initial correlations are believed to be responsible for the plasma polarization shifts, which for lines of the Lyman series of these ions are observed to be to shorter wavelengths,[7–11] i.e., opposite in direction to the electron-produced shifts calculated below.

One purpose of the present paper, which is based on the same methods as recent calculations[12] of shifts of ionized helium lines from distant electron collisions and quadrupole interactions with ions, is to investigate the question whether or not such shifts are sufficient to describe the observations in the case of hydrogen. Agreement within theoretical and experimental errors would then increase our confidence in the corresponding calculations for ionized helium and other one-electron ions and support the conclusion[12] that another effect, e.g., plasma polarization, is responsible for the observed blue shifts of Lyman lines of such ions. Corresponding effects for hydrogen have been found theoretically to be negligibly small.[13]

Any plasma shifts of hydrogen and hydrogenic ion lines are also of interest for wavelength calibrations. However, in such cases densities and temperatures are often low enough for fine structure to be resolved and for effective lifetimes to be long so that broadening and shifts caused by both ions and electrons must be described by a dynamical theory. For hydrogen, experiments and calculations in this low density regime have been done[14] and were found to be reasonably consistent. These calculations were concerned with shifts of resolved fine-structure components, whereas the present paper deals mostly with shifts of the center of gravity of fine-structure components caused by dynamical quadratic Stark effects due to interactions between level groups of principal quantum numbers $n$ and $n+1$.

These dynamical Stark effects caused by plasma electrons are calculated in Sec. II. Then shifts caused by inhomogeneities in ion-produced fields are estimated. The paper concludes with a comparison with laboratory measurements of Balmer line shifts in dense plasmas.

## II. SHIFTS FROM ELECTRON COLLISIONS

As in the calculations for ionized helium,[12] the starting point of the present calculations is the impact-parameter method, semiclassical impact approximation for level shifts $d_{nl}$ in a plasma of electron density $N$

$$d_{nl}=2\pi Nv\int[2\phi(\rho,v)]\rho\,d\rho$$
$$\approx 2\pi Nv\int\rho\,d\rho\left[\frac{2}{3}\left[\frac{\hbar}{m\rho v}\right]^2\sum_{n',l'}\frac{l_>}{2l+1}\left[n',l'\left|\frac{r}{a_0}\right|n,l\right]^2 B(z')\right.$$
$$\left.+\frac{2}{15}\left[\frac{\hbar a_0}{m\rho^2 v}\right]^2\sum_{n',l'}(2l'+1)\begin{bmatrix}l' & 2 & l\\ 0 & 0 & 0\end{bmatrix}^2\left[n',l'\left|\left[\frac{r}{a_0}\right]^2\right|n,l\right]^2 B_q(z')\right]. \quad (1)$$

 

TABLE I. Characteristic values of the adiabaticity parameter $z_n$ for $n'=n+1$ contributions and the corresponding integral dipole shift functions $b(z_n)$ according to Eqs. (2)—(4), the latter applied as described in the text. Asymptotic values $b(z)\sim\pi/4z$ are given in parentheses. All values are for Lyman-series lines. Shift functions for Balmer-series lines are larger by $\lesssim 5\%$.

| | $n=2$ | | $n=3$ | | $n=4$ | | $n=5$ | | $n=6$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| $e^2/\hbar v$ | $z_n$ | $b(z_n)$ | $z_n$ | $b(z_n)$ | $z_n$ | $b(z_n)$ | $z_n$ | $b(z_n)$ | $z_n$ | $b(z_n)$ |
| 6 | 7.49 | 0.106(0.105) | 6.88 | 0.115(0.115) | 5.94 | 0.136(0.13) | 5.18 | 0.115(0.15) | 4.55 | 0.179(0.17) |
| 5 | 5.20 | 0.152(0.15) | 4.78 | 0.169(0.17) | 4.13 | 0.197(0.19) | 3.60 | 0.227(0.22) | 3.16 | 0.263(0.25) |
| 4 | 3.33 | 0.248(0.24) | 3.06 | 0.271(0.26) | 2.65 | 0.321(0.30) | 2.30 | 0.376(0.34) | 2.02 | 0.429(0.39) |
| 3 | 1.87 | 0.467(0.42) | 1.72 | 0.508(0.46) | 1.49 | 0.587(0.53) | 1.30 | 0.667(0.61) | 1.14 | 0.746(0.69) |
| 2 | 0.83 | 0.949(0.94) | 0.76 | 0.987(1.03) | 0.66 | 1.06(1.19) | 0.58 | 1.15(1.36) | 0.51 | 1.19(1.55) |
| 1 | 0.21 | 1.45(3.8) | 0.19 | 1.47(4.1) | 0.17 | 1.49(4.8) | 0.14 | 1.50(5.5) | 0.13 | 1.51(6.2) |

This is Eq. (151) of Ref. 15, which is appropriate as long as the phase shift $2\phi$ and its imaginary counterpart $2\eta$ and higher multipole than dipole interactions are small. The validity of the latter assumption will be checked by also calculating quadrupole contributions due to interactions with levels $n',l'$, which involve the characteristic shift function $B_q(z')$ instead of the dipole function $B(z')$. The argument $z'$ is a measure of the adiabaticity of the interactions

$$z'=(E_n-E_{n'})\rho/\hbar v \ . \tag{2}$$

It depends on the unperturbed energy levels $E_n$ and $E_{n'}$, the impact parameter $\rho$, and the velocity $v$ of the incoming electron. The other quantities in Eq. (1), besides the radial matrix elements, are $l_>=\max(l,l')$ and the velocity $v$ of the perturbing electron.

The integral over impact parameters can be written in terms of integral shift functions, i.e.,

$$\int_{\rho_{\min}}^{\infty}\frac{d\rho}{\rho}B(z')=\int_{z_n}^{\infty}\frac{dz'}{z'}B(z')=b(z_n)\ , \tag{3a}$$

$$\int_{\rho_{\min}}^{\infty}\frac{d\rho}{\rho^3}B_q(z')=\left[\frac{E_n-E_{n'}}{\hbar v}\right]^2\int_{z_n}^{\infty}\frac{dz'}{z'^3}B_q(z')=\left[\frac{E_n-E_{n'}}{\hbar v}\right]^2 b_q(z_n)\ , \tag{3b}$$

where $z_n$ is a cutoff parameter necessitated either by the failure of the dipole approximation for the electron-atom interaction, or by the breakdown of perturbation theory at small impact parameters. In contrast to the case of He II, only the latter is important here, being indicated by $2\phi$ or $2\eta$ approaching unity, i.e., by

$$\frac{2}{3}\left[\frac{e^2}{\hbar v}\right]\sum_{n',l'}\left[\frac{E_n-E_{n'}}{2E_H z_n}\right]^2\frac{l_>}{2l+1}\times\left[n',l'\left|\frac{r}{a_0}\right|n,l\right]^2 B(z_n)\approx 1\ , \tag{4}$$

or a relation with $B(z_n)$ replaced by $A(z_n)$ for $2\eta$, whichever is larger. The quadrupole term is first neglected, and only the most important contributions to the sum, $n'=n(A=1)$, are considered in the estimates of $z_n$ and $b(z_n)$ for $n'=n+1$ presented in Table I. The radial matrix elements (see Table II) were averaged over magnetic and angular momentum quantum numbers with $m=0$ and $\pm 1$ as appropriate for Lyman lines. For Balmer lines, $m=\pm 2$ states contribute as well, but their inclusion in the average over radial matrix elements reduces $z_n$ only by $\lesssim 5\%$ and increases the integral shift functions at most by these amounts. Also, values of the integral shift function for the $z_n$ values corresponding to Eq. (4) are usually not

TABLE II. Average values of dipole and quadrupole matrix elements

$$D_n=\sum_{l'}[l_>/(2l+1)]\left[n+1,l'\left|\frac{r}{a_0}\right|n,l\right]^2$$

and

$$Q_n=\tfrac{1}{5}\sum_{l'}(2l'+1)\begin{pmatrix}l' & 2 & l\\ 0 & 0 & 0\end{pmatrix}^2\left[n+1,l'\left|\left[\frac{r}{a_0}\right]^2\right|n,l\right]^2,$$

of $n'=n$ dipole matrix elements $D_{nn}$ and of the ratios $R_n$ of averaged quadratic Stark-effect coefficients $E_2'$ and $\Delta E_2$ from Eqs. (5b) and (5a). The averages were calculated for the Lyman series, with those for the Balmer series given in parentheses.

| | $D_n$ | $D_{nn}$ | $Q_n$ | $R_n$ |
|---|---|---|---|---|
| 2 | 13.78 | 13.5 | 2.28 | 0.850 |
| 3 | 48.64 (51.92) | 92.6 (81) | $4.18\times10^3(3.38\times10^3)$ | 0.825(1.007) |
| 4 | 124.8(132.4) | 324.0 (293.1) | $3.41\times10^4(3.08\times10^4)$ | 0.851(0.946) |
| 5 | 265.5(279.5) | 830.8 (769.7) | $1.71\times10^5(1.61\times10^5)$ | 0.905(0.959) |
| 6 | 500.2(522.8) | 1771.9(1784.1) | $6.47\times10^5(6.23\times10^5)$ | 0.955(1.007) |

TABLE III. Dipole ($n'\neq n+1$) and quadrupole ($n'=n+1$) corrections $d_n$ and $q_n$ to the shifts according to Eqs. (6) and (7). Also listed are the ratios $r_n$ of integral (dipole) shift functions $b(z_n')$ and $b(z_n)$, with $z_n'$ related to $z_n$ by Eq. (8), and the correction factors $f_n$ to the $n'=n+1$ dipole shifts according to Eq. (9) and based on the functions $b(z_n)$ for the Lyman series in Table I.

| | $n=2$, $L-\alpha$ | | | $n=3$, $L-\beta$ | | | $n=4$, $L-\gamma$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $e^2/\hbar w$ | $d_n$ | $q_n$ | $r_n$ | $f_n$ | $d_n$ | $q_n$ | $r_n$ | $f_n$ | $d_n$ | $q_n$ | $r_n$ | $f_n$ |
| 5 | 0.175 | 0.060 | 0.949 | 1.184 | 0.206 | 0.045 | 0.947 | 1.198 | 0.169 | 0.044 | 0.965 | 1.178 |
| 3 | 0.159 | 0.150 | 0.809 | 1.118 | 0.191 | 0.106 | 0.882 | 1.179 | 0.157 | 0.083 | 0.907 | 1.147 |
| 2 | 0.176 | 0.119 | 0.829 | 1.124 | 0.222 | 0.081 | 0.910 | 1.213 | 0.197 | 0.057 | 0.934 | 1.188 |
| | $n=5$, $L-\delta$ | | | | $n=6$, $L-\epsilon$ | | | | $n=3$, $H-\alpha$ | | | |
| 5 | 0.101 | 0.042 | 0.963 | 1.106 | 0.045 | 0.040 | 0.957 | 1.042 | −0.007 | 0.035 | 1.029 | 1.057 |
| 3 | 0.095 | 0.065 | 0.922 | 1.082 | 0.044 | 0.051 | 0.935 | 1.030 | −0.006 | 0.080 | 0.945 | 1.019 |
| 2 | 0.124 | 0.041 | 0.930 | 1.095 | 0.061 | 0.031 | 0.955 | 1.047 | −0.007 | 0.061 | 0.949 | 1.003 |
| | $n=4$, $H-\beta$ | | | | $n=5$, $H-\gamma$ | | | | $n=6$, $H-\delta$ | | | |
| 5 | 0.055 | 0.037 | 1.009 | 1.081 | 0.041 | 0.038 | 1.000 | 1.079 | −0.007 | 0.037 | 0.983 | 1.013 |
| 3 | 0.051 | 0.071 | 0.954 | 1.076 | 0.039 | 0.058 | 0.954 | 1.051 | −0.006 | 0.047 | 0.955 | 0.996 |
| 2 | 0.064 | 0.049 | 0.961 | 1.074 | 0.050 | 0.037 | 0.949 | 1.036 | −0.009 | 0.029 | 0.966 | 0.986 |

very different from the adiabatic ($z\gg 1$) approximation $b(z)=\pi/4z$ (see entries in parentheses in Table I).

To assess the importance of dipole interactions with other than $n'=n+1$ levels and of shifts of the lower levels of the lines, it is useful to compare calculations of (static) quadratic Stark effects of these lines using the exact expression[16]

$$E_2=-(\tfrac{1}{2}n)^4[17n^2-3(n_1-n_2)^2-9m^2+19]\,, \tag{5a}$$

with the $n'=n+1$ approximation implied so far

$$E_2'=-\tfrac{2}{3}\sum_{l'}\frac{l_>}{2l+1}\left[n+1,l'\left|\frac{r}{a_0}\right|n,l\right]^2 \times\left[\frac{1}{n^2}-\frac{1}{(n+1)^2}\right]^{-1}. \tag{5b}$$

(Both expressions are in atomic energy units and for a fieldstrength $e/a_0^2$.) For this comparison, differences $\Delta E_2$ of $E_2$ values for upper and lower levels were averaged over $m$ and parabolic quantum numbers $n_1$ and $n_2$ according to the relative intensities[17] of the various Stark components, whereas the $E_2'$ values for the upper levels of the lines were averaged as discussed below Eq. (4). As can be seen from Table II, $n'=n+1$ accounts for 83–95% of the average quadratic Stark effect for Lyman lines and for 95% of the effect for $H_\beta$ and $H_\gamma$. In the cases of $H_\alpha$ and $H_\delta$, the $n'=n+1$ approximation even slightly exceeds the exact values. This is possible because the latter involve differences of level shifts. For other than $n'=n+1$ interactions, one can use the adiabatic approximation for all velocities contributing significantly to the Maxwell average. The characteristic dipole shift functions and dipole matrix elements combine, therefore, in the same manner as in the quadratic Stark-effect coefficients, and the relative correction $d_n$ for dipole interactions with $n'\neq n+1$ levels and for dipole shifts of the lower level is given by the product of $(\Delta E_2-E_2')/E_2'$ and the ratio of the adiabatic limit $b(z_n)\sim\pi/4z_n$ and $b(z_n)$ as listed in Table I. This relative correction may therefore be written as

$$d_n=\frac{\pi(R^{-1}-1)}{4z_n b(z_n)}\,, \tag{6}$$

with $R_n$ being the ratio of $E_2'$ and $\Delta E_2$ (see Table II). Values of $d_n$ for some typical electron velocities are listed in Table III.

Another correction to $n'=n+1$ dipole shifts arises from higher multipole interactions. According to Eqs. (1)–(3), the relative correction from, e.g., $n'=n+1$ quadrupole interactions is

$$q_n=\frac{1}{4}\left[\frac{1}{n^2}-\frac{1}{(n+1)^2}\right]^2\frac{Q_n}{D_n}\left[\frac{e^2}{\hbar w}\right]^2\frac{b_q(z_n)}{b(z_n)}\,, \tag{7}$$

where $Q_n$ and $D_n$ are the values of the corresponding quadrupole and dipole matrix elements (see Table II). Values of $q_n$ are listed in Table III. They are so small that contributions from other quadrupole and higher multipole interactions may be neglected.

A final correction to the $n'=n+1$ dipole shift functions obtained so far arises from other than $n'=n$ contri-

butions in Eq. (4). Clearly $n'=n+1$ will by far be the largest correction, and improved values $z_n'$ for $z_n$ may therefore be calculated from

$$z_n' = \left[1+\frac{D_n}{D_{nn}}\max(A,B)\right]^{1/2} z_n \ . \tag{8}$$

Here $D_n$ is the $n'=n+1$ dipole matrix element, $D_{nn}$ the $n'=n$ matrix element, and the width and shift functions are taken at $z_n$. For Balmer lines, an additional factor given by the square root of the ratio of $D_{nn}$ matrix elements arises if the $b(z_n)$ functions of Lyman lines are used as a reference. Such $z_n'$ values can be used to calculate improved values $b(z_n')$ of the integral dipole shift function. The entire correction factor to be applied to the shifts calculated accounting for $n'=n+1$ dipole interactions only, is finally

$$f_n = \frac{b(z_n')}{b(z_n)} + d_n + q_n \ . \tag{9}$$

As can be seen from Table III, which also contains values of $b(z_n')/b(z_n)=r_n$, these correction factors are so close to unity that no significant errors should arise from this part of the calculation. However, it is well known[15,18] that in the limit $z \gg 1$, an additional factor of 1.16 arises from collisions within the impact parameter $\rho_m$, while the recent calculations[12] for He II and comparison with $R$-matrix calculations[19] suggest a factor of $\sim 1.25$.

Shifts of Lyman and Balmer lines are therefore calculated from

$$\begin{aligned}\overline{d_{nl}} &= 1.2\frac{4\pi}{3}\left[\frac{\hbar}{m}\right]^2 N D_n \left[\frac{1}{v} b(z_n) f_n\right]_{\rm av} \\ &= 1.2\frac{4\pi}{3}\left[\frac{\hbar}{m}\right]^2 \left[\frac{2m}{\pi kT}\right]^{1/2} N D_n \int e^{-x} b(z_n) f_n dx \ ,\end{aligned} \tag{10}$$

with

$$x = \frac{mv^2}{2kT} = \left[\frac{\hbar v}{e^2}\right]^2 \frac{E_H}{kT} \ , \tag{11}$$

introduced in order to facilitate the Maxwell average. Values of the integrals and corresponding wavelength shifts can be found in Table IV. These shifts are very insensitive to temperature and are linear in electron density. The increase with temperature is approximately $T^{1/3}$ rather than $T^{1/6}$ as would be predicted by the adiabatic (phase shift) limit for dipole interactions. Deviations from the latter approximation are not very large for typical conditions. This can be seen by comparison with an estimate for Balmer $\gamma$ based on this approximation in Eq. (21) of Ref. 3, namely, $\Delta\lambda = 2.5\times 10^{17} N$ at $T = 1.2\times 10^4$ K. Similar reductions by factors of $\sim 2$ below the adiabatic limit are quite typical also for lines from neutral atoms[15,18] other than hydrogen for which most perturbing levels are either higher or lower than the upper state of the line.

## III. SHIFTS FROM QUADRUPOLE INTERACTIONS WITH IONS

As mentioned in the Introduction, hydrogen lines from dense plasmas are very nearly symmetrical. In the case of quasistatic broadening by ions, this symmetry is simply a consequence of the symmetrical linear Stark splitting in fields large enough for fine structure to be negligible and small enough for quadratic Stark effects to be relatively small. However, the local fields acting on the atoms are not strictly homogeneous so that besides dipole-field interactions, quadrupole-field gradient interactions must be considered as well. Following Demura and Sholin,[20] the corresponding profile asymmetry can be analyzed and be used to calculate (truncated) first moments $\overline{\Delta\lambda}$ of the profiles, namely,[12]

$$\overline{\Delta\lambda} = -\frac{\alpha}{2}\lambda^2 a_0^2 N \frac{1}{\sum_{k'} I_{k'}} \left[\sum_k{}'' I_k \Delta_k^q \int_0^{\beta_k} \beta\Lambda(\beta)d\beta + \sum_k{}' I_k \left[\epsilon_k^{(1)}\Delta_k^d \int_0^{\beta_k} \beta\Lambda(\beta)d\beta + \Delta_k^q \int_0^{\beta_k} \beta\chi(\beta)d\beta\right]\right] . \tag{12}$$

TABLE IV. Thermal averages of corrected (see text) integral dipole shift functions $b(z_n)f_n$ and wavelength (red) shifts $\Delta\lambda$ due to electron collisions (at an electron density of $10^{17}$ cm$^{-3}$).

| | | | | Lyman lines | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | | | $b(z_n)f_n$ | | | | | $\Delta\lambda$ (mÅ) | | |
| ($10^3$ K) | $n=2$ | $n=3$ | $n=4$ | $n=5$ | $n=6$ | $n=2$ | $n=3$ | $n=4$ | $n=5$ | $n=6$ |
| 7.9 | 0.229 | 0.257 | 0.289 | 0.307 | 0.326 | 3.8 | 10.8 | 28.1 | 60.6 | 118 |
| 10.5 | 0.302 | 0.339 | 0.377 | 0.397 | 0.418 | 4.4 | 12.4 | 31.8 | 67.9 | 131 |
| 15.8 | 0.427 | 0.480 | 0.525 | 0.543 | 0.565 | 5.1 | 14.3 | 36.1 | 75.8 | 145 |
| | | | | Balmer lines | | | | | | |
| | | | $b(z_n)f_n$ | | | | | $\Delta\lambda$ (Å) | | |
| 7.9 | | 0.222 | 0.266 | 0.298 | 0.315 | | 0.409 | 0.686 | 1.29 | 2.28 |
| 10.5 | | 0.291 | 0.347 | 0.383 | 0.402 | | 0.465 | 0.776 | 1.44 | 2.53 |
| 15.8 | | 0.406 | 0.479 | 0.520 | 0.540 | | 0.529 | 0.874 | 1.59 | 2.77 |

TABLE V. Wavelength (blue) shifts $\overline{\Delta\lambda}$ from ion quadrupole interactions at an ion density[a] of $N=10^{17}$ cm$^{-3}$. The shift values are averaged over portions of the line profiles between $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ intensity points corresponding to calculated fractional widths[b] $\alpha_{1/2}$, etc., designated here as ranges $\frac{1}{2}$, etc.

| Range | $n=2$ | $n=3$ | $n=4$ | $n=5$ | $n=6$ |
|---|---|---|---|---|---|
| | | $-\overline{\Delta\lambda}$ (mÅ) for Lyman lines | | | |
| $\frac{1}{2}$ | 0.91 | 1.41 | 8.6 | 19.1 | 43 |
| $\frac{1}{4}$ | 1.37 | 2.99 | 16.9 | 37.2 | 76 |
| $\frac{1}{8}$ | 1.75 | 4.22 | 21.4 | 47.3 | 83 |
| | | $-\overline{\Delta\lambda}$ (Å) for Balmer lines | | | |
| $\frac{1}{2}$ | | 0.074 | 0.125 | 0.35 | 0.74 |
| $\frac{1}{4}$ | | 0.107 | 0.239 | 0.69 | 1.43 |
| $\frac{1}{8}$ | | 0.138 | 0.333 | 0.90 | 1.84 |

[a]The calculated shifts are approximately linear in ion density and depend only weakly on temperature (see text).

[b]From P. Kepple and H. R. Griem, Phys. Rev. 173, 317 (1968) or extrapolations. See also Table AIIIa of Ref. 15.

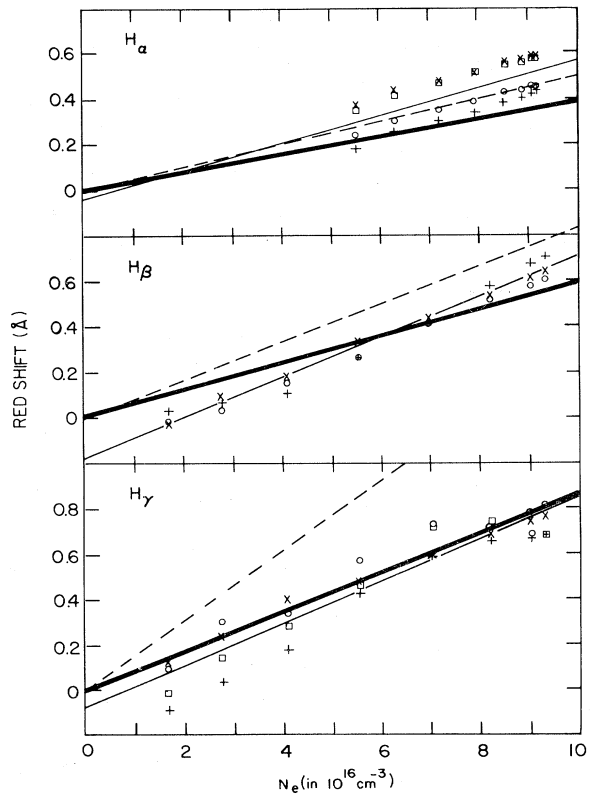


FIG. 1. Comparison between measured shifts (from Ref. 5) of the Balmer $\alpha$, $\beta$, and $\gamma$ lines with the present calculations of the combined electron- and ion-produced shifts (solid lines) and of the electron-produced shifts (dashed lines). The intercepts are indications of uncertainties in the absolute wavelength scale, which was determined using Fe I lines as standards.

These mean values of shifts calculated over symmetric wavelength ranges $\pm\Delta\lambda'$ from the unperturbed line center arise from three physical effects: the quadrupole shift of unshifted Stark components ( $\sum''$ ), the intensity changes of shifted components due to field gradients (first term in $\sum'$ ), and the asymmetry of the profiles of shifted components from field gradients (second term in $\sum'$ ). These effects are characterized by dimensionless quadrupole splitting factors $\Delta_k^q$ and relative intensity correction factors $\epsilon_k^{(1)}$, while $\Delta_k^d$ is the usual linear Stark-effect (dipole) splitting factor, all in Sholin's notation.[21] The $I_k$ are the relative intensities of the various Stark components, and the functions $\Lambda(\beta)$ and $\chi(\beta)$ are special functions[20] of the reduced field strength $\beta=F/F_0$ ($F_0\approx e/r_0^2$, $4\pi N r_0^3/3=1$).

The limits of the integrals over $\beta$ must be chosen such that the corresponding (first-order) displacements are within $\pm\Delta\lambda'$. For unshifted components, this condition gives

$$\beta_k=\left[\frac{4\pi r_0 e\,\Delta\lambda'}{3\alpha\lambda^2 F_0 a_0^2\,|\,\Delta_k^q\,|}\right]^{2/3}, \tag{13a}$$

for shifted components

$$\beta_k=\frac{4\pi e\,\Delta\lambda'}{3\alpha\lambda^2 F_0 a_0\,|\,\Delta_k^d\,|}\,. \tag{13b}$$

These limits are, of course, well defined only to the extent that the electron broadening is smaller than $\Delta\lambda'$ and that the quasistatic approximation can be used to describe effects produced by ions. Both of these assumptions are readily justified if $\Delta\lambda'$ is well beyond the half-width of the profile. However, they are not true within the half-width, especially not for lines with strong unshifted components.

Calculated values of $\overline{\Delta\lambda}$ for $\Delta\lambda'=\Delta\lambda_{1/2}$, $\Delta\lambda_{1/4}$, and

$\Delta\lambda_{1/8}$, where the $\Delta\lambda_{1/n}$ are fractional intensity widths,[15] are presented in Table V. To allow for the effects of ion-ion correlations and Debye shielding, the integrands in Eq. (12) were multiplied with the ratio of fieldstrength distribution functions[22] $H_a(\beta)$, which include these effects, and the Holtsmark distribution[1] $H_0(\beta)$. Here $a$ is the ratio of $r_0$ and the electron Debye radius, $\rho_D=(kT/4\pi Ne^2)^{1/2}$. Since $a$ varies only slowly with density and temperature, namely, $a=2.2(e^2N^{1/3}/kT)^{1/2}$, results are presented only for $a=0.6$, a typical value for arc plasmas. Shifts due to ion-atom quadrupole interactions for, e.g., $a=0.4$ are larger by $\sim 10\%$.

## IV. COMPARISON WITH EXPERIMENTS

Although the asymmetry of the profiles is rather weak, a convention regarding the definition of line shifts is required. For example, one may[5] define the line center as the average wavelength of the points bisecting the $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ widths, and take the difference of this average and the theoretical wavelength, averaged over fine structure, as the shift. This convention is closely equivalent to using ion quadrupole shifts $\overline{\Delta\lambda}$ from Table V for $\Delta\lambda'=\pm\Delta\lambda_{1/4}$, which was, therefore, done to obtain the (solid) theoretical curves in Fig. 1. The electron-produced shifts were taken from Table IV, scaled for temperature ($\sim 13\times 10^3$ K) from the calculated values for $T=10.5\times 10^3$ K by a factor of $\sim 1.074$.

Agreement with measurements[5] is quite satisfactory, in particular if one remembers that especially for the $\alpha$ line the fine structure is not entirely negligible[23] ($\sim 0.14$ Å), with the strongest component being at about $+0.05$ Å from the mean wavelength of the fine-structure components. Measurements[6] of $H_\alpha$ and $D_\alpha$ relative to the Doppler and fine-structure broadened Geissler-tube spectrum gave $\Delta\lambda=(0.43\pm 0.05)$ Å at $N=10^{17}$ cm$^{-3}$ and $T=13\times 10^3$ K for both lines and measured at one-half intensity. This compares with $\Delta\lambda=0.50-0.07$ Å$=0.43$ Å from Tables IV and V. The absence of any mass dependence is, of course, consistent with the smallness of the ion contribution. For $H_\gamma$, the earlier and less accurate measurement[3] gave red shifts between 0.5 and 1.4 Å at densities approaching $10^{17}$ cm$^{-3}$ according to observed linewidths.

We, therefore, conclude that red shifts from distant electron-atom collisions combined with smaller but significant blue shifts from ion-atom quadrupole interactions account very satisfactorily for the observed red shifts of hydrogen lines in dense plasmas. This result supports the conclusion of an earlier paper[12] that in the case of ionized helium Lyman-series lines other effects, e.g., the plasma polarization shift, play an important role, and that the linearized Debye model for this additional shift is inconsistent with the observed shifts[24] of He II lines which are analogous to the Paschen series.

Returning to hydrogen lines, we also note that an earlier interpretation[25] of the observed plasma red shifts entirely in terms of ion-atom quadrupole interactions led to red shifts similar to observed shifts rather than to the smaller blue shifts calculated here. This discrepancy and erroneous interpretation can be traced to the use of unphysically large $\beta_k$ values in Ref. 25 in a relation corresponding to Eq. (12). Such large $\beta_k$ values do not contribute significantly to the regions of the profile involved in the shift measurements. However, neglecting ion quadrupole effects altogether would be incorrect as well. This can be seen from the significant overestimate obtained for $H_\gamma$ if the calculated electron-produced shifts were taken alone (see dashed curves in Fig. 1).

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