PHYSICAL REVIEW A

GENERAL PHYSICS

THIRD SERIES, VOLUME 28, NUMBER 1 JULY 1983

Neutron phase shift in moving matter

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(Received 28 December 1982)

The effects of a moving medium on the phase of a massive particle are studied theoretically. The discussion explains a recently published Fizeau-type neutron-interferometer experiment. It is shown explicitly that the physics of this experiment can be described either as a manifestation of a time-dependent potential in the Schrödinger equation or as a verification of the Galilean-transformation properties of Schrödinger waves. An intriguing consequence of these considerations is that for most materials no Fizeau phase shift occurs if the boundaries of the moving medium are at rest. However, we point out that such would not be the case in an experiment involving a medium whose potential is velocity dependent.

I. INTRODUCTION

In a recent paper,[1] the results of an experiment were presented on the effects of a moving medium on the phase shifts observed in a neutron interferometer. The experiment bore a close resemblance to the historic work of Fizeau[2] in which phase shifts in light waves were produced by water flowing in stationary tubes. However, in the case of the neutron experiment, the observed phase shifts were directly connected to the property that the boundaries of the medium chosen, a rotating square quartz rod, were moving. For light, the experiment with moving boundaries had been performed by Zeeman.[3] In contrast to the original Fizeau experiment (or the later work of Macek *et al.*,[4] who used a rotating disk inside an interferometer), the experiment with neutrons would have given a null result[5] had the motion been contained inside stationary boundaries. This somewhat surprising conclusion, though not obvious, depends on the specific form of the dispersion relation for neutrons in most materials, as we will discuss in this paper. Our aim is to analyze the phase shifts caused by the motion of a slab of material inside a two-beam interferometer, from several different viewpoints.

In Sec. II we present the general result on the basis of a global argument, using the relativistic invariance of phase differences, and we derive its special form applicable to slow neutrons. In Sec. III we show that this result follows equally well from the motion of a finite-potential barrier representing the material medium. In Sec. IV we consider explicitly the Doppler-shifted waves as scattered by the moving nuclei in the medium showing the microscopic basis for the result of Sec. II. Section V examines various heuristic arguments based on the effective distance traveled or the effective time spent in the medium, and shows that great care is required when specific particle or wave kinematic models are used. Section VI discusses the relation of the neutron Fizeau phase shift to the transformation laws of wave vector and frequency. We conclude with a proposal for a Fizeau-type experiment with resonance neutrons, where a phase shift would be observed even with stationary boundaries.

II. MOVING REFERENCE FRAME

We first present an argument based on the relativistic invariance of phase differences at a space-time point, because the argument is equally valid for light and de Broglie waves. Consider the arrangement shown in Fig. 1, where a parallel-faced plate of thickness D and index of

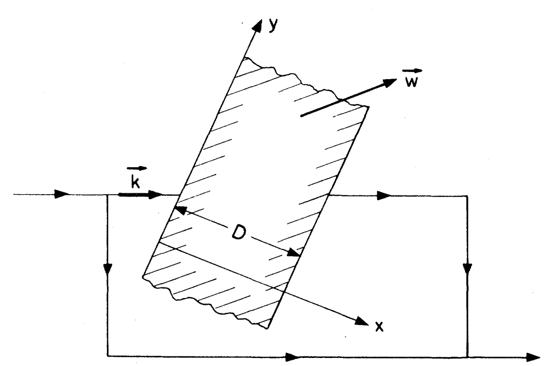

FIG. 1. Phase plate of thickness D moving with velocity $\vec{w}$ in one beam path of an interferometer. x-y coordinate system is at rest but $x=0$ coincides with the plate entrance surface at $t=0$ as shown.

refraction $n(k)$ is inserted in one beam of either a photon or a neutron interferometer. If initially the plate is at rest, the relative phase of the two interfering beams (top beam phase minus bottom beam phase) is

$$\phi(\vec{k})=D\{[n^2(k)k^2-k_y^2]^{1/2}-k_x\} , \qquad (1)$$

where k is the magnitude of the incident wave vector $\vec{k}$ and k_x (k_y) is the component perpendicular (parallel) to the plate surface. Suppose the plate is now set in motion with a laboratory velocity $\vec{w}$. Then in the rest frame of the plate the relative phase will be $\phi(\vec{k}\,')$, where $\vec{k}\,'$ is the incident wave vector in that frame and ϕ is the function (1). Since relative phase is a relativistically invariant quantity, $\phi(\vec{k}\,')$ must be the relative phase in every frame, including the laboratory frame. Therefore, the phase shift caused by the motion of the plate— the Fizeau effect—is

$$\Delta\phi=\phi(\vec{k}\,')-\phi(\vec{k}) . \qquad (2)$$

The detailed evaluation of (2) depends first on how $\vec{k}\,'$ is related to $\vec{k}$ and second on the specific form of the function $n(k)$.

From the Einsteinian transformation for energy momentum, one finds that if a particle of rest mass m_0 has a wave vector $\vec{k}$ in the laboratory frame, then the wave vector in a frame moving at velocity $\vec{w}$ is

$$\vec{k}\,'=\vec{k}-\left[(1-\gamma)\frac{\vec{k}\cdot\vec{w}}{w}+\gamma\frac{w}{c}\left[k^2+\frac{m_0^2c^2}{\hbar^2}\right]^{1/2}\right]\frac{\vec{w}}{w} , \qquad (3)$$

where $\gamma\equiv[1-(w^2/c^2)]^{-1/2}$. The specific form of $n(k)$ can be stated only after specifying both the type of radiation and the material of the plate. Equation (2) with (1) and (3) is relativistically exact and applies equally well to photons and neutrons.

For slow neutrons and low plate velocity as used in the experiment, Eq. (3) reduces to the Galilean approximation

$$\vec{k}\,'=\vec{k}-\frac{m}{\hbar}\vec{w} \qquad (3')$$

and the index of refraction is

$$n(k)=\left[1-\frac{2mV(k)}{\hbar^2k^2}\right]^{1/2} , \qquad (4)$$

where $V(k)$ is the neutron optical potential of the medium. It follows, from Eqs. (1), (3′), and (4), that the Fizeau phase shift [Eq. (2)] is explicitly

$$\Delta\phi=\{[(1-\alpha)^2-\beta']^{1/2}+\alpha-(1-\beta)^{1/2}\}k_xD , \qquad (5)$$

where we have introduced the dimensionless parameters $\alpha\equiv mw_x/\hbar k_x$, $\beta\equiv 2mV(k)/\hbar^2k_x^2$, and $\beta'\equiv 2mV(k')/\hbar^2k_x^2$.

We point out, that for most materials the neutron optical potential $V(k)$ is independent of wavelength ($\beta'=\beta$) at thermal or subthermal neutron energies. Assuming this to be the case, we note for future reference that the Fizeau phase shift to first order in β is

$$\Delta\phi=-\frac{\alpha\beta k_xD}{2(1-\alpha)} . \qquad (5')$$

III. MOVING POTENTIAL

Now we will show, for the case of a wavelength-independent potential, that the result of Eq. (5) can be obtained without invoking relativity considerations. Consider the two-dimensional Schrödinger equation with the following time-dependent potential:

$$V(x,y,t)=\begin{cases}V, & \text{for } w_xt<x<w_xt+D\\ 0, & \text{elsewhere}\end{cases} \qquad (6)$$

which describes a constant potential barrier of height V, moving with a velocity $\vec{w}$, occupying the same space-time region as the plate in Fig. 1. Assuming a wave incident from the left with wave vector $\vec{k}$ and frequency ω, the solution of Schrödinger's equation with the above potential may be written in the form

$$\psi_{\rm I}=Ae^{i(\vec{k}\cdot\vec{r}-\omega t)}+Be^{i(\vec{k}_1\cdot\vec{r}-\omega_1t)}, \quad \text{for } x<w_xt$$

$$\psi_{\rm II}=Ce^{i(\vec{K}\cdot\vec{r}-\Omega t)}+Ee^{i(\vec{K}_1\cdot\vec{r}-\Omega_1t)}, \quad \text{for } w_xt<x<w_xt+D \qquad (7)$$

$$\psi_{\rm III}=Fe^{i(\vec{k}_2\cdot\vec{r}-\omega_2t)}, \quad \text{for } w_xt+D<x .$$

Boundary conditions require that these wave functions agree at the moving boundaries, which lead to

$$\vec{k}_2=\vec{k}, \quad \omega_2=\omega ,$$

and

$$(K_x-k_x)w_x=\Omega-\omega, \quad K_y=k_y , \qquad (8)$$

and in addition, values for the Doppler-shifted reflected wave vectors $\vec{k}_1$ and $\vec{K}_1$. These wave vectors and the specific values of the amplitudes, which follow from the continuity of the derivative of ψ at the space-time boundaries, are not needed for finding the Fizeau phase shift.

Referring to $\psi_{\rm II}$ in Eq. (7), we see that the phase of the forward wave inside the potential region at position $\vec{r}$ and time t is $(\vec{K}\cdot\vec{r}-\Omega t)$, whenever the phase at the equivalent space-time point in the second beam of the interferometer is $(\vec{k}\cdot\vec{r}-\omega t)$. Therefore, the relative phase of these beams is

$$\phi(\vec{r},t)=(\vec{K}-\vec{k})\cdot\vec{r}-(\Omega-\omega)t . \qquad (9)$$

Evaluating this at $x=w_xt+D$ and any y (i.e., at the back-face of the moving barrier) and using Eqs. (8) we obtain

$$\phi(\vec{w})=(K_x-k_x)D , \qquad (10)$$

where the $\vec{w}$ dependence of ϕ is implicitly contained in K_x. To evaluate (10) we must find $K_x(\vec{k},\vec{w},V)$. The Schrödinger equation requires

$$\frac{\hbar^2k^2}{2m}=\hbar\omega, \quad \frac{\hbar^2K^2}{2m}+V=\hbar\Omega , \qquad (11)$$

which together with Eq. (8) leads to a quadratic equation for K_x whose solution is

$$K_x(\vec{k},\vec{w},V)=\{\alpha+[(1-\alpha)^2-\beta]^{1/2}\}k_x , \qquad (12)$$

where we have used the dimensionless parameters α and β

defined in Sec. II.

Since Eq. (10) is the relative phase of the upper and lower interferometer beams while the plate is in motion, we must subtract the relative phase while the plate is at rest to find the phase shift produced by the motion alone:

$$\Delta\phi=\phi(\vec{w})-\phi(0)\ . \tag{13}$$

When Eqs. (10) and (12) are employed to explicitly evaluate Eq. (13), the phase shift obtained agrees with the phase shift determined using Eq. (5) for the case of a potential independent of the wavelength of the neutron. This agreement justifies the assumption [implicit in Eq. (6)] that the height of such a potential barrier is independent of its speed.

IV. MOVING SCATTERERS

The feature that the phase shift $\Delta\phi$, as given in Eq. (5), vanishes for a wavelength-independent potential when the plate velocity is parallel to the plate surface, deserves discussion. This feature is not surprising within the framework of each of the derivations presented above, since they employ macroscopic descriptions of the medium: an index of refraction function $n(k)$ in Sec. II, and a potential barrier of constant height V in Sec. III. However, when one adopts a microscopic view of the medium the vanishing of $\Delta\phi$ is not immediately understandable, for then the radiation behind the plate is obtained as a superposition of wavelets scattered from individual scattering centers. Clearly, the radiation scattered from an individual scatterer is radically modified by the motion of the scatterer. And yet the total radiation field behind the plate produced by all moving scatterers acting in concert is not affected by the motion when the scatterers move parallel to the plate surface. Thus, in this section we wish to show explicitly how this comes about.

First, we must find the field produced by a *single* moving scatterer. Consider a scatterer of scattering length b whose position in the laboratory frame at time t is $\vec{a}+\vec{w}t$ and which is bathed in a laboratory plane wave $\exp[i(\vec{k}\cdot\vec{r}-\omega t)]$. Then, in the rest frame of the scatterer the scattered radiation is the spherical wave

$$\Psi'_{\text{scatt}}(\vec{r}\,',t)=-\frac{b}{|\vec{r}\,'-\vec{a}\,|}\times\exp[i(\vec{k}\,'\cdot\vec{a}+|\vec{k}\,'|\ |\vec{r}\,'-\vec{a}\,|)-i\omega' t]\ , \tag{14}$$

where the coordinate transformation between the moving primed frame and the laboratory unprimed frame is

$$\vec{r}=\vec{r}\,'+\vec{w}t \tag{15}$$

and the incident wave vector transforms according to Eq. (3′). Consequently, in the moving frame the scattered wave and the incident plane wave have a time-independent phase difference

$$\Delta\phi'(\vec{r}\,')\equiv\phi'_{\text{scatt}}(\vec{r}\,',t)-\phi'_{\text{plane}}(\vec{r}\,',t)$$
$$=-\vec{k}\,'\cdot(\vec{r}\,'-\vec{a})+|\vec{k}\,'|\ |\vec{r}\,'-\vec{a}\,|\ .$$

Since phase differences are relativistically invariant, that relative phase must be the same in the laboratory frame at the same space-time point

$$\Delta\phi(\vec{r},t)=\Delta\phi'[\vec{r}\,'(t)]\ .$$

In laboratory-frame quantities this phase difference is

$$\Delta\phi(\vec{r},t)=KR-\vec{K}\cdot\vec{R}$$

and the amplitude of the scattered wave [Eq. (14)] is

$$\frac{b}{|\vec{r}\,'-\vec{a}\,|}=\frac{b}{R}\ ,$$

where

$$\vec{K}\equiv\vec{k}-\frac{m}{\hbar}\vec{w} \tag{16}$$

and

$$\vec{R}\equiv\vec{r}-\vec{a}-\vec{w}t\ . \tag{16′}$$

Thus, finally, in the laboratory frame the scattered wave is obtained as

$$\psi_{\text{scatt}}(\vec{r},t)=-b\frac{e^{i(KR-\vec{K}\cdot\vec{R})}}{R}e^{i(\vec{k}\cdot\vec{r}-\omega t)}\ . \tag{17}$$

In this derivation of Eq. (17) we analyzed the scattering in the rest frame of the scatterer and used relativity arguments to obtain the scattered field in the laboratory frame. This is analogous to the macroscopic relativistic discussion in Sec. II. Nevertheless, the same result must be obtainable via an argument made entirely within the laboratory frame. Such an argument is given in Appendix A.

Some features of the scattered wave are worth noting. First, expression (17) cannot be obtained from Eq. (14) using just the transformation of Eq. (15), i.e., the wave function $\psi(\vec{r},t)$ in the laboratory frame is not simply the wave function $\psi'(\vec{r}\,',t)$ in the moving frame evaluated at the space-time point $(\vec{r},t)$. Second, the scattered wave in the laboratory frame is neither a spherical wave nor is its frequency at a given position independent of time, a feature related to Doppler shift.

The phase shift by a moving medium may now be obtained by integrating Eq. (17) over all scattering centers of the medium. For simplicity we assume that the scattering centers are arranged in an infinitesimally thin slab which permits us to ignore multiple scattering. Let $\vec{a}$ again be an arbitrary position inside the slab at $t=0$ and let the slab be bathed in the plane incident wave $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$. Then, from Eq. (17) the total radiation field at the space-time point $(\vec{r},t)$ is

$$\psi(\vec{r},t)=e^{i(\vec{k}\cdot\vec{r}-\omega t)}\times\left[1-bN\int_{\text{slab}}d\vec{a}\,\frac{e^{i(KR-\vec{K}\cdot\vec{R})}}{R}\right]\ , \tag{18}$$

where N is the number of scattering centers per unit volume. This integral is evaluated in Appendix B for coplanar, but otherwise arbitrary $\vec{k}$, $\vec{w}$ and plate surface normal. It gives for a plate of infinitesimal thickness δD the wave function at a normal distance x behind the plate at time $t=0$ as

$$\psi(x)=e^{ik_x x}\left[1-2\pi i\frac{Nb\delta D}{K_x}\right] . \tag{19}$$

Note, that the effect of the infinitesimal plate is to multiply the original wave function by the expression in large parentheses. The wave behind a slab with finite thickness D can be obtained by repeated applications of this factor if multiple scattering is still neglected. Now using the well-known expression $e^x=\lim_{n\to\infty}(1+x/n)^n$, the wave function behind a finite plate is obtained as

$$\psi(x)=e^{ik_x x}e^{-2\pi i NbD/K_x} . \tag{20}$$

Note that the phase of ψ depends on $\vec{w}$ only via K_x [Eq. (16)] and thus does not depend on w_y. Therefore, the wave field behind a plate moving parallel to its boundary is independent of its speed, despite the property that this wave field is a superposition of individual wavelets, each one depending strongly on the speed of the moving scatterers [Eq. (17)].

Subtracting the phase shift due to a plate at rest from the phase shift

$$\phi(\vec{w})=-2\pi\frac{NbD}{k_x-\frac{m}{\hbar}w_x} ,$$

due to a moving plate, the Fizeau phase shift

$$\Delta\phi=\phi(\vec{w})-\phi(0)=-2\pi NbD\frac{mw_x}{\hbar k_x\left[k_x-\frac{m}{\hbar}w_x\right]} \tag{21}$$

is obtained. This is identical to the Fizeau phase shift of Eq. (5′) since a medium consisting of N scatterers per unit volume each of coherent scattering length b has a mean neutron optical potential

$$V=\frac{2\pi\hbar^2}{m}Nb . \tag{22}$$

It can be shown that the terms to higher order in β and therefore the exact expression of Eq. (5) are obtained if one considers the effects of multiple scattering in the moving medium.

V. MOVING PARTICLE

Having given several derivations of the phase shift of Eq. (5), we can safely comment on some heuristic arguments that have been brought to our attention. These arguments, which appeal explicitly to the velocity of the neutron as a particle, can be misleading unless done carefully, for one can fortuitously obtain approximately the correct result via arguments that are conceptually inadequate. To simplify the discussion, we will assume here that the incident wave vector $\vec{k}$ and the plate velocity $\vec{w}$ are normal to the plate surface, so that $k_x=|\vec{k}|$ and $w_x=|\vec{w}|$.

One argument focuses on extra distance. That is, when the plate of thickness D moves at speed w the neutron must travel the extra distance

$$\Delta D=D\frac{w}{u-w} \tag{23}$$

to reach the back surface, where u is the speed of the neutron while in the moving plate. One may conjecture that the shift (5) is due simply to this extra material, and hence, propose that the following equality may be true:

$$\Delta\phi=(?)\frac{m}{\hbar}(u-v)\Delta D , \tag{24}$$

where v is the neutron speed in vacuum. The conjecture is certainly consistent with the fact, as discussed in Sec. IV, that (5) vanishes when the rear surface does not recede. And, when evaluated to first order in α and β, Eq. (24) does give

$$\Delta\phi=-\tfrac{1}{2}\alpha\beta kD , \tag{25}$$

in agreement with (5) to this order, apparently confirming the conjecture.

The conceptual inadequacy of this argument becomes clear when one attempts to rigorously derive Eq. (5) from Eq. (24). In the course of this effort one realizes that the speed of the neutron inside the moving plate (u) is not the same as the speed inside the plate at rest (u_0), and that neither u nor u_0 used in Eq. (24) gives the exact Eq. (5). Moreover, the fact that u and u_0 differ implies that Eq. (24) is itself incomplete and this seems to suggest that it should be replaced by

$$\Delta\phi=(?)\frac{m}{\hbar}(u-v)\Delta D+\frac{m}{\hbar}(u-u_0)D . \tag{24′}$$

The additional term is the phase accumulated over the distance D due to the difference between u and u_0. Still, Eq. (24′) does not yield Eq. (5), but for small α and β twice the correct result is obtained. In fact, in Eq. (24′) the second term alone is the exact Fizeau phase shift, but this term does not depend at all on the extra thickness ΔD.

The reason we arrived at the wrong result is that Eq. (24′) is still incomplete since it does not reflect the fact that the particle experiences a change $\Delta E=m(u-v)w$ of its *total* energy upon entering the moving plate. This causes a third contribution to Eq. (24′)

$$-\frac{1}{\hbar}\Delta E\,T=-\frac{m}{\hbar}(u-v)\Delta D , \tag{24″}$$

since $T=\Delta D/w$ is the time spent by the neutron inside the moving plate. This term now completely cancels the original term which was the original focus of the conjecture, leaving the correct result. Clearly, the energy change involved in this argument corresponds to the frequency change [Eq. (8)] found in Sec. III by imposing moving boundary conditions on the Schrödinger wave function.

Another argument focuses on the extra time the neutron spends inside the moving plate as compared to the plate at rest. Whatever the details, one can dismiss this conjecture by considering a modified arrangement with phase plates of different thickness in each beam, one plate at rest and the other one moving. By suitably choosing the thicknesses of the plates, the time spent by the neutron inside the medium can be made equal for both beams. One can show that, despite the equality of the travel times, the two beams still differ in phase by

$$\Delta\phi=\frac{m}{\hbar}v\left[\left[1-\frac{u_0}{v}\right]-\left[1-\frac{u}{v}\right]\left[\frac{u-w}{u_0}\right]\right]D , \tag{26}$$

where D is the thickness of the plate at rest. This difference, though small, is conceptually significant.

Thus, though heuristic arguments of the kind discussed in this section may give results in agreement with the accuracy of present experiments, they can be shown to miss conceptually interesting points unless done with proper care. Similarly, the interpretation that the phase shift, observed in the experiment using a rotating square quartz rod, was due to an effective wedge seen by the neutron,[1] is only valid if all the individual effects discussed above are taken into account.

VI. CONCLUDING COMMENTS

A remarkable consequence of the considerations presented above is the fact that the wavelength of a neutron in a moving plate is different from that in the same plate at rest [Eq. (12)]. With suitable caution this may be interpreted as a dependence of the index of refraction on the velocity of the material. Equally remarkable is the change of frequency that the particle experiences upon entering the moving plate [Eq. (8)]. We also found that the contribution of the frequency change to the Fizeau phase shift is of the same order of magnitude as that due to the wavelength change. Both the wavelength and frequency changes could be found directly by using the Galilean transformation properties of Schrödinger waves.[6] Hence, the experimental verification of the Fizeau phase shift [Eq. (5)] may be viewed as a demonstration of these transformation laws.

Another feature deserves comment. We found that the neutron Fizeau phase shift $\Delta\phi$ vanishes when the motion of the plate is such that its boundaries are stationary. However, $\Delta\phi$ does not vanish for an analogous arrangement with visible light in glass.[5] This difference in behavior is neither due entirely to the mass difference of the radiation used in the two arrangements nor is it attributable simply to the difference in relativities involved. As a specific example to the contrary, one finds that $\Delta\phi$ vanishes for x rays in many materials. Similarly, we find from Eqs. (5) and (22) that the neutron Fizeau phase shift for the stationary boundary is

$$\Delta\phi = 2\pi \frac{mN}{\hbar k}\frac{k_y}{k_x}\frac{d}{dk}b(k)\,wD \tag{27}$$

if $\beta \ll 1$ and $|\vec{w}| \ll \hbar k/m$. This phase shift does not vanish if the scattering length b is a function of wavelength or, equivalently, if the mean potential of the medium is wavelength dependent. We propose that this can experimentally be tested at or near resonances, where the scattering length can be strongly dependent on wavelength. A measurement of the phase shift due to a Sm plate at rest has recently[7] demonstrated the feasibility of neutron-interferometry experimentation at a resonance.

ACKNOWLEDGMENTS

We wish to thank Professor C. G. Shull and Professor S. A. Werner for discussions and encouragement and Professor A. Peres for a challenging exchange of correspondence. This work was supported by the National Science Foundation Grant No. DMR-80-21057-A01.

APPENDIX A

We calculate the field of a moving pointlike scatterer in a plane neutron wave $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$. The scattered field obeys the equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - i\hbar\frac{\partial}{\partial t}\right]\psi(\vec{r},t) = -\frac{\hbar^2}{2m}u_0\delta(\vec{r}-\vec{r}_s(t))e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \tag{A1}$$

where u_0 is a length characteristic of the scatterer and $\vec{r}_s(t)$ is the position of the moving scatterer. Assuming the scatterer moves at constant velocity $\vec{r}(t)=\vec{a}+\vec{w}t$, multiplying both sides of Eq. (A1) by $e^{-i(\vec{q}\cdot\vec{r}-\Omega t)}$ and integrating over $d\vec{r}$ and dt, we find the momentum-space wave function

$$\Phi(\vec{q},\Omega) = \frac{-2\pi u_0\delta(\Omega-\omega+(\vec{k}-\vec{q})\cdot\vec{w})e^{i(\vec{k}-\vec{q})\cdot\vec{a}}}{q^2-\frac{2m\Omega}{\hbar}-i\epsilon}, \tag{A2}$$

where $-i\epsilon$ with $\epsilon>0$ has been included in the denominator to ensure outgoing waves. Multiplying both sides of Eq. (A2) by $e^{i(\vec{q}\cdot\vec{r}-\Omega t)}/(2\pi)^4$, and integrating over $d\vec{q}$ and $d\Omega$, we obtain the scattered field

$$\psi(\vec{r},t) = -u_0\int\frac{d\vec{Q}}{(2\pi)^3}\frac{e^{i(\vec{Q}-\vec{K})\cdot\vec{R}}}{Q^2-K^2-i\epsilon}e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \tag{A3}$$

where $\vec{Q}\equiv\vec{q}-(m\vec{w}/\hbar)$, $\vec{K}$ and $\vec{R}$ are as defined in Eq. (16), and the $d\Omega$ integral has already been performed. Performing the remaining integration over $d\vec{Q}$, we obtain

$$\psi(\vec{r},t) = -\frac{u_0}{4\pi}\frac{e^{i(KR-\vec{K}\cdot\vec{R})}}{R}e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \tag{A4}$$

which, choosing $u_0=4\pi b$, is identical to Eq. (17).

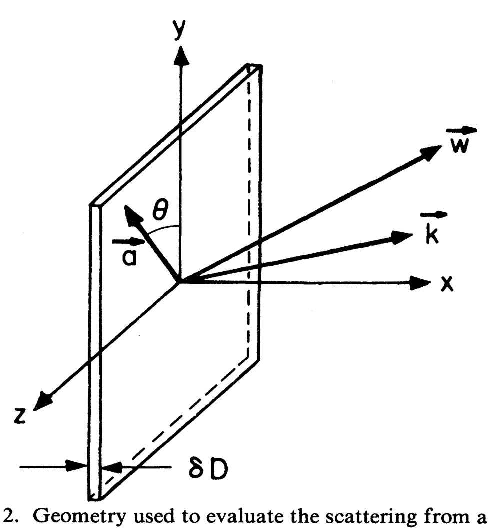

FIG. 2. Geometry used to evaluate the scattering from a plate of thickness δD moving with velocity $\vec{w}$.

APPENDIX B

We now derive the wave field behind a moving slab from the superposition of wavelets scattered by individual moving scatterers.

At time $t=0$, let the instantaneous position of the thin scattering slab coincide with the $x=0$ plane of a coordinate system at rest in the laboratory and let the incident $\vec{k}$ and the plate velocity $\vec{w}$ both lie in the x-y plane as shown in Fig. 2. Let the position $\vec{a}$ of a scattering element be given by the polar coordinates (a,θ) also shown in Fig. 2, so that the scattering volume element $d\vec{a}$ is $\delta D\, a\, da\, d\theta$, where δD is the thickness of the slab. Finally, for simplicity, choose $t=0$ for the field-evaluation time and choose a point x on the x axis for the field-evaluation position $\vec{r}$. With these choices and the definitions (16) and (16′), the integral in Eq. (18) is

$$bN\,\delta D\, e^{-iK_x x}\int_x^{\infty} dR\, e^{iKR} \times \int_{-\pi}^{+\pi} d\theta \exp[iK_y \sin\theta (R^2-x^2)^{1/2}]\ . \tag{B1}$$

The integral over θ gives a zeroth-order Bessel function[8]

$$2\pi bN\,\delta D\, e^{-iK_x x}\int_x^{\infty} dR\, e^{iKR} J_0(K_y(R^2-x^2)^{1/2})\ . \tag{B2}$$

The remaining integral gives[8]

$$2\pi bN\,\delta D\, e^{-iK_x x}\left\{\frac{1}{\pm(K_y^2-K^2)^{1/2}}\exp[\mp(K_y^2-K^2)^{1/2}x]\right\}\ . \tag{B3}$$

Physically, the upper signs correspond to a Doppler-shifted backscattered wave present in the region $x<0$. Choosing, therefore, the lower signs, we have

$$\frac{2\pi i bN\,\delta D}{K_x} \tag{B4}$$

as the forward-scattered wave in the region $x>0$.

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