

Scattering of 1.1732- and 1.3325-MeV gamma rays through small angles by carbon, aluminum, copper, tin, and lead

P. P. Kane, J. Mahajani, G. Basavaraju, and A. K. Priyadarsini

Department of Physics, Indian Institute of Technology, Powai, Bombay 400 076, India

(Received 20 October 1982)

With the help of Ge(Li) detectors, elastic and Compton scattering cross sections of copper, tin, and lead were determined in the double-shadow-cone geometry from 4.51° to 12.05° in the case of 1.1732- and 1.3325-MeV γ rays of ^{60}Co . A new method was developed for the accurate measurement of cross sections. The measured elastic scattering cross sections are determined predominantly by Rayleigh scattering from bound atomic electrons. The nuclear Thomson and the Delbrück scattering amplitudes have also been considered in the interpretation of data. The relativistic modified form-factor theory of Rayleigh scattering leads to good agreement with the data for $x < 12 \text{ \AA}^{-1}$, where $x = \sin(\theta/2)/\lambda$, and where θ is the scattering angle and λ the photon wavelength. The measured Compton scattering cross sections for $x > 4.5 \text{ \AA}^{-1}$ are in agreement with those calculated on the basis of tabulated nonrelativistic incoherent scattering functions, although there are deviations at smaller x values.

I. INTRODUCTION

In a previous paper,¹ hereafter referred as paper I, we reported a new method for the determination of Compton and elastic scattering cross sections of lead at small angles to an accuracy of 3% to 4%. The method relied in an essential way on the separation, with the help of a Ge(Li) detector, of the Compton and the elastic scattering photopeaks at small scattering angles and a direct comparison of the respective photopeak areas with the Compton peak area obtained with a small atomic number target such as carbon or aluminum. We have mentioned an approximate agreement of the experimental values up to about 8° of lead elastic scattering cross sections with the predictions based on nonrelativistic form-factor theory² of Rayleigh scattering. We have indicated the possibility of a disagreement between experimental results and calculations based on relativistic form factors.³ The desirability of performing relativistic modified form-factor calculations was also pointed out.

Since then, we extended the lead measurements to 9.95° and 12.05° , obtained additional results with carbon, copper, tin, and lead targets over the angular range of 4.51° to 12.05° , used another Ge(Li) detector with better energy resolution, reduced the background with the help of a foam plastic target holder of less than 0.05 g/cm^2 thickness and performed the necessary relativistic form-factor and relativistic modified form-factor calculations. An analysis of the different lead results was presented⁴ earlier. The

calculations mentioned here were made with the help of relativistic self-consistent or Dirac-Hartree-Fock (DHF) wave functions kindly supplied by Mann.⁵

For references to earlier work as well as many details pertaining to our previous experimental work, paper I, should be consulted. Important features of our later work are described in Sec. II. The theoretical calculations are briefly explained in Sec. III. A comparison of the final experimental results with theory and with other recently reported Ge(Li) detector measurements is presented in Sec. IV.

II. EXPERIMENTAL DETAILS

The experimental method has been described in paper I. Considerations of intensity, background, and convenient target size led us to adopt two separate double-shadow-cone geometrical arrangements for covering the angular range from 4.51° to 12.05° . The use of a foam plastic target holder of 0.05 g/cm^2 thickness and additional shielding resulted in a decrease of background by a factor varying between 2 and 3 at different angles. Figure 1 indicates the results obtained at 9.95° in the case of 1.3325-MeV γ rays with a ^{60}Co source of about 250 mCi strength. The target thickness t was chosen in such a way that μt was less than 0.4, where μ is the attenuation coefficient. Thus, secondary effects such as multiple scattering and bremsstrahlung production in the target were made negligible. The new Ge(Li) detector had a photopeak full width at half

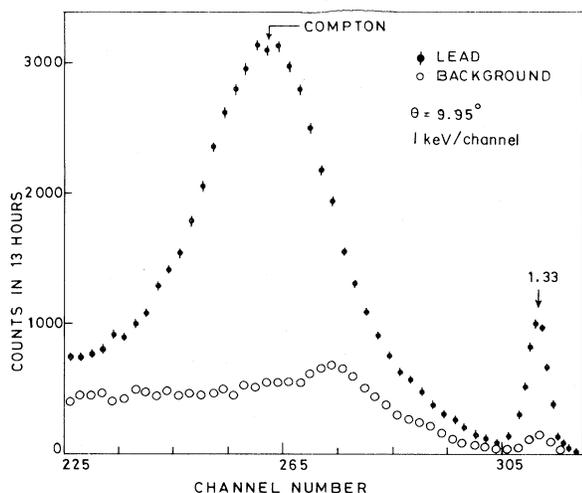


FIG. 1. Data obtained at 9.95° in the neighborhood of the channels corresponding to elastic and Compton scattering of 1.3325-MeV γ rays. A biased amplifier was used in the experiment. Except in the case of the lead scatterer data in channels above 306, counts in alternate channels are shown.

maximum of about 4 keV for 1.3325-MeV γ rays. As discussed in paper I, we performed auxiliary measurements with thinner and thicker targets and found that the target-dependent background correction to the measured counts was negligible within the experimental error of about 3%.

For the sake of completeness and easy understanding of the subsequent discussion, an important equation from paper I is reproduced here. The new method of accurate measurement of small-angle scattering cross sections is based on the following equation:

$$\frac{d\sigma_{\text{el}}}{d\Omega} = \frac{d\sigma_{\text{KN}}}{d\Omega} S(x, Z=13) \frac{n_{\text{el}}}{n_{\text{Al}}} \frac{T^{\text{Al}}}{T} \frac{N^{\text{Al}}}{N} \frac{\epsilon_c}{\epsilon}, \quad (1)$$

where $d\sigma_{\text{el}}/d\Omega$ is the differential cross section of the target atom for elastic scattering through angle θ , $d\sigma_{\text{KN}}/d\Omega$ is the Klein-Nishina prediction for Compton scattering cross section at the same angle per electron assumed free and at rest, $S(x, Z=13)$ is the well-known² incoherent scattering function for aluminum, $x = \sin(\theta/2)/\lambda$ and is proportional to momentum transfer, λ is the wavelength of the incident γ radiation, n_{el} is the count rate in the elastic scattering photopeak with the target under study, n_{Al} is the Compton scattering count rate with a low- Z target such as aluminum, T^{Al} and T are, respectively, the transmission factors for aluminum at Compton energy and for the target under study at the incident gamma energy, N^{Al} and N are the num-

ber of scattering atoms in the aluminum target and the target under study, and ϵ_c and ϵ are the detector photopeak efficiencies for Compton and elastically scattered γ rays, respectively. An equation of a similar form but without the ϵ_c/ϵ factor is used to determine Compton scattering cross sections. The errors in the ratios T^{Al}/T , N^{Al}/N , and ϵ_c/ϵ are less than 1% and $S(x, Z=13)$ is known to about 0.5% for x values of interest in this work. Thus, the error in the cross section is determined mainly by the error in the count rate ratio $n_{\text{el}}/n_{\text{Al}}$.

Although in most cases comparison targets of aluminum were used, we performed additional measurement at 4.51° with comparison targets of graphite as well as aluminum. The density of the graphite target was 1.62 g/cm^3 and was larger than the value 1.545 g/cm^3 mentioned by Ramanathan *et al.*⁶ in their recent work. Transmission factors and accurately known incoherent scattering functions for carbon and aluminum were used to normalize the Compton scattering count rates obtained with targets of graphite and aluminum containing the same number of electrons. The normalized graphite count rate was $(5 \pm 4)\%$ larger than that obtained with the aluminum target. This difference, albeit small, could be due to either nonuniform porosities in the graphite target or to inaccuracies in the attenuation coefficients used in the estimation of transmission factors. In view of the expected accuracies of about 1% in the attenuation coefficients^{7,8} at energies of about 1 MeV, the latter possibility is, in fact, unlikely. The lead elastic scattering cross section deduced with the help of measurements with the comparison graphite target thus turns out to be $(5 \pm 4)\%$ smaller than that deduced with the use of the comparison aluminum target. This possible graphite target porosity effect needs to be borne in mind when experimental data from different laboratories are compared with each other or with theory.

The elastic and the Compton scattering counts were determined from the observed pulse-height distributions by procedures described in detail in an unpublished thesis.⁹ At the smaller angles of 4.51° and 5.33° , the widths of the Compton peaks are nearly equal to the Compton energy shifts. The width of the Compton peak is much larger than that of the elastic scattering peak on account of the finite angular spread and the momentum distribution of the atomic electrons. A combination of two Gaussian functions along with a linear term representing an underlying continuum was fitted by the least-squares method to the net scattered counts under the elastic and the Compton peaks in order to determine the elastic and the Compton intensities.

In the case of 1.3325-MeV measurements at larger angles, a simpler, direct procedure was followed

with the help of a calibration source of about 0.2-mCi strength placed at the target position. The elastic scattering intensity was first determined by the technique of normalization of the calibration source 1.3325-MeV photopeak to the elastic scattering photopeak. The elastic scattering counts so determined were subtracted from the measured pulse-height distributions in order to determine the Compton component. In the case of 1.1732-MeV measurements, the underlying continuum arising from 1.3325-MeV γ rays was first subtracted from the measured pulse-height distribution in order to determine the contribution due only to 1.1732-MeV γ rays. The 1.1732-MeV elastic and Compton scattering intensities were determined in a manner similar to the one described in the case of 1.3325-MeV measurements.

As shown, for example, in Fig. 5 of paper I, the underlying continuum arising from 1.3325-MeV γ rays could be fitted by a linear function in most cases. At the two larger angles of 9.95° and 12.05°, the Compton peaks of 1.1732-MeV γ rays occur at 1.131 MeV and 1.116 MeV, respectively, whereas the Compton edge in the detector response to 1.3325-MeV γ rays occurs at 1.118 MeV. At these angles, the Compton cross sections are about 20 times larger than the lead elastic scattering cross sections and, therefore, the dominant Compton edge contribution is actually shifted towards energies slightly lower than 1.118 MeV. The known curvature in the detector pulse-height distribution near the Compton edge necessitates a quadratic fitting function for the underlying continuum in the case of 1.1732-MeV measurements at these angles. The fitting function had the form given in Eq. (2) as follows:

$$y_i = C_0 + C_1(x_i - x_0) + C_2(x_i - x_0)^2, \quad (2)$$

where y_i indicates the counts in channel x_i , x_0 is the channel corresponding to the Compton peak of 1.1732-MeV γ rays, and the coefficients C_0 , C_1 , and C_2 are determined by the least-squares method from the counts in channels just outside the elastic and the Compton photopeaks arising from 1.1732-MeV γ rays. The underlying continuum so determined at 12.05° is represented by the solid curve in Fig. 2. Thus, whenever a multienergy source is used, care is required in the choice of a fitting function for the underlying continuum.

Electron binding effects are negligible in the case of an atom of low atomic number such as aluminum. Compton scattering measurements with such a low- Z target lead to estimates of the angular spread as well as the possible error in the setting of the mean angle. The relevant data are given in Table I of paper I. The measured Compton shift agreed within about 1.5% with the calculated shift

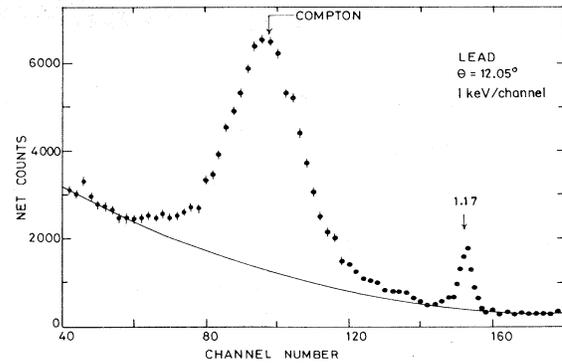


FIG. 2. Net counts obtained at 12.05° with a lead scatterer in the neighborhood of channels corresponding to elastic and Compton scattering of 1.1732-MeV γ rays. Except in the case of data near channel 153, net counts in alternate channels are shown. Solid line represents a fit to the underlying continuum arising from higher-energy components (see Sec. II).

and indicated an angle setting error of less than 1%. After an unfolding of the detector response width, the full width at half maximum of the aluminum scattering pulse-height distribution gives a spread in scattering angles of 1.2°. As mentioned in paper I, the finite angular spread has to be known in order to apply the angular acceptance correction to the measured scattering cross sections. This downward correction to the elastic scattering cross sections can be as high as 12% in the case of the small angles.

The corrections and errors have been discussed in detail in paper I. The statistical errors in the case of the small copper elastic scattering cross sections were rather large and varied between 5% and 13%. The statistical errors in the case of lead and tin elastic scattering measurements varied between 3% and 7%. The different errors were combined in quadrature in order to obtain the total errors quoted in Sec. IV in connection with cross-section values.

III. THEORETICAL CALCULATIONS

In the range of momentum transfers under discussion, namely, x less than 12 \AA^{-1} , the nuclear Thomson scattering amplitude can be accurately calculated in the point-charge approximation and shown to contribute less than 2.5% of the Rayleigh scattering amplitude for Z larger than 28. Accurate calculations of the Delbrück scattering amplitude in the first nonvanishing order are now available at selected photon energies.¹⁰ At energies of about 1.4 MeV or lower the imaginary Delbrück amplitudes make a negligible contribution to the elastic scattering cross sections. In the small-angle regime, the real Delbrück amplitudes interfere destructively with the

corresponding Rayleigh amplitudes and the cross sections are determined essentially by the no-spin-flip amplitudes. In the cases under discussion, the Delbrück amplitudes are less than about 4% of the Rayleigh amplitudes and the nuclear resonance scattering amplitudes are usually negligible.

In the case of γ energies of about 1 MeV, accurate calculations of Rayleigh amplitudes are based on expressions of relativistic second-order S matrix in terms of multipole expansions and have been performed only in the case of K and L shells.¹¹ In the small-angle regime, K -, L -, M -, and N -shell Rayleigh amplitudes need to be calculated accurately. It has been shown in Ref. 11 that, for x less than about 10 \AA^{-1} and γ energies larger than about ten times the electron binding energy, Rayleigh amplitudes for large- Z atoms are predicted to an accuracy of about 1% by the simpler relativistic modified form-factor formalism. This formalism was suggested by Franz¹² as an improvement over the usual form-factor approach which neglects electron binding in intermediate states. It was later recommended by Brown and Mayers¹³ particularly for the no-spin-flip amplitudes. However, as shown, for example, in Ref. 14, which revealed the presence of real Delbrück amplitudes at energies of about 1 MeV and large angles, rather elaborate procedures are necessary in order to obtain reliable estimates of Rayleigh amplitudes for large x .

It was shown in paper I that the lead elastic scattering cross sections up to 8° were in surprisingly good agreement with the nonrelativistic form-factor

theory, even though *a priori* it is not expected to be valid for high- Z elements. In Sec. IV, a detailed comparison will be made of the available Ge(Li) detector results with the predictions of elastic scattering cross sections on the basis of nonrelativistic form factors, relativistic form factors, and relativistic modified form factors. The nonrelativistic form factors were obtained from the work of Hubbell *et al.*² The relativistic form factors and modified form factors were computed for each subshell with the help of DHF wave functions kindly supplied by Mann. These computations were done on a DEC-10 computer, in double precision in critical cases in order to assess possible errors in rounding off. Several values of relativistic form factors were compared with those of Hubbell and Øverbø.³ The agreement was within 0.5%. The required relativistic modified form factors are not available in published reports.

Whole atom Compton scattering cross sections were also determined in the present work. The relativistic calculations of Ribberfors¹⁵ within the framework of an impulse approximation are not particularly suitable for application to small scattering angles. Therefore, the experimental values are compared in Sec. IV with cross-section values calculated from nonrelativistic incoherent scattering functions tabulated by Hubbell *et al.*²

IV. RESULTS AND DISCUSSION

Table I gives the final experimental values of cross sections for the elastic scattering of 1.1732-

TABLE I. Experimental values of elastic scattering cross sections of lead, tin, and copper for 1.1732- and 1.3325-MeV γ rays. x is equal to $\sin(\theta/2)/\lambda$, where λ is the wavelength of the incident radiation.

Energy (MeV)	θ (deg)	x (\AA^{-1})	$\frac{d\sigma_{el}}{d\Omega}$ ($10^{-24} \text{ cm}^2/\text{sr}$)		
			Lead	Tin	Copper
1.1732	4.51	3.72	3.86 ± 0.232	1.07 ± 0.064	
	5.33	4.40	2.42 ± 0.133	0.609 ± 0.040	0.096 ± 0.015
	6.22	5.13	1.54 ± 0.078	0.372 ± 0.025	0.072 ± 0.009
	7.11	5.88	1.20 ± 0.062	0.248 ± 0.020	0.062 ± 0.009
	8.00	6.60	0.796 ± 0.045	0.141 ± 0.009	0.050 ± 0.007
	9.95	8.21	0.512 ± 0.032	0.093 ± 0.007	0.023 ± 0.0025
	12.05	9.93	0.264 ± 0.017	0.061 ± 0.005	0.015 ± 0.002
1.3325	4.51	4.23	2.60 ± 0.122	0.765 ± 0.048	
	5.33	5.00	1.64 ± 0.071	0.382 ± 0.028	0.108 ± 0.011
	6.22	5.83	1.13 ± 0.044	0.253 ± 0.020	0.066 ± 0.005
	7.11	6.66	0.913 ± 0.035	0.156 ± 0.018	0.053 ± 0.006
	8.00	7.50	0.618 ± 0.025	0.121 ± 0.011	0.033 ± 0.003
	9.95	9.32	0.303 ± 0.016	0.069 ± 0.005	0.016 ± 0.001
	12.05	11.28	0.146 ± 0.008	0.045 ± 0.003	0.009 ± 0.001

MeV and 1.3325-MeV γ rays through angles between 4.51° and 12.05° by lead, tin, and copper. It should be noted that the values given in Table 2 of paper I for lead at the two smallest angles are now revised as a result of additional work mentioned in Sec. III.

Rayleigh amplitudes were combined with small nuclear Thomson and Delbrück amplitudes in order to calculate the theoretical values of elastic scattering cross sections. In Fig. 3 we present a comparison between experimental results and theoretical calculations of elastic scattering cross sections for the medium-Z tin target. The solid curve is based on the use of relativistic form factors for the evaluation of Rayleigh amplitudes. The dashed curve shows what happens when relativistic modified form factors are used instead of relativistic form factors. It is quite clear that relativistic modified form-factor theory predicts systematically too large cross sections, the percentage deviations increasing with x . On the other hand, the relativistic form-factor theory is in very good agreement with experimental data. Thus if corrections for relativity are included

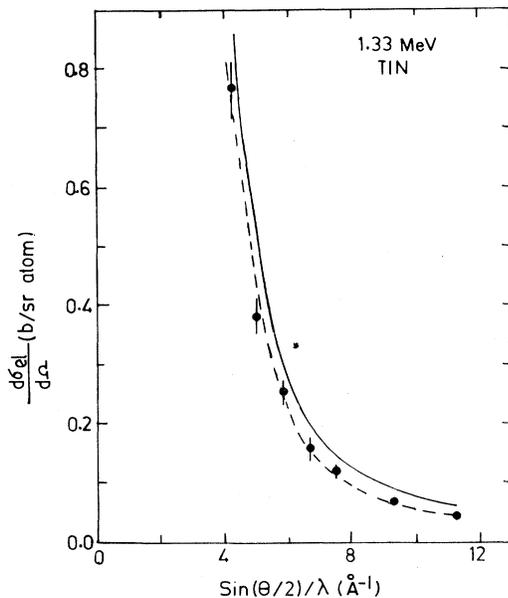


FIG. 3. Differential cross section of tin for elastic scattering of 1.3325-MeV γ rays at different values of $x = \sin(\theta/2)/\lambda$ (\AA^{-1}). Errors are either shown or are smaller than the sizes of the respective points. Solid line represents theoretical predictions based on relativistic form factors for the calculation of Rayleigh amplitudes. Dashed line represents similar predictions based on relativistic modified form factors. Nuclear Thomson and Delbrück amplitudes are included in the theoretical estimates of elastic scattering cross sections.

through the use of relativistic wave functions, it is necessary to include appropriate corrections for electron binding in intermediate states. We do not show here a similar comparison in the case of lead but the corresponding differences between the solid and the dashed curves are much larger than those in the case of tin. As shown later in this section, the relativistic modified form-factor theory is in excellent agreement with our lead data. The experimental data are also in fair agreement with nonrelativistic form-factor theory of Rayleigh scattering. Thus, for momentum transfer up to about 12\AA^{-1} , the increase in cross section due to relativistic effects seems to be compensated almost entirely by the decrease due to binding effects. In order to give a quantitative idea of the extent of agreement, in the case of the largest and the smallest atomic number cases studied, between the three theoretical approaches and the experimental data, we define χ^2 as follows:

$$\chi^2 = \frac{1}{n} \sum_{i=1}^n \frac{(\sigma_{\text{expt}}^i - \sigma_{\text{theory}}^i)^2}{(\Delta\sigma_i)^2}, \quad (3)$$

where $\Delta\sigma_i$ is the error only in the experimental value σ_{expt}^i for the cross section, σ_{theory}^i is the calculated elastic scattering cross section, and n is the number of measurements. With the theoretical Rayleigh amplitudes based on nonrelativistic form factors, relativistic form factors, and relativistic modified form factors, the values of χ^2 are 1.90, 83.8, and 0.84, respectively, for lead, and 1.87, 2.17, and 1.24, respectively, for copper. Although the differences between the χ^2 values obtained with the three approaches are not very large for a smaller atomic number element such as copper, the relativistic modified form-factor formalism is uniformly the best one and in very good agreement with the data up to an x value of about 12\AA^{-1} .

Having established the validity of the relativistic modified form-factor approach for the calculation of Rayleigh amplitudes up to about 12\AA^{-1} , we present now a comparison between the predictions based on this approach and the Ge(Li) detector data available from different laboratories. The ^{152}Eu γ rays of energies between 0.2447 and 1.408 MeV were used by Ramanathan *et al.* with targets of copper, cadmium, tantalum, and lead in the scattering angle range from 2.4° to 10° . The same γ energies were used by the Australian group between 3° and 45° with a lead target,¹⁶ and between 7° and 45° with a target of tungsten.¹⁷ The measurements mentioned in Refs. 6, 16, 17, and 18 were made in the usual scattering geometry and not in the double-shadow-cone geometry. Since our primary interest is to focus attention on the dominant Ray-

leigh contribution for $3 < x < 12 \text{ \AA}^{-1}$, and since the relative Delbrück contribution increases with x as well as γ energy, we discuss the above-mentioned measurements in an energy range around 1 MeV, namely, that between 0.7789 and 1.408 MeV. For the same reasons, we do not include in the analysis the recent results of de Barros *et al.*,¹⁸ obtained at the lower energy of 0.468 MeV from 5° to 40° . In Fig. 4 we show the variation with x of the ratio of the experimental lead elastic scattering cross section to the calculated Rayleigh cross section. If the effect of the nuclear Thomson and Delbrück amplitudes on the actual cross sections is also considered, the ratios are expected to lie in the regions indicated by the slanting lines. Our data are seen to be in excellent agreement with theory. There is a possibility that the slightly smaller values of Ramanathan *et al.* may be due to the graphite porosity effect mentioned in Sec. II. For $7 < x < 12 \text{ \AA}^{-1}$, the values of Chitwattanagorn *et al.* are in overall agreement with theoretical expectations. The report of Chitwattanagorn *et al.* does not mention angular acceptance corrections which become relatively more important at smaller angles, that is, at smaller x values. Judging from the results mentioned in paper I and Ref.

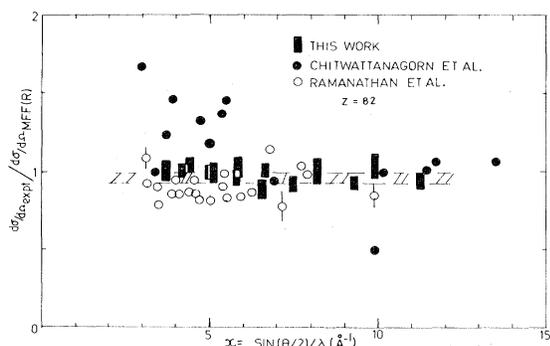


FIG. 4. Cross section $d\sigma/d\Omega_{\text{MFF(R)}}$ is the Rayleigh scattering cross section calculated in the relativistic modified form-factor approach. Ratio of the lead experimental elastic scattering cross section $d\sigma/d\Omega_{\text{expt}}$ to $d\sigma/d\Omega_{\text{MFF(R)}}$ is shown at different values of x . Rectangles represent values obtained in the present work along with errors. Other values have been obtained with γ rays of different energies (see text). Errors in the results of Ramanathan *et al.* are shown on only a few representative points. Errors in the results of Chitwattanagorn *et al.* are comparable but are not shown (see text). With the inclusion of nuclear Thomson and Delbrück scattering effects, the points are expected to lie within the region of slanting lines.

6, and the description of the experimental arrangement given in Ref. 16, these corrections are expected to be of the order of 10% to 15% for the smaller x values. If these corrections were to be made to the data of Chitwattanagorn *et al.*, the resulting values for small x will lie in better, but still not good, agreement with theory. In this connection, it is worth noting that the Australian group has mentioned explicitly in Ref. 17 that angular acceptance corrections were not applied. Further, it is stated in Ref. 17 that an angle setting error gave rise to a possible error of about 10% in the absolute cross sections. This error is also likely to be more important at smaller angles.

Comparisons somewhat similar to those in Fig. 4 are shown in Figs. 5(a) and 5(b) for tin and copper, respectively. Since the experimental errors are larger than 6.5% in these cases, and since the slanting line regions will extend only from near unity to about 0.97 in the case of tin and to about 0.995 in the case of copper, these are not shown here.

Thus, for a wide variation in atomic number, the relativistic modified form-factor theory of Rayleigh scattering has been shown to give a good account of the experimental data for $x < 12 \text{ \AA}^{-1}$. If for purposes of radiation shielding design or radiation dosimetry extrapolations to other energies and targets are needed, the convenient but slightly less accurate

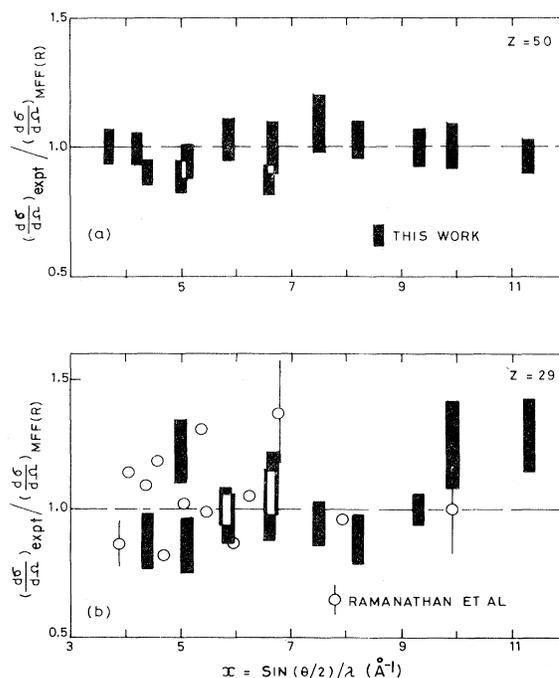


FIG. 5. Similar to Fig. 4. (a) and (b) are for tin and copper, respectively.

TABLE II. Experimental values of Compton scattering cross sections of lead, tin, and copper for 1.1732- and 1.3325-MeV γ rays. Compton cross sections are expressed in units of 10^{-24} cm²/sr. Theoretical value of the Compton scattering cross section is obtained from the product of the Klein-Nishina cross section per electron and the incoherent scattering function $S(x,z)$ calculated in a nonrelativistic treatment by Hubbell *et al.* (Ref. 2). Ratio of the experimental value to the theoretical value is designated as Ratio in the table. Relevant values of x are listed in Table I. Klein-Nishina cross section depends upon energy and angle, and $S(x,Z)$ approaches unity with increasing x but more slowly with larger Z .

Energy (MeV)	Angle (deg)	Lead		Tin		Copper	
		$\left[\frac{d\sigma_c}{d\Omega}\right]_{\text{expt}}$	Ratio	$\left[\frac{d\sigma_c}{d\Omega}\right]_{\text{expt}}$	Ratio	$\left[\frac{d\sigma_c}{d\Omega}\right]_{\text{expt}}$	Ratio
1.1732	4.51	4.80±0.22	0.88±0.040	3.07±0.14	0.87±0.040	2.01±0.10	0.94±0.047
	5.33	5.27±0.24	0.94±0.043	3.55±0.16	0.99±0.045	2.06±0.09	0.95±0.041
	6.22	5.53±0.25	0.97±0.044	3.60±0.16	0.99±0.044	2.07±0.09	0.96±0.041
	7.11	5.40±0.24	0.94±0.042	3.51±0.16	0.96±0.044	2.11±0.095	0.98±0.044
	8.00	5.02±0.22	0.87±0.038	3.51±0.16	0.97±0.044	2.11±0.09	0.98±0.042
	9.95	5.77±0.31	1.00±0.055	3.48±0.19	0.97±0.053	2.08±0.11	0.99±0.052
	12.05	5.45±0.30	0.97±0.054	3.39±0.18	0.98±0.052	2.05±0.11	1.01±0.054
1.3325	4.51	5.21±0.18	0.93±0.032	3.38±0.12	0.94±0.033	2.07±0.07	0.96±0.033
	5.33	5.88±0.20	1.03±0.035	3.55±0.12	0.97±0.033	2.12±0.07	0.97±0.032
	6.22	5.71±0.20	0.99±0.035	3.63±0.13	0.99±0.036	2.22±0.08	1.02±0.037
	7.11	5.80±0.20	0.99±0.035	3.79±0.13	1.03±0.035	2.24±0.08	1.03±0.037
	8.00	5.69±0.20	0.98±0.035	3.56±0.12	0.98±0.033	2.15±0.07	1.00±0.033
	9.95	5.57±0.19	0.97±0.033	3.43±0.12	0.96±0.034	2.05±0.07	0.98±0.034
	12.05	5.38±0.19	0.96±0.034	3.38±0.12	0.98±0.035	1.93±0.06	0.96±0.030

nonrelativistic form-factor theory may be used. However, relativistic form factors predict unduly large Rayleigh amplitudes in the case of targets of medium and high atomic number and should therefore not be used.

If the experimental accuracy can be improved to the 1% level, it would be possible to provide a confirmation of even the relatively small Delbrück contribution at small angles under the assumption that Rayleigh scattering amplitudes are accurately known. Alternatively, if nonvanishing lowest-order Delbrück calculations can be considered adequate, it will be possible to probe the adequacy of present Rayleigh scattering calculations.

The Compton scattering cross sections are summarized in Table II. Values of nonrelativistic, incoherent scattering functions $S(x,Z)$ are obtained from the work of Hubbell *et al.*² $S(x,Z)/Z$ is significantly lower than unity and approaches unity with increasing x . The product of the Klein-Nishina prediction $d\sigma_{\text{KN}}/d\Omega$ and $S(x,Z)$ gives the theoretical value of the Compton scattering cross section per atom. From Table II, the experimental values of Compton scattering cross sections are seen to be in

agreement within the error of 3% to 4% with nonrelativistic predictions in the case of x larger than about 4.5 \AA^{-1} . The experimental values for smaller x tend to be slightly lower than nonrelativistic predictions. Thus, there is a suggestion that reliable relativistic calculations of Compton scattering cross sections are desirable for the smaller x values where binding effects are more important.

ACKNOWLEDGMENTS

We are extremely grateful to Dr. J. B. Mann of Los Alamos Scientific Laboratory for kindly sending us DHF wave functions in the case of copper, tin, and lead. We thank Dr. J. H. Hubbell of the National Bureau of Standards and Professor R. H. Pratt of the University of Pittsburgh for continuing interest in our work, the workshop personnel under K. Sridhara for construction of the apparatus, and Dr. M. P. Navalkar of the Bhabha Atomic Research Centre for graphite targets. The work was supported in part by a grant from the National Bureau of Standards, Washington, D.C., under the Special Foreign Currency Program.

- ¹P. P. Kane, G. Basavaraju, J. Mahajani, and A. K. Priyadarsini, *Nucl. Instrum. Methods* **155**, 467 (1978).
- ²J. H. Hubbell, Wm. J. Veigele, E. A. Briggs, R. T. Brown, D. T. Cromer, and R. J. Howerton, *J. Phys. Chem. Ref. Data* **4**, 471 (1975).
- ³J. H. Hubbell and I. Øverbø, *J. Phys. Chem. Ref. Data* **8**, 69 (1979).
- ⁴P. P. Kane, J. Mahajani, G. Basavaraju, and A. K. Priyadarsini, *Abstracts of the XII International Conference on the Physics of Electronic and Atomic Collisions, Gatlinburg, Tennessee, 1981*, edited by S. Datz (North-Holland, Amsterdam, 1981); P. P. Kane, invited talk presented at the 4th National Symposium on Radiation Physics, Patiala, India, 1981 (unpublished).
- ⁵Private communication from Dr. J. B. Mann.
- ⁶N. Ramanathan, T. J. Kennett, and W. V. Prestvitch, *Can. J. Phys.* **57**, 343 (1979).
- ⁷J. H. Hubbell, report No. NSRDS-NBS-29 1969 (unpublished); J. H. Hubbell, H. A. Gimm, and I. Øverbø, *J. Phys. Chem. Ref. Data* **9**, 1023 (1980).
- ⁸P. P. Kane, G. Basavaraju, and K. M. Varier, *Nucl. Instrum. Methods* **147**, 507 (1977).
- ⁹J. Mahajani, Ph.D. thesis, Indian Institute of Technology, Bombay, 1982 (unpublished).
- ¹⁰P. Papatzacos and K. Mork, *Phys. Rev. D* **12**, 206 (1975); *Phys. Rep. C* **21**, 81 (1975).
- ¹¹L. Kissel, R. H. Pratt, and S. C. Roy, *Phys. Rev. A* **22**, 1970 (1980); **27**, 285 (1983).
- ¹²W. Franz, *Z. Phys.* **95**, 652 (1935); **98**, 314 (1936).
- ¹³G. E. Brown and D. F. Mayers, *Proc. R. Soc. London, Ser. A* **242**, 89 (1957) and earlier papers mentioned therein.
- ¹⁴G. Basavaraju, P. P. Kane, and K. M. Varier, *Pramāna* **12**, 665 (1979).
- ¹⁵R. Ribberfors, *Phys. Rev. B* **12**, 2067 (1975); **12**, 3136 (1975).
- ¹⁶W. Chitwattanagorn, R. B. Taylor, P. Teansomprasong, and I. B. Whittingham, *J. Phys. (London)* **G6**, 1147 (1980).
- ¹⁷R. B. Taylor, P. Teansomprasong, and I. B. Whittingham, *Aust. J. Phys.* **34**, 125 (1981).
- ¹⁸S. de Barros, J. Eichler, M. Gaspar, and O. Gonsalves, *Phys. Rev. C* **24**, 1765 (1981).