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Theory of resonance fluorescence in a fluctuating laser field

W. Vogel and D.-G. Welsch
Sektion Physik, Friedrich-Schiller—Universität Jena, DDR-6900 Jena, Max-Wien-Platz 1, German Democratic Republic

K. Wódkiewicz
Institute of Theoretical Physics, Warsaw University, Warsaw 00-681, Hóza 69, Poland
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A method of calculation of multiple-time correlation functions of the resonance fluorescence light from a two-level atom driven by a realistic laser field with phase and amplitude fluctuations is developed. Both kinds of fluctuations are assumed to be Gaussian. The phase fluctuations are treated in the phase-diffusion model. The averaging over the amplitude fluctuations can be performed in a closed way when the relative mean-square amplitude fluctuation is small compared with the ratio of the amplitude correlation decay rate to the characteristic atomic relaxation rate. There is no restriction to Markovian amplitude fluctuations. Arbitrary values for the mean Rabi frequency are allowed. Results are presented for the intensity, the intensity correlation function, and the spectrum of the fluorescent light. It is shown that even in the case when the relative strength of the amplitude fluctuations is small, significant effects can occur due to the finite correlation length of the fluctuations. The results for fast amplitude correlation decay correspond to the situation without amplitude fluctuations. When the correlation time of the amplitude fluctuations becomes comparable with the atomic relaxation times, the Rabi oscillations of the intensity and the intensity correlation function are nearly washed out, and non-Lorentzian line shapes for the side peaks in the spectrum are observed. It is found that the ratio of the heights of the central peak to the side peak increases with increasing correlation time.

I. INTRODUCTION

For the last few years the theory of resonant interaction between atomic systems and fluctuating light fields has been a subject of increasing interest.[1–18] Various effects have been studied, such as the spectrum and the intensity correlation function of the resonance fluorescence light from a two-level atom, optical double resonance, and multiphoton ionization.

A simple case of a fluctuating light field is a laser field the phase of which fluctuates according to a Wiener-Levy process. For such a phase-diffusion field the statistical averaging of the atomic equations of motion can be performed exactly.[1–3] The effect of amplitude fluctuations was first studied by Eberly[4] on the basis of decorrelation assumptions.

Recently, methods have been developed to study the effects of Markovian field fluctuations.[5–9] The averaging of the atomic equations of motion leads, in general, to an infinite set of coupled differential or integral equations, which are solved, after truncation, by numerical methods. In particular, calculations have been performed for chaotic fields,[5–8] Gaussian-amplitude fields,[7,8] and phase-diffusion fields.[9] In a recent work[17] the problem of resonant interaction between a fluctuating single-mode laser field with a two-level atom has been treated by utilizing a kind of multiple—time-scale method. Since this method gives an expansion in powers of the ratio of the atomic decay constant to the Rabi frequency the expression derived for the fluorescent spectrum is valid as a high—driving-field approximation. All the results of investigation of the spectrum of resonance fluorescence show that the side peaks appearing in the case of a resonant, intense, coherent, driving field[19] are washed out due to the amplitude fluctuations of the exciting field.

Besides the spectrum there has been a growing interest in the intensity correlation function of the fluorescent light. This interest results partly from the possibility of observing the effect of photon antibunching[20–30] and testing the quantum nature of light and partly from possibilities of spectroscopic applications.[31–34] Both aspects require a careful investigation of the effects of fluctuating, driving fields.

As has been shown,[1,2,10] the intensity correlation function of the resonance fluorescence light from a two-level atom can be decomposed into two intensities when the exciting field is a coherent or a phase-diffusion field. For other kinds of driving fields the situation can drastically change. Analytical calculations[11–13] performed for various stochastic, driving fields in the weak-field limit show that for chaotic pump fields[11] and laser fields with amplitude fluctuations[12,13] the factorization of the intensity correlation function is, in general, impossible. It is worth noting that in the case of laser fields with phase fluctuations leading to non-Lorentzian line shapes the factorization is a very good approximation.[13] The effects of extremely strong, stochastic, driving fields have been studied as well.[14] However, the validity of the analytical results presented for Gaussian-amplitude fields and chaotic fields are restricted by the condition that the mean-square deviation of the Rabi frequency and the bandwidth of the exciting field must be large compared with the line width of the atomic transition because the atomic relaxation is disregarded in the theory. The results are only valid as long as the time remains small compared with atomic relaxation time. Therefore the attempt[9,35] has been made to utilize the methods[5–9] outlined above. Numerical calculations are in preparation.[36]

An alternative, analytical method for treating the effects of fluctuating light fields in optical processes has recently been developed by making use of Feynman's operator-algebra and path-integration technique.[3,15,16] This method, which is equivalent to van Kampen's cumulant expansion,[18] requires small and/or fast fluctuations. Furthermore, it is, in general, restricted to one-time averages and cannot be applied to the calculation of the spectrum, the intensity correlation function, and other higher-order correlation properties of the resonance fluorescence light in a straightforward way.

In this paper we present a theory of resonance fluorescence from a two-level atom driven by a realistic single-mode laser field with Gaussian phase and amplitude fluctuations, the latter satisfying the condition that the relative mean-square deviation of the Rabi frequency is small compared with the ratio of a characteristic atomic relaxation time to the correlation time of the amplitude fluctuations. We note that for a laser with intensity fluctuations of the order of a percent this condition is no restriction to fast correlation decay. The phase fluctuations are taken into account within the phase-diffusion model usually used. We present closed solutions for the intensity correlation function (Sec. II) and for the spectrum (Sec. III) of the fluorescent light, without assumptions about the value of the Rabi frequency, without assumptions about the time scale, and without the restriction to Markovian amplitude fluctuations.

II. INTENSITY CORRELATION OF THE FLUORESCENCE

The intensity $I(t)$ and the intensity correlation function $G^{(2)}(t,t+\tau)$ of a given light field (at fixed space point $\vec{r}$) are defined by the following equations:

$$I(t)=\sum_i \langle E_i^{(-)}(t)E_i^{(+)}(t)\rangle \ , \tag{2.1}$$

$$G^{(2)}(t,t+\tau)=\sum_{i,j} \langle E_i^{(-)}(t)E_j^{(-)}(t+\tau) \times E_j^{(+)}(t+\tau)E_i^{(+)}(t)\rangle \ , \tag{2.2}$$

where the vector components E_i of the operator of the electric field strength are decomposed into positive- ($E_i^{(+)}$) and negative- ($E_i^{(-)}$) frequency parts. In the case of resonant interaction between a two-level atom and a pump light field the intensity and the intensity correlation function of the scattered light can be expressed in terms of atomic correlation functions[10,12,19]:

$$I\left[t+\frac{r}{c}\right]=f(\vec{r})\sigma_{22}(t) \ , \tag{2.3}$$

$$G^{(2)}\left[t+\frac{r}{c},t+\frac{r}{c}+\tau\right]=[f(\vec{r})]^2 G_{22}(t,t+\tau) \ , \tag{2.4}$$

where the spatial function is given by

$$f(\vec{r})=\left[\frac{\omega_{21}}{c}\right]^4 \left|\frac{\vec{D}_{21}}{r}-\frac{(\vec{D}_{21}\cdot\vec{r})\vec{r}}{r^3}\right|^2 , \tag{2.5}$$

ω_{21} and $\vec{D}_{21}$, respectively, being the atomic transition frequency and the transition matrix element of the electric dipole operator. The occupation probability for the excited atomic quantum state $\sigma_{22}(t)$ and the correlation function $G_{22}(t,t+\tau)$ can be written as expectation values of flip operators $A_{nm}(t)$ [$A_{nm}(t=0)=|n\rangle\langle m|$ with $n,m=1,2$ and with $|1\rangle$ and $|2\rangle$, respectively, being the atomic ground-state vector and the excited-state vector]:

$$\sigma_{22}(t)=\langle A_{22}(t)\rangle \ , \tag{2.6}$$

$$G_{22}(t,t+\tau)=\langle A_{21}(t)A_{22}(t+\tau)A_{12}(t)\rangle \ . \tag{2.7}$$

In order to calculate $G_{22}(t,t+\tau)$ we turn to the Bloch equations of motion. For this purpose we write the excitation laser field in the form

$$E_L(t)=E_L^{(+)}(t)+E_L^{(-)}(t)\ ,$$
$$E_L^{(+)}(t)=\tfrac{1}{2}[\hat{E}_L+\delta\hat{E}_L(t)]e^{-i[\omega_L t+\varphi_L(t)]}\ , \tag{2.8}$$
$$E_L^{(-)}(t)=[E_L^{(+)}(t)]^*\ ,$$

where $\delta\hat{E}_L(t)$ and $\varphi_L(t)$, respectively, are real Gaussian random variables for the amplitude and the phase fluctuations of the laser field. The Yariv-Caton laser model[37] used in this paper is outlined in Appendix A. We now introduce slowly varying atomic operators defined by

$$\tilde{A}_{12}(t)=e^{i[\omega_L t+\varphi_L(t)]}A_{12}(t)\ ,\quad \tilde{A}_{21}(t)=\tilde{A}^{\dagger}_{12}(t)\ , \tag{2.9}$$

and write the Bloch equations of motion in the compact vector form

$$\frac{d}{d\tau}\,|\,\Psi(t,t+\tau))=M(t+\tau)\,|\,\Psi(t,t+\tau))\ , \tag{2.10}$$

where the components of the four-dimensional vector $|\,\Psi(t,t+\tau))$ are defined by

$$(1\,|\,\Psi(t,t+\tau))=\langle\tilde{A}_{21}(t)A_{22}(t+\tau)\tilde{A}_{12}(t)\rangle\ , \tag{2.11}$$
$$(2\,|\,\Psi(t,t+\tau))=\langle\tilde{A}_{21}(t)A_{11}(t+\tau)\tilde{A}_{12}(t)\rangle\ , \tag{2.12}$$
$$(3\,|\,\Psi(t,t+\tau))=\langle\tilde{A}_{21}(t)\tilde{A}_{12}(t+\tau)\tilde{A}_{12}(t)\rangle\ , \tag{2.13}$$
$$(4\,|\,\Psi(t,t+\tau))=\langle\tilde{A}_{21}(t)\tilde{A}_{21}(t+\tau)\tilde{A}_{12}(t)\rangle\ , \tag{2.14}$$

and the 4×4 matrix $M(t)$ is given by

$$M(t)=\begin{bmatrix} -\Gamma_1 & 0 & -\frac{i}{2}[\Omega_R+\delta\Omega_R(t)] & \frac{i}{2}[\Omega_R+\delta\Omega_R(t)] \\ \Gamma_1 & 0 & \frac{i}{2}[\Omega_R+\delta\Omega_R(t)] & -\frac{i}{2}[\Omega_R+\delta\Omega_R(t)] \\ -\frac{i}{2}[\Omega_R+\delta\Omega_R(t)] & \frac{i}{2}[\Omega_R+\delta\Omega_R(t)] & -i\,\delta\omega-\Gamma_2+i\dot{\varphi}_L(t) & 0 \\ \frac{i}{2}[\Omega_R+\delta\Omega_R(t)] & -\frac{i}{2}[\Omega_R+\delta\Omega_R(t)] & 0 & i\,\delta\omega-\Gamma_L-i\dot{\varphi}_L(t) \end{bmatrix}\ . \tag{2.15}$$

Here the amplitude and the amplitude fluctuation of the driving laser field are expressed in terms of Rabi frequencies $[\Omega_R,\delta\Omega_R(t)]$. Γ_1 and Γ_2, respectively, are the rates of energy and phase relaxation, $\delta\omega=\omega_{21}-\omega_L$. From Eqs. (2.11)–(2.14) the initial condition for $|\,\Psi(t,t+\tau))$ is seen to be

$$(i\,|\,\Psi(t,t))=\delta_{i,2}\langle A_{22}(t)\rangle\ . \tag{2.16}$$

In order to calculate $\langle A_{22}(t)\rangle$ we again make use of the Bloch equations:

$$\frac{d}{dt}\,|\,\Phi(t))=M(t)\,|\,\Phi(t))\ , \tag{2.17}$$

where

$$(1\,|\,\Phi(t))=\langle A_{22}(t)\rangle\ , \tag{2.18}$$
$$(2\,|\,\Phi(t))=\langle A_{11}(t)\rangle\ , \tag{2.19}$$
$$(3\,|\,\Phi(t))=\langle\tilde{A}_{12}(t)\rangle\ , \tag{2.20}$$
$$(4\,|\,\Phi(t))=\langle\tilde{A}_{21}(t)\rangle\ . \tag{2.21}$$

We now assume that at time $t=0$ the atom is in the ground state. This implies the following initial condition for $|\,\Phi(t))$:

$$(i\,|\,\Phi(0))=\delta_{i,2}\ . \tag{2.22}$$

The solutions of Eqs. (2.10) and (2.17), which satisfy the initial conditions given by Eqs. (2.16) and (2.22), can be found by formal integration. The result is

$$|\,\Psi(t,t+\tau))=S(t,t+\tau)\,|\,\Psi(t,t))$$
$$=S(t,t+\tau)\,|\,2)(1\,|\,\Phi(t))\ , \tag{2.23}$$
$$|\,\Phi(t))=S(0,t)\,|\,2)\ , \tag{2.24}$$

where $S(t_1,t_2)$ is the time-ordered exponential matrix

$$S(t_1,t_2)=T\exp\left[\int_0^{t_2-t_1}d\tau\,M(t_1+\tau)\right]. \tag{2.25}$$

Combining Eqs. (2.7), (2.9), (2.11), (2.23), and (2.24) the correlation function $G_{22}(t,t+\tau)$ can be represented in the following form:

$$G_{22}(t,t+\tau)=(1\,|\,S(t,t+\tau)\,|\,2)(1\,|\,S(0,t)\,|\,2)\ . \tag{2.26}$$

We note that $G_{22}(t,t+\tau)$ is equal to the product of two occupation probabilities for the upper atomic quantum state. In Eq. (2.26), $(1\,|\,S(0,t)\,|\,2)=\sigma_{22}(t)$ is the probability that at time t the upper atomic level is occupied provided that at time $t=0$ the ground state has been occupied. Analogously, $(1|S(t,t+\tau)|2)$ is the occupation probability for the upper atomic quantum state at time $t+\tau$ when at time t the atom has been in the ground state.

In the case of a stochastic, driving field we have to average Eq. (2.26) over the fluctuations of the field:

$$\langle G_{22}(t,t+\tau)\rangle_{\rm st} = \langle (1|S(t,t+\tau)|2)(1|S(0,t)|2)\rangle_{\rm st} . \quad (2.27)$$

The average excited-state population is given by

$$\langle \sigma_{22}(t)\rangle_{\rm st} = \langle (1|S(0,t)|2)\rangle_{\rm st} . \quad (2.28)$$

It is seen that, in general, the average correlation function given by Eq. (2.27) cannot be decomposed into two average excited-state populations.

From the derivation of Eq. (2.27) it is seen that this equation holds for arbitrary, stochastic, driving fields. It is not restricted to the stochastic model considered in this paper. Equation (2.27) was first derived for the case of vanishing atomic relaxation.[14] More general conditions have recently been considered.[35,38]

We now turn to the problem of performing in Eq. (2.27) the averaging over the amplitude and the phase fluctuations of the driving laser field under consideration. For this purpose we assume the phase of the laser field is a Wiener-Levy process. In this phase-diffusion model $\dot{\varphi}_L$ is a Gaussian random variable with

$$\langle \dot{\varphi}_L(t)\rangle_{\rm st}=0 , \qquad \langle \dot{\varphi}_L(t)\dot{\varphi}_L(t')\rangle_{\rm st}=2\Gamma_L\delta(t-t') , \quad (2.29)$$

where $2\Gamma_L$ is the line width of the laser light. Owing to the δ correlation assumed the averaging over the phase fluctuations in Eq. (2.27) can be performed by averaging $(1|S(t,t+\tau)|2)$ and $(1|S(0,t)|2)$ independently of each other. Standard methods[1–3] lead to the result that in the matrices $M(t+\tau)$ and $M(t)$ defined by Eq. (2.15) both $i\dot{\varphi}_L$ and $-i\dot{\varphi}_L$ must be replaced by $-\Gamma_L$. The effect of the phase fluctuations therefore is the modification of the dephasing rate: $\Gamma_2\rightarrow\Gamma_2+\Gamma_L$.

The remaining problem of performing the averaging over the amplitude fluctuations is, in general, more complicated because of their finite correlation time. In order to find a suitable approximation we subdivide the matrix M given by Eq. (2.15) into three parts (after substituting $-\Gamma_L$ for the $i\dot{\varphi}_L$ and $-i\dot{\varphi}_L$):

$$M(t)=M_0\left[1+\frac{\delta\Omega_R(t)}{\Omega_R}\right]-\frac{\delta\Omega_R(t)}{\Omega_R}M_1 , \quad (2.30)$$

where the matrices M_0 and M_1 are defined by

$$M_0=\begin{bmatrix} -\Gamma_1 & 0 & -\frac{i}{2}\Omega_R & \frac{i}{2}\Omega_R \\ \Gamma_1 & 0 & \frac{i}{2}\Omega_R & -\frac{i}{2}\Omega_R \\ -\frac{i}{2}\Omega_R & \frac{i}{2}\Omega_R & -i\,\delta\omega-\Gamma_2-\Gamma_L & 0 \\ \frac{i}{2}\Omega_R & -\frac{i}{2}\Omega_R & 0 & i\,\delta\omega-\Gamma_2-\Gamma_L \end{bmatrix} , \quad (2.31)$$

$$M_1=\begin{bmatrix} -\Gamma_1 & 0 & 0 & 0 \\ \Gamma_1 & 0 & 0 & 0 \\ 0 & 0 & -i\,\delta\omega-\Gamma_2-\Gamma_L & 0 \\ 0 & 0 & 0 & i\,\delta\omega-\Gamma_2-\Gamma_L \end{bmatrix} . \quad (2.32)$$

For our laser model we can assume that the condition

$$\frac{\langle(\delta\Omega_R)^2\rangle_{\rm st}}{\Omega_R^2}\Gamma\tau_A \ll 1, \quad \Gamma=\max(\Gamma_1,\Gamma_2+\Gamma_L) \quad (2.33)$$

is fulfilled, τ_A being the correlation time of the amplitude fluctuations. For simplicity we confine ourselves to the case of exactly resonant excitation, that is, $\delta\omega=0$. If the amplitude fluctuations are not too strong, as should be realistic for many cases, the condition (2.33) is no restriction to fast correlation decay. In consequence of the condition (2.33) we can now disregard in Eq. (2.30) the term proportional to M_1 (see Appendix B):

$$M(t)=M_0\left[1+\frac{\delta\Omega_R(t)}{\Omega_R}\right] . \quad (2.34)$$

Combining the results from Eqs. (2.25), (2.27), and (2.34) we obtain

$$\langle G_{22}(t,t+\tau)\rangle_{\rm st} = \left\langle (1\,|\exp\left[M_0\left[\tau+\int_0^\tau d\tau_1\frac{\delta\Omega_R(t+\tau_1)}{\Omega_R}\right]\right]|\,2)(1\,|\exp\left[M_0\left[t+\int_0^t d\tau_2\frac{\delta\Omega_R(\tau_2)}{\Omega_R}\right]\right]|\,2)\right\rangle_{\rm st} . \quad (2.35)$$

It is convenient to perform the statistical averaging in the M_0 representation

$$M_0\,|\,\lambda_k) = \lambda_k\,|\,\lambda_k)\ . \quad (2.36)$$

Standard methods yield

$$\langle G_{22}(t,t+\tau)\rangle_{\rm st} = \sum_{k,l} C_k^{1,2}C_l^{1,2}e^{\lambda_k\tau+\lambda_l t}\exp\left[\frac{1}{\Omega_R^2}[\lambda_k^2 g^{(1)}(t,\tau)+\lambda_l^2 g^{(1)}(0,t)+\lambda_k\lambda_l g^{(2)}(t,\tau)]\right] , \quad (2.37)$$

where

$$g^{(1)}(t_1,t_2) = \int_0^{t_2} d\tau \int_0^\tau d\tau' \langle \delta\Omega_R(t_1+\tau)\delta\Omega_R(t_1+\tau')\rangle_{\rm st} , \quad (2.38)$$

$$g^{(2)}(t_1,t_2) = \int_0^{t_2} d\tau \int_0^{t_1} d\tau' \langle \delta\Omega_R(t_1+\tau)\delta\Omega_R(\tau')\rangle_{\rm st} , \quad (2.39)$$

and

$$C_k^{i,j} = (i\,|\,\lambda_k)(\lambda_k\,|\,j)\ . \quad (2.40)$$

The eigenvalues λ_k and the coefficients $C_k^{1,2}$ can be found by straightforward calculation. The result is

$$\lambda_1=0,\quad \lambda_2=-(\Gamma_2+\Gamma_L)\ ,$$
$$\lambda_3=-\tfrac{1}{2}(\Gamma_1+\Gamma_2+\Gamma_L) +[\tfrac{1}{4}(\Gamma_1-\Gamma_2-\Gamma_L)^2-\Omega_R^2]^{1/2}\ , \quad (2.41)$$
$$\lambda_4=-\tfrac{1}{2}(\Gamma_1+\Gamma_2+\Gamma_4) -[\tfrac{1}{4}(\Gamma_1-\Gamma_2-\Gamma_L)^2-\Omega_R^2]^{1/2}\ ,$$

and

$$C_1^{1,2}=\tfrac{1}{2}\left[1-\frac{\Gamma_1(\Gamma_2+\Gamma_L)}{\lambda_3\lambda_4}\right],\quad C_2^{1,2}=0\ ,$$
$$C_3^{1,2}=-\frac{1}{2}\frac{(\Gamma_1+\lambda_4)(\Gamma_1+\lambda_3)}{\lambda_3(\lambda_4-\lambda_3)}\ , \quad (2.42)$$
$$C_4^{1,2}=-\frac{1}{2}\frac{(\Gamma_1+\lambda_3)(\Gamma_1+\lambda_4)}{\lambda_4(\lambda_3-\lambda_4)}\ .$$

It should be noted that when the amplitude fluctuates according to a stationary Gaussian process $g^{(1)}(t_1,t_2)$ does not depend on t_1. We further emphasize that the result (2.37) is valid for arbitrary values of the (mean) Rabi frequency Ω_R and that there is no restriction to Markovian amplitude fluctuations. As can easily be seen the average excited-state population is given by

$$\langle \sigma_{22}(t)\rangle_{\rm st} = \sum_k C_k^{1,2}e^{\lambda_k t}\exp\left[\frac{\lambda_k^2}{\Omega_R^2}g^{(1)}(0,t)\right] . \quad (2.43)$$

When the Rabi frequency becomes large compared with the atomic damping rates and the laser linewidth we can simplify Eqs. (2.37) and (2.43) and obtain

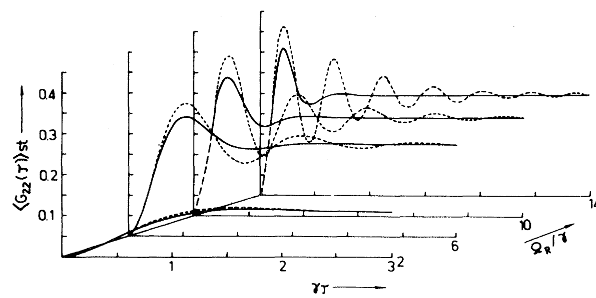

FIG. 1. Time development of the stationary fluorescent light intensity correlation function, which is proportional to $\langle G_{22}(\tau)\rangle_{\rm st}$, for various values of the Rabi frequency Ω_R. Radiative damping ($\Gamma_2=\gamma$, $\Gamma_1=2\gamma$) and small laser linewidth ($\Gamma_L=0.01$) are assumed. Rate of amplitude correlation decay is chosen to be $\Gamma_A=2\gamma$. Behavior in the case of a realistic laser with small mean-square amplitude fluctuation ($\epsilon=0.1$, full lines) is compared with the behavior in the case without amplitude fluctuations ($\epsilon=0$, dotted curves). Note that the stationary fluorescent light intensity correlation function is proportional to the fluorescent intensity because

$$\langle G_{22}(\tau)\rangle_{\rm st} = \langle \sigma_{22}(\infty)\rangle_{\rm st}\langle \sigma_{22}(\tau)\rangle_{\rm st}\ .$$

$$\langle G_{22}(t,t+\tau)\rangle_{st}=\langle \sigma_{22}(t)\rangle_{st}\langle \sigma_{22}(\tau)\rangle_{st}+\langle \Delta G_{22}(t,t+\tau)\rangle_{st} , \quad (2.44)$$

$$\langle \sigma_{22}(t)\rangle_{st}=\tfrac{1}{2}\{1-\cos(\Omega_R t)\exp[-\tfrac{1}{2}(\Gamma_1+\Gamma_2+\Gamma_L)t-g^{(1)}(0,t)]\} , \quad (2.45)$$

$$\langle \Delta G_{22}(t,t+\tau)\rangle_{st}=\tfrac{1}{4}\exp[-\tfrac{1}{2}(\Gamma_1+\Gamma_2+\Gamma_L)(t+\tau)-g^{(1)}(t,\tau)-g^{(1)}(0,t)]$$
$$\times(\cos(\Omega_R t)\cos(\Omega_R \tau)\{\cosh[g^{(2)}(t,\tau)]-1\}+\sin(\Omega_R t)\sin(\Omega_R \tau)\sinh[g^{(2)}(t,\tau)]) . \quad (2.46)$$

As is seen $\langle \Delta G_{22}(t,t+\tau)\rangle_{st}$ is the deviation of $\langle G_{22}(t,t+\tau)\rangle_{st}$ from the factorized value $\langle \sigma_{22}(t)\rangle_{st}\langle \sigma_{22}(\tau)\rangle_{st}$.

Assuming exponential correlation decay for the amplitude fluctuations and inserting Eq. (A5) into Eq. (2.45) we find by comparison that in the limiting case $\Gamma/\Omega_R \to 0$ the resulting expression for $\langle \sigma_{22}(t)\rangle_{st}$ is in agreement with the result obtained by the multiple—time-scale method.[17] Note that the latter as a high-field approximation yields practicable results in the limit $\Gamma/\Omega_R \to 0$ only. Since for fixed values of $\langle (\delta\Omega_R)^2\rangle_{st}$, τ_A, and Γ in this limit our approximation condition (2.33) is also fulfilled, both methods are expected to lead to equal results. From our theory we see, however, that for any small but finite value of Γ/Ω_R the high-field results given in Eqs. (2.44)—(2.46) remain valid provided that the values of $\langle (\delta\Omega_R)^2\rangle_{st}$, τ_A, and Γ fulfill the inequality (2.33).

From an inspection of the results given in Eqs. (2.37) and (2.43) we find that the intensity correlation function and the intensity of the fluorescent light depend on the correlation function of the amplitude fluctuations of the driving laser field $\langle \delta\Omega_R(t_1)\delta\Omega_R(t_2)\rangle_{st}$ via the time-integrated correlation functions $g^{(1)}(t',t)$ and $g^{(2)}(t',t)$ defined in Eqs. (2.38) and (2.39) (examples are presented in Appendix A). We thus expect that for a wide class of correlation functions $\langle \delta\Omega_R(t_1)\delta\Omega_R(t_2)\rangle_{st}$ the properties of the fluorescent light do not differ substantially. Let us consider, for example, the dependence of the intensity correlation function on $g^{(1)}$ and assume that the amplitude of the exciting laser field fluctuates according to a stationary process: $g^{(1)}(t',t)=g^{(1)}(0,t)$. From Eq. (2.38) both the short-time behavior of $g^{(1)}(0,t)$ and the asymptotic long-time behavior are easily seen to be independent of the concrete form of the correlation function $\langle \delta\Omega_R(t_1)\delta\Omega_R(t_2)\rangle_{st}$:

$$g^{(1)}(0,t)=\begin{cases}\tfrac{1}{2}\langle(\delta\Omega_R)^2\rangle_{st}t^2 & \text{for } t\ll\tau_A ,\\ \tilde{\Gamma}t & \text{for } t\gg\tau_A ,\end{cases} \quad (2.47)$$

where

$$\tilde{\Gamma}=\int_0^\infty d\tau\langle \delta\Omega_R(\tau)\delta\Omega_R(0)\rangle_{st} . \quad (2.48)$$

Some results of Eqs. (2.37) and (2.43) are presented in Figs. 1—3. In all figures radiative damping ($\Gamma_2=\gamma$, $\Gamma_1=2\gamma$) and small laser linewidth ($\Gamma_L=10^{-2}\gamma$) are assumed. The value of the relative mean-square amplitude fluctuation is chosen to be

$$\epsilon=\langle(\delta\Omega_R)^2\rangle_{st}/\Omega_R^2=0.1 .$$

The calculations were performed by utilizing the

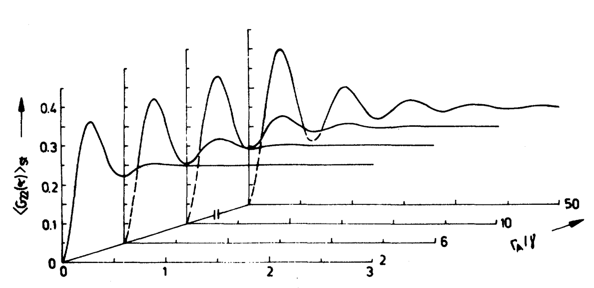

FIG. 2. Time development of the stationary fluorescent light intensity correlation function, which is proportional to $\langle G_{22}(\tau)\rangle_{st}$, for various values of the rate of amplitude correlation decay Γ_A of the exciting laser field in the case of small mean-square amplitude fluctuation $\epsilon=0.1$. Radiative damping ($\Gamma_2=\gamma$, $\Gamma_1=2\gamma$) and small laser linewidth ($\Gamma_L=0.01\gamma$) are assumed, and the Rabi frequency $\Omega_R=10$.

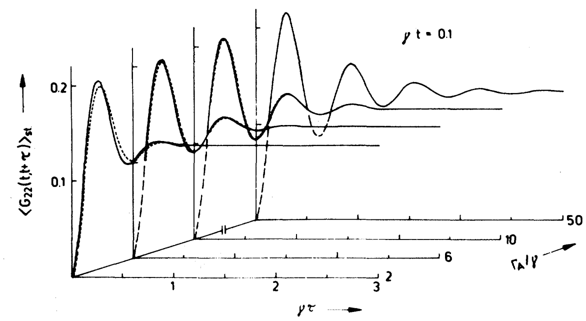

FIG. 3. Time development of the nonstationary fluorescent light intensity correlation function, which is proportional to $\langle G_{22}(t,t+\tau)\rangle_{st}$, for various values of the rate of amplitude correlation decay Γ_A of the exciting laser field in the case of small mean-square amplitude fluctuation $\epsilon=0.1$. Radiative damping ($\Gamma_2=\gamma$, $\Gamma_1=2\gamma$) and small laser linewidth ($\Gamma_L=0.01\gamma$) are assumed, and the Rabi frequency $\Omega_R=10\gamma$. Dotted curves represent the factorization result $\langle \sigma_{22}(t)\rangle_{st}\langle \sigma_{22}(\tau)\rangle_{st}$.

correlation functions given by Eqs. (A4), (A8), and (A12). In accordance with the above, the results were nearly equal. There were no deviations larger than 1%.

In Fig. 1 the steady-state intensity correlation function of the fluorescent light is shown for certain values of the Rabi frequency Ω_R and for relatively slow correlation decay ($\Gamma_A = 2\gamma$). It is seen that the amplitude fluctuations are responsible for damping the Rabi oscillations. This effect is further seen to increase with increasing value of Ω_R/γ. In the other case when the Rabi frequency becomes comparable with the damping rate γ or smaller than γ ($\Omega_R/\gamma \lesssim 1$) the intensity correlation function shows the same behavior as if there were no amplitude fluctuations. Indeed, from an inspection of Eqs. (2.37)–(2.39), (2.41), and (2.43) we see that when $\Omega_R/\gamma \lesssim 1$, the last exponential functions in Eqs. (2.37) and (2.43) give rise to contributions of the order

$$[\langle(\delta\Omega_R)^2\rangle_{\mathrm{st}}/\Omega_R^2](\Gamma/\Gamma_A)\ ,$$

which can be neglected due to the condition (2.33). In particular, in the limit of the weak pump field ($\Omega_R/\gamma \ll 1$) the effect of the amplitude fluctuations is unimportant provided that

$$[\langle(\delta\Omega_R)^2\rangle_{\mathrm{st}}/\Omega_R^2](\Gamma/\Gamma_A) \ll 1\ .$$

This result is in full agreement with results derived by means of perturbation theory.[13]

In Figs. 2 and 3 the intensity correlation function of the fluorescent light is shown for certain values of the rate of amplitude decay $\Gamma_A = \tau_A^{-1}$, the value of the Rabi frequency being $\Omega_R = 10\gamma$. In Fig. 2 the steady-state intensity correlation function is shown, whereas in Fig. 3 an example for nonstationary behavior is presented. Although the strength of the amplitude fluctuation is assumed to be small ($\epsilon = 0.1$), the effect of finite correlation length can become considerable. From Figs. 2 and 3 it is seen that in the case of relatively slow correlation decay ($\Gamma_A = 2\gamma$) the Rabi oscillations are quickly damped out. With decreasing correlation length, that is, with increasing value of Γ_A, this damping effect obviously vanishes. The intensity correlation function tends to become increasingly oscillatory with time as the amplitude correlation decay becomes fast. The curves presented for the fast correlation decay $\Gamma_A = 50\gamma$ are nearly the same as in the case of coherent, driving field. Furthermore, from Figs. 2 and 3 it is seen that within our theory, which is based on the condition given by Eq. (2.33), the factorization of the intensity correlation function is a very good approximation:

$$\langle G_{22}(t,t+\tau)\rangle_{\mathrm{st}} \approx \langle\sigma_{22}(t)\rangle_{\mathrm{st}}\langle\sigma_{22}(\tau)\rangle_{\mathrm{st}}\ .$$

Small deviations can only be observed under nonstationary conditions when the amplitude correlation decay is not too fast (see Fig. 3). They are negligibly small in the steady-state case. This is within the validity of the approximation condition (2.33), in agreement with the results found for the weak-field limit.[12,13]

It is worth noting that the applicability of our theory is not restricted to the case of weak amplitude fluctuations considered above. From the condition (2.33) it is easily seen that the case of stronger amplitude fluctuations with sufficiently fast correlation decay can be treated as well. In this case the effect of amplitude fluctuations can be described by the (nonvanishing) rate

$$[\,|\Omega_R^2 - \tfrac{1}{4}(\Gamma_1-\Gamma_2-\Gamma_L)^2|\,/\Omega_R^2]\tilde{\Gamma}$$

as is seen from an inspection of Eqs. (2.33), (2.37), (2.43), (2.47), and (2.48). This rate is simply given by $\tilde{\Gamma}$ provided that the driving field is sufficiently strong,

$$\tfrac{1}{4}(\Gamma_1-\Gamma_2-\Gamma_L)^2/\Omega_R^2 \ll 1\ .$$

The different behavior of the steady-state intensity correlation function for weak and for strong amplitude fluctuations is shown in Fig. 4, the value of the rate $\tilde{\Gamma}$ being fixed. Radiation damping is assumed ($\Gamma_2 = \gamma$, $\Gamma_1 = 2\gamma$) and the value of the Rabi frequency is $\Omega_R = 10\gamma$. It can be seen that in the case of strong amplitude fluctuations the Rabi oscillations are rapidly damped out with the rate $\tilde{\Gamma}$. We emphasize that this result is valid for both Markovian and non-Markovain amplitude fluctuations. The value of $\tilde{\Gamma}$ depends, of course, on the model used. In the case of weak amplitude fluctuations the

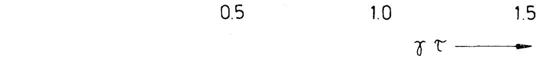

FIG. 4. Time development of the stationary fluorescent light intensity correlation function, which is proportional to $\langle G_{22}(\tau)\rangle_{\mathrm{st}}$, in the case of weak and slow amplitude fluctuations of the exciting laser field ($\epsilon = 0.1$, $\Gamma_A = 2\gamma$) and in the case of strong and fast amplitude fluctuations ($\epsilon = 2.5$, $\Gamma_A = 50\gamma$), the rate $\tilde{\Gamma} = \epsilon\Omega_R^2/\Gamma_A$ being fixed in both cases ($\tilde{\Gamma} = 5\gamma$). Radiative damping ($\Gamma_2 = \gamma$, $\Gamma_1 = 2\gamma$) and small laser linewidth ($\Gamma_L = 0.01\gamma$) are assumed, and the Rabi frequency $\Omega_R = 10\gamma$.

damping behavior is more complicated. It is determined by the full function $g^{(1)}$ defined by Eq. (2.38).

We finally note that from our theory the results that have recently been derived for a Gaussian-amplitude field[14] can easily be obtained. After disregarding the laser linewidth ($\Gamma_L=0$) and the relaxation rates ($\Gamma_1=\Gamma_2=0$) we set $\Omega_R=0$ and find from Eqs. (2.44)–(2.46)

$$\langle G_{22}(t,t+\tau)\rangle_{\rm st} = \tfrac{1}{4}\{1-e^{-g^{(1)}(0,t)}-e^{-g^{(1)}(t,\tau)} + e^{-[g^{(1)}(0,t)+g^{(1)}(t,\tau)]}\cosh[g^{(2)}(t,\tau)]\}\ , \quad (2.49)$$

$$\langle \sigma_{22}(t)\rangle_{\rm st}=\tfrac{1}{2}(1-e^{-g^{(1)}(0,t)})\ . \quad (2.50)$$

III. STATIONARY SPECTRUM OF THE FLUORESCENCE

We now turn to the calculation of the stationary spectrum of the fluorescent light from a two-level atom driven by a laser field with phase and amplitude fluctuations. As is well known, the Wiener-Khintchin spectrum of a given light field is obtained by a Fourier transformation of the two-time correlation function

$$G^{(1)}(t,t+\tau)=\sum_i \langle E_i^{(-)}(t)E_i^{(+)}(t+\tau)\rangle\ . \quad (3.1)$$

Taking into consideration that in the case of resonance fluorescence from a two-level atom the $G^{(1)}$ correlation function of the scattered light is proportional to the atomic correlation function $\langle A_{21}(t)A_{12}(t+\tau)\rangle$ (Refs. 10, 12, and 19) we define the spectrum by

$$S(\omega)=\lim_{t\to\infty}\int_{-\infty}^{+\infty} d\tau\, e^{i\omega\tau}\langle A_{21}(t)A_{12}(t+\tau)\rangle\ . \quad (3.2)$$

In order to calculate $\langle A_{21}(t)A_{12}(t+\tau)\rangle$ we follow the procedure outlined in Sec. II. After introducing slowly varying atomic operators defined by Eq. (2.9) we define the four-dimensional vector $|\Psi(t,t+\tau))$ by its components $(i\,|\,\Psi(t,t+\tau))$ as follows:

$$(1\,|\,\Psi(t,t+\tau))=\langle \tilde{A}_{21}(t)A_{22}(t+\tau)\rangle\ , \quad (3.3)$$

$$(2\,|\,\Psi(t,t+\tau))=\langle \tilde{A}_{21}(t)A_{11}(t+\tau)\rangle\ , \quad (3.4)$$

$$(3\,|\,\Psi(t,t+\tau))=\langle \tilde{A}_{21}(t)\tilde{A}_{12}(t+\tau)\rangle\ , \quad (3.5)$$

$$(4\,|\,\Psi(t,t+\tau))=\langle \tilde{A}_{21}(t)\tilde{A}_{21}(t+\tau)\rangle\ . \quad (3.6)$$

$|\Psi(t,t+\tau))$ satisfies the Bloch equation

$$\frac{d}{d\tau}\,|\,\Psi(t,t+\tau))=M(t+\tau)\,|\,\Psi(t,t+\tau))\ , \quad (3.7)$$

the initial condition being

$$(1\,|\,\Psi(t,t))=0,\quad (2\,|\,\Psi(t,t))=\langle \tilde{A}_{21}(t)\rangle\ ,$$
$$(3\,|\,\Psi(t,t))=\langle A_{22}(t)\rangle,\quad (4\,|\,\Psi(t,t))=0\ . \quad (3.8)$$

The matrix $M(t)$ is defined by Eq. (2.15). The solution of Eq. (3.7) which satisfies the initial condition (3.8) can be found by formal integration. The result is

$$|\,\Psi(t,t+\tau))=S(t,t+\tau)\,|\,2)(2\,|\,\Psi(t,t))+S(t,t+\tau)\,|\,3)(3\,|\,\Psi(t,t))$$
$$=S(t,t+\tau)\,|\,2)\langle \tilde{A}_{21}(t)\rangle+S(t,t+\tau)\,|\,3)\langle A_{22}(t)\rangle\ , \quad (3.9)$$

where the time-ordered exponential matrix S is defined by Eq. (2.25). $\langle \tilde{A}_{21}(t)\rangle$ and $\langle A_{22}(t)\rangle$ are calculated by means of Eqs. (2.20), (2.21), and (2.24). We obtain

$$\langle \tilde{A}_{21}(t)\rangle=(4\,|\,S(0,t)\,|\,2),\quad \langle A_{22}(t)\rangle=(1\,|\,S(0,t)\,|\,2)\ . \quad (3.10)$$

Combining the results of Eqs. (2.9), (3.5), (3.9), and (3.10) we now express the correlation function $\langle A_{21}(t)A_{12}(t+\tau)\rangle$ in the form

$$\langle A_{21}(t)A_{12}(t+\tau)\rangle=\exp[-i\omega_L\tau-i\varphi_L(t+\tau)+i\varphi_L(t)]$$
$$\times[(3\,|\,S(t,t+\tau)\,|\,2)(4\,|\,S(0,t)\,|\,2)+(3\,|\,S(t,t+\tau)\,|\,3)(1\,|\,S(0,t)\,|\,2)]\ . \quad (3.11)$$

In order to take into account the stochastic features of the driving laser field we have to average the quantum-mechanical correlation function $\langle A_{21}(t)A_{12}(t+\tau)\rangle$ over the phase and amplitude fluctuations, before calculating the spectrum of the fluorescent light. In the first place, we perform the averaging over the phase fluctuations by making use of the phase-diffusion model [cf. Eq. (2.29)]. If the strength of the phase fluctuations is sufficiently small the result simply consists in substituting in Eq. (3.11) for $-i\varphi_L(t+\tau)+i\varphi_L(t)$ the quantity $-\Gamma_L\tau$ and in substituting in the matrix $M(t)$ defined by Eq. (2.15) for $i\dot{\varphi}_L$ and $-i\dot{\varphi}_L$ the quantity $-\Gamma_L$. In the second place, we perform the averaging over the amplitude fluctuations in the approximation given by Eqs. (2.33) and (2.34). In a way analogous to that of Sec. II we finally obtain the result

$$\langle\langle A_{21}(t)A_{12}(t+\tau)\rangle\rangle_{st}$$
$$=e^{-i\omega_L\tau-\Gamma_L\tau}\sum_{k,l}e^{\lambda_k\tau+\lambda_l t}\exp\{\Omega_R^{-2}[\lambda_k^2 g^{(1)}(t,\tau)+\lambda_l^2 g^{(1)}(0,t)+\lambda_k\lambda_l g^{(2)}(t,\tau)]\}(C_k^{3,2}C_l^{4,2}+C_k^{3,3}C_l^{1,2}) , \tag{3.12}$$

which reads in the limit $t\rightarrow\infty$ as

$$\lim_{t\rightarrow\infty}\langle\langle A_{21}(t)A_{12}(t+\tau)\rangle\rangle_{st}=e^{-i\omega_L\tau-\Gamma_L\tau}\sum_k e^{\lambda_k\tau}\exp\left[\frac{\lambda_k^2}{\Omega_R^2}g^{(1)}(\infty,\tau)\right](C_k^{3,2}C_1^{4,2}+C_k^{3,3}C_1^{1,2}) . \tag{3.13}$$

The coefficients $C_k^{i,j}$ are defined by Eq. (2.40). In particular, we obtain

$$C_1^{3,2}=\frac{i}{2}\Omega_2\frac{\Gamma_1}{\lambda_3\lambda_4},\quad C_2^{3,2}=0 ,$$
$$C_3^{3,2}=-\frac{i}{2}\frac{\Omega_R(\Gamma_1+\lambda_3)}{\lambda_3(\lambda_4-\lambda_3)}, \tag{3.14}$$
$$C_4^{3,2}=-\frac{i}{2}\frac{\Omega_R(\Gamma_1+\lambda_4)}{\lambda_4(\lambda_3-\lambda_4)} ,$$

$$C_1^{3,3}=0,\quad C_2^{3,3}=\tfrac{1}{2} ,$$
$$C_3^{3,3}=-\frac{\Omega_R^2}{2(\lambda_4-\lambda_3)(\Gamma_1+\lambda_4)} , \tag{3.15}$$
$$C_4^{3,3}=-\frac{\Omega_R^2}{2(\lambda_3-\lambda_4)(\Gamma_1+\lambda_3)} .$$

$C_1^{1,2}$ is given by Eq. (2.42) and $C_1^{4,2}=-C_1^{3,2}$. The λ_i values are given by Eq. (2.41). In accordance with the arguments given in Sec. II we found the correlation function $\langle\langle A_{21}(t)A_{12}(t+\tau)\rangle\rangle_{st}$ is nearly equal for the three kinds of correlation functions $\langle\delta\Omega_R(t_1)\delta\Omega_R(t_2)\rangle_{st}$ studied in Appendix A. In order to derive an explicit expression for the stationary spectrum we will therefore confine ourselves to the particular case of Markovian amplitude correlation. Making use of Eqs. (A5) and (A7) we insert Eq. (3.13) into Eq. (3.2) and perform the Fourier transform, after expanding $\exp(\cdots g^{(1)})$ into a power series. This yields the (averaged) spectrum of the fluorescent light for arbitrary values of the Rabi frequency in the form

$$S(\omega)=\sum_{n=0}^{\infty}\frac{1}{n!}\left[\frac{\lambda_k^2}{\Omega_R^2}\frac{\tilde{\Gamma}}{\Gamma_A}\right]^n\sum_k\exp\left[-\frac{\lambda_k^2}{\Omega_R^2}\frac{\tilde{\Gamma}}{\Gamma_A}\right]\frac{c_k^{3,2}C_1^{4,2}+C_k^{3,3}C_1^{1,2}}{i(\omega_L-\omega)+\Gamma_L+n\Gamma_A-\lambda_k-\dfrac{\lambda_k^2}{\Omega_R^2}\tilde{\Gamma}}+\text{c.c.} , \tag{3.16}$$

where $\tilde{\Gamma}$ is given by Eq. (A7).

In the strong-field limit Eq. (3.16) can be simplified to

$$S(\omega)=S_0(\omega)+S_+(\omega)+S_-(\omega) , \tag{3.17}$$

$$S_0(\omega)=\frac{1}{2}\frac{(\Gamma_2+2\Gamma_L)}{(\omega-\omega_L)^2+(\Gamma_2+2\Gamma_L)^2} , \tag{3.18}$$

$$S_\pm(\omega)=\tfrac{1}{8}e^{\tilde{\Gamma}/\Gamma_A}\sum_{n=0}^{\infty}\frac{1}{n!}\left[-\frac{\tilde{\Gamma}}{\Gamma_A}\right]^n\frac{(\Gamma_1+\Gamma_2+3\Gamma_L+2\tilde{\Gamma}+2n\Gamma_A)}{[\omega-(\omega_L\mp\Omega_R)]^2+\frac{1}{4}(\Gamma_1+\Gamma_2+3\Gamma_L+2\tilde{\Gamma}+2n\Gamma_4)^2} . \tag{3.19}$$

This result corresponds to the lowest order in the expansion in powers of γ/Ω_R, as has also been obtained by means of the multiple–time-scale method.[17] This method as a high-field approximation has the disadvantage that it is only practicable when the driving field is sufficiently strong. Our method, however, does not exhibit this problem.

We note that the results recently derived for a Gaussian-amplitude field[14] are involved in our more general theory. They can be obtained with the help of the procedure outlined at the end of Sec. II.

If we examine the result given by Eq. (3.16) and remember the condition (2.33) we observe that the effect of the amplitude fluctuations is a modification of the side peaks of the fluorescent spectrum, whereas the central peak remains unchanged. When the Rabi frequency becomes comparable with or smaller than the atomic relaxation rates the effect of

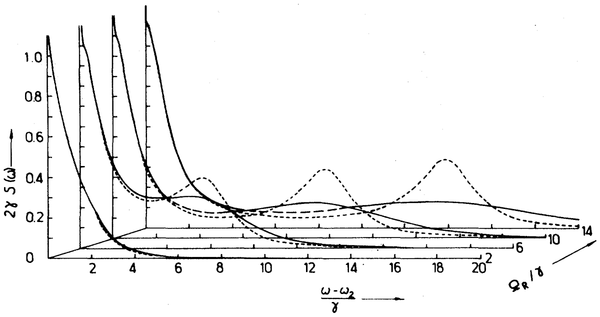

FIG. 5. Full stationary spectrum of the fluorescence for various values of the Rabi frequency Ω_R. Radiative damping ($\Gamma_2=\gamma$, $\Gamma_1=2\gamma$) and small laser linewidth ($\Gamma_L=0.01\gamma$) are assumed. Rate of amplitude correlation decay is chosen to be $\Gamma_A=2\gamma$. Behavior in the case of a realistic laser with small mean-square amplitude fluctuation ($\epsilon=0.1$, full lines) is compared with the behavior in the case without amplitude fluctuations ($\epsilon=0$, dotted curves).

the amplitude fluctuations obviously vanishes. The dependence of the stationary spectrum on the Rabi frequency is shown in Fig. 5 for relatively slow decay of amplitude correlation ($\Gamma_A=2\gamma$), and the relative mean-square amplitude fluctuation $\epsilon=0.1$. In this figure and the following ones radiative damping ($\Gamma_2=\gamma$, $\Gamma_1=2\gamma$) and small laser linewidth ($\Gamma_L=10^{-2}\gamma$) are assumed.

Some spectra for certain values of the amplitude correlation decay rate Γ_A are presented in Fig. 6, the value of the Rabi frequency being $\Omega_R=10\gamma$. The side peaks (superpositions of Lorentzians) are non-Lorentzians. The deviation of the line shape from a Lorentzian increases with increasing value of $\tilde{\Gamma}/\Gamma_A$, that is, for fixed strength of the amplitude fluctuations the deviation becomes larger as the amplitude correlation decay becomes slower. It is further seen that with increasing value of Γ_A the spectrum tends to the Mollow spectrum[19] observed in the case of a coherent, driving field. The spectrum shown for the fast correlation decay $\Gamma_A=50\gamma$ is, in very good approximation, the Mollow spectrum. Substantial deviations from the Mollow spectrum are seen in the case of relatively slow correlation decay ($\Gamma_A=2\gamma$). The side peaks are broadened at the expense of their heights. Non-Lorentzian line shapes for the side peaks are observed.

We note that in the case of strong amplitude fluctuations with fast correlation decay the situation becomes quite different, as can be seen from Fig. 7. In this case the only important term in the expansion (3.19) is that with $n=0$. Therefore, in contrast to the case of weak amplitude fluctuations, the side peaks remain Lorentzians which are, however, extremely broadened.

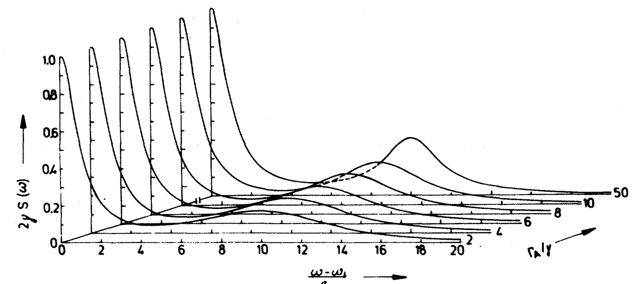

FIG. 6. Stationary spectrum of the fluorescence for various values of the rate of amplitude correlation decay Γ_A of the exciting laser field in the case of small mean-square amplitude fluctuation $\epsilon=0.1$. Radiative damping ($\Gamma_2=\gamma$, $\Gamma_1=2\gamma$) and small laser linewidth ($\Gamma_L=0.01\gamma$) are assumed, and the Rabi frequency $\Omega_R=10\gamma$. Rayleigh scattering term is omitted.

IV. SUMMARY

We have examined the resonant interaction of a two-level atom undergoing energy and phase relaxation with an external laser field with Gaussian phase and amplitude fluctuations. We have presented closed solutions for the intensity, the intensity correlation function, and the stationary spectrum of the fluorescent light, without the introduction of factorization conditions and without the restriction to Markovian amplitude fluctuations. The method used is based on the realistic assumptions that, first, the phase fluctuations can be treated in the phase-diffusion model and that, second, the product of the relative mean-square amplitude fluctuation, the amplitude correlation decay time, and the characteristic

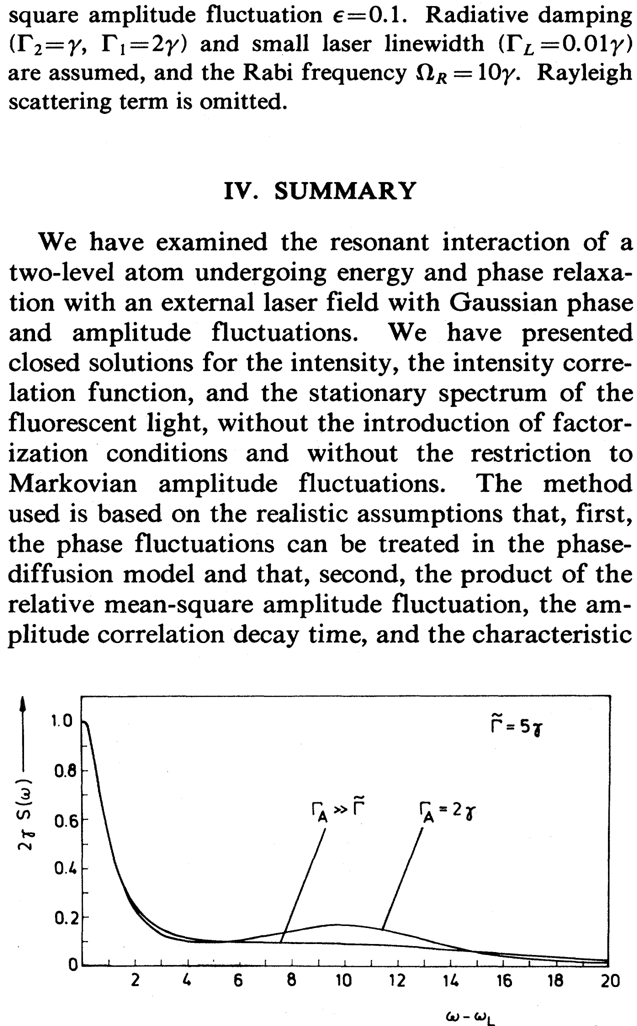

FIG. 7. Stationary spectrum of the fluorescence in the case of weak and slow amplitude fluctuations of the exciting laser field ($\epsilon=0.1$, $\Gamma_A=2\gamma$) and in the case of strong and fast amplitude fluctuations ($\epsilon=2.5$, $\Gamma_A=50\gamma$), the rate $\tilde{\Gamma}=\epsilon\Omega_R^2/\Gamma_A$ being fixed in both cases ($\tilde{\Gamma}=5\gamma$). Radiative damping ($\Gamma_2=\gamma$, $\Gamma_1=2\gamma$) and small laser linewidth ($\Gamma_L=0.01\gamma$) are assumed, and the Rabi frequency $\Omega_R=10\gamma$. Rayleigh scattering term is omitted.

atomic relaxation rate is small compared to unity:

$$[\langle \delta\Omega_R)^2\rangle_{st}/\Omega_R^2]\tau_A\Gamma \ll 1$$

[Eq. (2.33)]. The main advantage of the theory is that it can be applied under the following various conditions.

(i) The inequality (2.33) is fulfilled in the limit of vanishing atomic relaxation ($\Gamma\to 0$). Especially in the case of a Gaussian-amplitude field ($\Omega_R\to 0$) the results of Le Berre-Rousseau *et al.*[14] are recovered.

(ii) The inequality (2.33) is fulfilled in the limit of fast correlation decay ($\tau_A\to 0$). The fluctuations can become strong and the effect of fluctuations is simply described by means of a rate as has been found for the case of one-time averages.[15,16]

(iii) The inequality (2.33) is fulfilled in the saturation limit ($\Gamma/\Omega_R\to 0$). The results agree with the recent ones of Chaturvedi and Gardiner.[17]

(iv) In the case of a realistic single-mode laser with relatively small amplitude fluctuations ($\langle(\delta\Omega_R)^2\rangle_{st}/\,\Omega_R^2 \ll 1$) the theory is neither restricted to fast correlation decay nor to the strong-field limit.

We have found that the effect of amplitude fluctuations increases with increasing Rabi frequency. Even when the strength of the amplitude fluctuations is small the effect of the length of the amplitude fluctuations can become considerable. Substantial modifications, in comparison with the case of coherent, driving fields, occur when the amplitude correlations decay slowly. Then the Rabi oscillations of the intensity and the intensity correlation function of the fluorescent light are rapidly damped out and the side peaks of the spectrum are broadened in a non-Lorentzian way. The modifications occurring due to non-Markovian amplitude fluctuations are found to be negligibly small. Finally, we would like to emphasize that our method is not restricted to the calculation of two-time correlation functions of the fluorescent light as has been considered in this paper. It can be applied to the calculation of higher-order correlation functions as well.

APPENDIX A: LASER MODEL

Within the framework of the Yariv-Caton laser model[37] the electric field strength obeys the differential equation

$$\frac{d^2E_L(t)}{dt^2}+[\nu-\beta+\zeta E_L^2(t)]\frac{dE_L(t)}{dt}+\omega_L^2E_L(t)=N(t)\ . \qquad \text{(A1)}$$

In this equation, ν is connected with the resonator losses; β and ζE_L^2, respectively, characterize the pump mechanism and the saturation behavior, ζ resulting from the third-order susceptibility. $N(t)$ is a Gaussian noise. Sufficiently high above threshold Eq. (A1) can be linearized by means of the ansatz (2.8). The result is

$$\frac{d\varphi_L(t)}{dt}=\frac{1}{2\omega_L}N_C(t)\ ,$$
$$\frac{d\,\delta\hat{E}_L(t)}{dt}+\gamma_s\,\delta\hat{E}_L(t)=\frac{1}{2\omega_L}N_S(t)\ , \qquad \text{(A2)}$$

where

$$\gamma_s=\tfrac{1}{4}\zeta\hat{E}_L^2\ ,$$
$$N(t)=N_C(t)\cos(\omega_Lt)+N_S(t)\sin(\omega_Lt)\ ,$$

$N_C(t)$ and $N_S(t)$ being uncorrelated. The power spectra of $N_C(t)$ and $N_S(t)$ are given by

$$P_{N_C}(\omega)=P_{N_S}(\omega)=A+Bg(\omega)\ , \qquad \text{(A3)}$$

where $g(\omega)$ is the fluorescence line-shape function of the atomic laser transition with weight B, and A describes an additional white-noise spectrum resulting from the resonator losses and the thermal background radiation.

When the value of A/B is large compared with unity or, as has been shown recently,[12] when the value of the laser linewidth Γ_L is small compared with the fluorescence linewidth $\gamma_{\rm fl}$ of the laser transition the phase $\varphi_L(t)$ can be assumed to fluctuate according to a Wiener-Levy process (note that for a He-Ne laser $\Gamma_L/\gamma_{\rm fl}\approx 10^{-5}$).

Let us now study the amplitude fluctuations.

(i) When $A/B \gg 1$, the amplitude fluctuates according to a stationary Gauss-Markov process:

$$\langle\delta\hat{E}_L(t)\delta\hat{E}_L(t')\rangle_{st}=\langle(\delta\hat{E}_L)^2\rangle_{st}\,e^{-\gamma_s|t-t'|}\ . \qquad \text{(A4)}$$

The functions $g^{(1)}(t',t)$ and $g^{(2)}(t',t)$ defined by Eqs. (2.38) and (2.39) are derived to be

$$g^{(1)}(t',t)=g^{(1)}(0,t)=\tilde{\Gamma}t+\frac{\tilde{\Gamma}}{\gamma_s}(e^{-\gamma_st}-1)\ , \qquad \text{(A5)}$$

$$g^{(2)}(t',t)=\frac{\tilde{\Gamma}}{\gamma_s}(e^{-\gamma_st}-1)(e^{-\gamma_st'}-1)\ , \qquad \text{(A6)}$$

$$\tilde{\Gamma}=\epsilon\frac{\Omega_R^2}{\gamma_s},\quad \epsilon=\frac{\langle(\delta\Omega_R)^2\rangle_{st}}{\Omega_R^2}\ . \qquad \text{(A7)}$$

The correlation time of the amplitude fluctuation $\tau_A=\Gamma_A^{-1}$ defined by

$$\langle\delta E_L(t)\delta E_L(t+\tau_A)\rangle_{st}=\langle(\delta\hat{E}_L)^2\rangle_{st}/e$$

is simply given by $\tau_A=\gamma_s^{-1}$.

(ii) When $A/B \ll 1$ and the fluorescence line shape function is Lorentzian the amplitude fluctuates in a non-Markovian way. This process, however, can be represented as a stationary two-dimensional Markov process:

$$\langle \delta\hat{E}_L(t)\delta E_L(t')\rangle_{\rm st}=\langle(\delta\hat{E}_L)^2\rangle_{\rm st}\frac{\gamma_{\rm fl}\gamma_s}{\gamma_s-\gamma_{\rm fl}}(\gamma_{\rm fl}^{-1}e^{-\gamma_{\rm fl}|t-t'|}-\gamma_s^{-1}e^{-\gamma_s|t-t'|})\ . \tag{A8}$$

The functions $g^{(1)}(t',t)$ and $g^{(2)}(t',t)$ are calculated to be

$$g^{(1)}(t',t)=g^{(1)}(0,t)=\tilde{\Gamma}t+\tilde{\Gamma}\frac{\gamma_{\rm fl}^2\gamma_s^2}{\gamma_s^2-\gamma_{\rm fl}^2}[\gamma_{\rm fl}^{-3}(e^{-\gamma_{\rm fl}t}-1)-\gamma_s^{-3}(e^{-\gamma_s t}-1)]\ , \tag{A9}$$

$$g^{(2)}(t',t)=\tilde{\Gamma}\frac{\gamma_{\rm fl}^2\gamma_s^2}{\gamma_s^2-\gamma_{\rm fl}^2}[\gamma_{\rm fl}^{-3}(e^{-\gamma_{\rm fl}t}-1)(e^{-\gamma_{\rm fl}t'}-1)-\gamma_s^{-3}(e^{-\gamma_s t}-1)(e^{-\gamma_s t'}-1)]\ , \tag{A10}$$

$$\tilde{\Gamma}=\epsilon\Omega_R^2\frac{\gamma_s+\gamma_{\rm fl}}{\gamma_{\rm fl}\gamma_s}\ . \tag{A11}$$

The correlation time $\tau_A=\Gamma_A^{-1}$ is a function on γ_s and $\gamma_{\rm fl}$. In the Markovian limiting cases $\gamma_{\rm fl}/\gamma_s \ll 1$ and $\gamma_s/\gamma_{\rm fl} \ll 1$, we have $\Gamma_A=\gamma_{\rm fl}$ and $\Gamma_A=\gamma_s$, respectively.

(iii) When $A/B \ll 1$ and the fluorescence line shape function $g(\omega)$ is assumed to be Gaussian with $\gamma_{\rm fl} \ll \gamma_s$, the amplitude fluctuation cannot be treated as a (n-dimensional) Markov process. The correlation function is now given by

$$\langle \delta\hat{E}_L(t)\delta\hat{E}_L(t')\rangle_{\rm st}=\langle(\delta\hat{E}_L)^2\rangle_{\rm st}\exp\left[-\frac{\gamma_{\rm fl}^2}{4}(t-t')^2\right]\ , \tag{A12}$$

and $\tau_A=\Gamma_A^{-1}=2/\gamma_{\rm fl}$. The functions $g^{(1)}(t',t)$ and $g^{(2)}(t',t)$ are derived to be

$$g^{(1)}(t',t)=g^{(1)}(0,t)=\tilde{\Gamma}\Phi\left[\frac{\gamma_{\rm fl}}{2}t\right]t+\frac{2\tilde{\Gamma}}{\sqrt{\pi}\gamma_{\rm fl}}\left[\exp\left[-\frac{\gamma_{\rm fl}^2}{4}t^2\right]-1\right]\ , \tag{A13}$$

$$\begin{aligned}g^{(2)}(t',t)=\frac{2\tilde{\Gamma}}{\gamma_{\rm fl}}\Bigg\{&\frac{\gamma_{\rm fl}}{2}t'\Phi\left[\frac{\gamma_{\rm fl}}{2}t'\right]-\frac{\gamma_{\rm fl}}{2}t\Phi\left[\frac{\gamma_{\rm fl}}{2}t\right]-\frac{\gamma_{\rm fl}}{2}(t-t')\Phi\left[\frac{\gamma_{\rm fl}}{2}(t-t')\right]\\&-\frac{1}{\sqrt{\pi}}\left[\exp\left[-\frac{\gamma_{\rm fl}^2}{4}(t-t')^2\right]+\exp\left[-\frac{\gamma_{\rm fl}^2}{4}t^2\right]-\exp\left[-\frac{\gamma_{\rm fl}^2}{4}t'^2\right]-1\right]\Bigg\}\ ,\end{aligned} \tag{A14}$$

$$\tilde{\Gamma}=\epsilon\Omega_R^2\frac{\sqrt{\pi}}{\gamma_{\rm fl}}\ , \tag{A15}$$

where Φ is the error function.

APPENDIX B: DERIVATION OF THE INEQUALITY (2.33)

Let us consider the matrix

$$M'(t)=M_0\left[1+\frac{\delta\Omega_R(t)}{\Omega_R}\right]\ , \tag{B1}$$

where the matrix M_0 is given by Eq. (2.31). From Eq. (2.30) the matrix $M'(t)$ is seen to be related to the matrix $M(t)$ according to

$$M'(t)=M(t)+\frac{\delta\Omega_R(t)}{\Omega_R}M_1\ , \tag{B2}$$

where M_1 is the so-called relaxation matrix given by Eq. (2.32). The main step in the derivation of the results of the foregoing sections is the substitution of $M'(t)$ for $M(t)$ in the equation of motion:

$$\frac{d\,|\Psi(t))}{dt}=M(t)\,|\,\Psi(t))\approx M'(t)\,|\,\Psi(t))\ . \tag{B3}$$

This approximation can be justified when the effect of $M_1\delta\Omega_R(t)/\Omega_R$ is sufficiently small. Since

$$M(t)=(M_0-M_1)\left[1+\frac{\delta\Omega_R(t)}{\Omega_R}\right]+M_1 \tag{B4}$$

we have

$$M'(t)=(M_0-M_1)\left[1+\frac{\delta\Omega_R(t)}{\Omega_R}\right]+M_1'(t)\ , \tag{B5}$$

where the new relaxation matrix

$$M_1'(t)=M_1\left[1+\frac{\delta\Omega_R(t)}{\Omega_R}\right] \tag{B6}$$

(note that M_0-M_1 only depends on Ω_R). We see that the matrices $M(t)$ and $M'(t)$ differ in the relaxation matrix. In order to study the relaxation behavior governed by the matrix $M_1'(t)$ we calculate

$$\left\langle T \exp\left[\int_0^t d\tau M_1'(\tau)\right]\right\rangle_{\text{st}} = \exp(\widetilde{M}_1(t)t)\ , \tag{B7}$$

$$\widetilde{M}_1(t) = M_1 + \frac{M_1^2}{t}\int_0^t d\tau_1 \int_0^{\tau_1} d\tau_2 \left\langle \frac{\delta\Omega_R(\tau_1)}{\Omega_R}\frac{\delta\Omega_R(\tau_2)}{\Omega_R}\right\rangle_{\text{st}} . \tag{B8}$$

The matrix $\widetilde{M}_1(t)$ can simply be derived from M_1 by substituting

$$\Gamma_1 \to \Gamma_1\left[1 - \frac{\Gamma_1}{\Gamma}\alpha(t)\right] , \tag{B9}$$

$$(\Gamma_2 + \Gamma_L \pm i\,\delta\omega) \to (\Gamma_2 + \Gamma_L \pm i\,\delta\omega)\left[1 - \frac{\Gamma_2 + \Gamma_L \pm i\,\delta\omega}{\Gamma}\alpha(t)\right] , \tag{B10}$$

where

$$\alpha(t) = \frac{\Gamma}{\Omega_R^2 t}\int_0^t d\tau_1 \int_0^{\tau_1} d\tau_2 \langle \delta\Omega_R(\tau_1)\delta\Omega_R(\tau_2)\rangle_{\text{st}}\ , \tag{B11}$$

$$\Gamma = \max(\Gamma_1, |\Gamma_2 + \Gamma_L \pm i\,\delta\omega|)\ . \tag{B12}$$

We thus find that when $\alpha(t)$ fulfills the inequality

$$\alpha(t) \ll 1 \tag{B13}$$

the matrices M_1 and $M_1'(t)$ describe equal relaxation behavior, and the matrix $M_1\delta\Omega_R(t)/\Omega_R$ in Eq. (B2) can be omitted. Moreover, the averaged matrix of time evolution

$$\langle S'(t)\rangle_{\text{st}} = \left\langle T \exp\left[\int_0^t d\tau M'(\tau)\right]\right\rangle_{\text{st}} \tag{B14}$$

can be written in the form

$$\langle S'(t)\rangle_{\text{st}} = \left\langle T \exp\left[\int_0^t d\tau M(\tau)\right]\right\rangle_{\text{st}} + \sum_{n=1}^{\infty} \alpha^n(t) S_n(t) \tag{B15}$$

as is seen from the expansion of the time-ordered exponential matrix in Eq. (B14). The substitution of $M'(t)$ for $M(t)$ is thus expected to be a good approximation provided that $\alpha(t)$ is sufficiently small. Making use of the estimation

$$\int_0^t d\tau_1 \int_0^{\tau_1} d\tau_2 \langle \delta\Omega_R(\tau_1)\delta\Omega_R(\tau_2)\rangle_{\text{st}} \lesssim \langle (\delta\Omega_R)^2\rangle_{\text{st}}\tau_A t\ , \tag{B16}$$

where τ_A is the correlation time of the amplitude fluctuations, the inequality (B13) can be written in the form

$$\frac{\langle (\delta\Omega_R)^2\rangle}{\Omega_R^2}\tau_a \Gamma \ll 1\ . \tag{B17}$$

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