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Geometrical effect on the measurement of stopping power: Angle-dependent energy loss of 7-MeV protons in Cu foils and computer simulation

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The emergence-angle dependence of energy loss of 7-MeV protons in Cu foils, the thicknesses of which are 3.743 and 7.576 mg/cm^2, has been measured with very high angular resolution. It was found that in both cases the energy loss increases as the emergence angle increases. The amount of the increase of the energy loss depends strongly on the thickness of the sample foils. Computer simulation was performed in order to investigate the origin of this phenomenon. Individual collision which protons undergo within the target was found to be restricted by the determination of detection angles. Considering properly the dependence of energy loss on impact parameter, we have obtained a good reproduction of the experimental results.

I. INTRODUCTION

In our previous work[1] it has been found that the energy loss of 7-MeV protons in metallic and organic thin foils increases with increasing emergence angle. As the amount of this increase could not be explained by known effects, the dependence of energy loss on the emergence angle was concluded to be due to a hitherto unknown effect. This effect was inferred to be the dependence of energy loss on the impact parameter with an atomic nucleus.

In a single collision between a heavy ion and an atom,[2,3] inelastic energy loss is known to have a strong dependence on the distance of closest approach. Hence it is quite natural to consider that the energy loss of protons in a single collision with an atomic nucleus should depend strongly on the impact parameter and be larger at small impact parameters owing to the high density of atomic electrons than at large impact parameters. If this is true, the dependence of energy loss on emergence angle will be explained by the following consideration. In the multiple scattering process which protons undergo within the target, the small emergence angle is more probably built up from accidental accumulation of rather small individual scattering angles than from accidental accumulation of rather large individual scattering angles. Also, the large emergence angle is more probably built up from accidental accumulation of large individual scattering angle than from accidental accumulation of small individual scattering angles. If the scattering angle with atomic nucleus is sufficiently small, which is the case of the present investigation, the impact parameter is roughly proportional to the inverse of the scattering angle. Hence a small scattering angle means a large impact parameter and also a small energy loss. Consequently, a small emergence angle corresponds to a smaller energy loss than a large emergence angle. This consideration implies that collisions with an atomic nucleus, which protons undergo within the target that are sampled in an experiment, may be influenced by the choice of emergence angle. This influence will become relatively weak if the number of collisions increases; in other words, if the target thickness increases. Consequently, it is important to study the dependence of the angle-dependent energy loss on the target thickness in order to understand this phenomenon well.

In the present work we have measured energy losses of 7-MeV protons in Cu targets, the thicknesses of which are 3.743 and 7.576 mg/cm^2, as a function of emergence angles. Since no theoretical work that deals with multiple scattering and the dependence of energy loss on impact parameter at the same time is available, we have performed computer simulations to investigate the origin of this phenomenon. The computer simulation is found to give a good agreement with experimental results.

II. EXPERIMENTAL PROCEDURE

The schematic diagram of the experimental arrangement is shown in Fig. 1. The beam of 7-MeV protons from the cyclotron of Kyoto University was collimated by the double diaphragm system of S1 and S2 with diameters of 0.7 mm each and 163 cm apart. The divergence of the incident beam was less than 0.05° before hitting the thin target. In order to prevent the protons scattered by the edges of diaphragms S1 and S2 from affecting the energy-loss measurement, a baffle S3 of 1.5 mm in diameter was

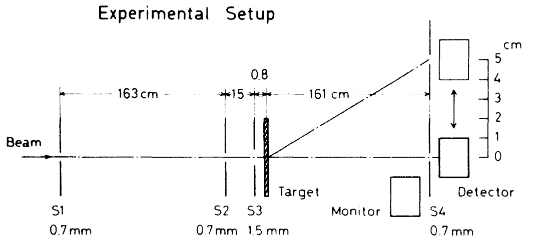

FIG. 1. Schematic diagram of experimental setup. The detector system was movable in the horizontal plane perpendicular to the direction of the incident proton beam in a range of 5 cm. The diameters of diaphragms S1, S2, and S4 were 0.7 mm and that of a baffle S3 was 1.5 mm.

placed 15 cm behind the diaphragm S2. The detecting system, which consisted of the diaphragm S4 of 0.7 mm in diameter and a surface barrier silicon detector, was placed 161 cm behind the target. This detecting system was movable perpendicular to the direction of the incident proton beam in a range of 5 cm. The displacement of 5 cm corresponds to the emergence angle of 1.78°. The detector subtended a solid angle of 1.5×10^{-7} sr as seen from the target. In this arrangement the energy-loss measurement was quite free from the edge scattered protons at 1-cm displacement.

The Cu foils were commercially obtained, and their thicknesses were determined by measuring area and weight, yielding 3.743 and 7.576 mg/cm^2.

The pulses from the detector were amplified with a low-noise amplifier and fed into a 4096-channel pulse-height analyzer. Another silicon detector was used to monitor the angular distribution due to multiple scattering. In order to monitor the stability of the detector-amplifier system, not only the pulses of protons which passed through the sample foil, but also those of protons which were scattered by a thin Au foil of 180 μg/cm^2, were measured simultaneously in one exposure. The pulse height of the incident protons was determined from that of scattered protons by the Au foil. The energy loss of 7-MeV protons in the Au foil was estimated to be 4 keV by using our previous stopping-power data.[4] The decrease of the energy due to elastic scattering was negligible. In the case of zero emergence angle this method could not be used because of the extremely high counting rate of protons which passed through the Au foil. When the measurement at zero emergence angle was made, only protons that passed through the sample target were detected. In this case the pulse height of the incident protons was determined from that of protons scattered by the Au foil at 0.33° in a separate measurement. The energy calibration of the pulse-height spectrum was performed with a very high precision pulse generator (ORTEC 448). The measurements of energy losses and angular distributions due to multiple scattering were repeated four times for one target at seven emergence angles between 0° and 1.78°. The details of the experimental method will be described elsewhere.[5]

III. PRINCIPLE OF THE CALCULATION

In this calculation we use a very simple model. Target atoms are assumed to be spheres of radius R and treated with a statistical model such as the Thomas-Fermi model. If protons enter the inside of these spheres, they are scattered by the target nuclei and lose a part of their energy due to collisions with atomic electrons. On the contrary, if protons traverse the outside of these spheres no scattering and no energy loss take place.

Let us consider the case of emergence of an angle α (Fig. 2). As protons experience a number of small-angle scatterings within the target with atomic nucleus, it is convenient to use two projected angles ϕ_x and ϕ_y instead of using the polar angle θ and the azimuth angle β of the track of a scattered proton (Fig. 3). The relation between the projected angles and the actual scattering angles is given by[6]

$$\tan\phi_x = \tan\theta \cos\beta \ , \qquad (1)$$

$$\tan\phi_y = \tan\theta \sin\beta \ . \qquad (2)$$

Under the small-angle approximation $\tan\theta$ can be replaced by θ, and then $\tan\phi_x$ and $\tan\phi_y$ can be also replaced by ϕ_x and ϕ_y, respectively:

$$\phi_x = \theta \cos\beta \ , \qquad (3)$$

$$\phi_y = \theta \sin\beta \ . \qquad (4)$$

After n collisions the direction of a proton can be given as

$$\sum_{i=1}^{n} \phi_{x,i} = \sum_{i=1}^{n} \theta_i \cos\beta_i \ , \qquad (5)$$

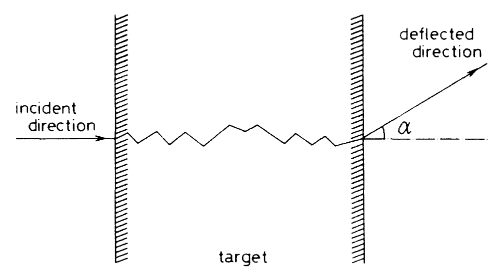

FIG. 2. Schematic representation of collisions through the target for emergence of an angle α.

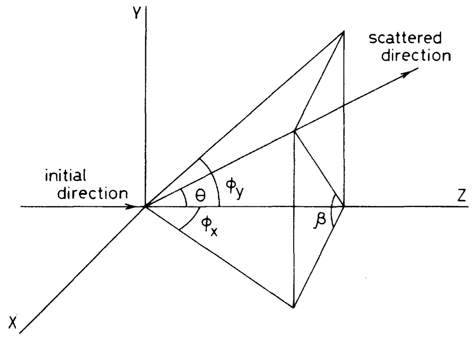

FIG. 3. Illustrating the spatial angles θ and β and the projected angles ϕ_x and ϕ_y.

$$\sum_{i=1}^{n} \phi_{y,i} = \sum_{i=1}^{n} \theta_i \sin\beta_i \ . \tag{6}$$

Since protons are detected after passing through the target at emergence angles of α and γ, where α denotes the polar angle and γ denotes the azimuth angle, the relation between these angles and the projected angles is expressed by

$$\tan\alpha \cos\gamma = \sum_{i=1}^{n} \phi_{x,i} \ , \tag{7}$$

$$\tan\alpha \sin\gamma = \sum_{i=1}^{n} \phi_{y,i} \ . \tag{8}$$

As seen in Eqs. (5)–(8), to decide on a pair of detection angles means to introduce a certain boundary condition to the collisions which protons undergo within the target. From these relations it is easily understood that this boundary condition has a strong effect on the individual collision if the number of collisions is small, in other words, if the target is thin.

Next, we consider the relation between scattering angle and impact parameter. The Coulomb potential of a nucleus which is partially screened by atomic electrons is described by

$$V(r) = -\frac{Z_1 Z_2 e^2}{r} u(r) \ , \tag{9}$$

where Z_1 and Z_2 are atomic numbers of the incident proton and the target atom, respectively. A good analytical approximation of the Thomas-Fermi screening function is given by Molière[7] and expressed as

$$u(r) = 0.1e^{-6r/a_{\mathrm{TF}}} + 0.55e^{-1.2r/a_{\mathrm{TF}}} + 0.35e^{-0.3r/a_{\mathrm{TF}}} \ . \tag{10}$$

The Thomas-Fermi screening radius a_{TF} is given by

$$a_{\mathrm{TF}} = 0.8853 a_0 Z_2^{-1/3} \ , \tag{11}$$

where a_0 is the Bohr radius. Supposing a scattering angle is sufficiently small, the scattering angle can be described by

$$\theta = \frac{p_\perp}{p_\parallel} \ , \tag{12}$$

where $p_\parallel$ denotes a proton momentum parallel to its initial direction, and $p_\perp$ denotes a momentum transfer perpendicular to the initial direction. Using Eqs. (9) and (10), we can calculate $p_\perp$ and finally find the relation between θ and the impact parameter b.[8]

$$\theta = \frac{2Z_1 Z_2 e^2}{M_1 v_1^2 b} g\left[\frac{b}{a_{\mathrm{TF}}}\right] \ , \tag{13}$$

$$g(\zeta) = \int_0^{2\pi} \cos\xi \, d\xi \left[u\left[\frac{\zeta}{\cos\xi}\right] - \frac{\zeta}{\cos\xi} u'\left[\frac{\zeta}{\cos\xi}\right]\right] \ , \tag{14}$$

where M_1 and v_1 are the mass and velocity of incident protons.

Considering the ionization energy loss, Kitagawa and Ohtsuki have derived an energy-loss formula of ions by collision with one atom as a function of the impact parameter b.[9] This formula has been shown to give the exact Bethe-Bloch formula. In the present calculation we use their expression

$$\Delta E(b) = -\frac{2Z_1^2 e^4}{m v_1^2} \int_{-\infty}^{\infty} dz \int_0^{2\pi} d\xi \int_{x_{\min}}^{\infty} x\, dx\, q^2 [K_0^2(qx) + K_1^2(qx)] \rho(r) \ , \tag{15}$$

where

$$r = (z^2 + x^2 + b^2 - 2bx\cos\xi)^{1/2} \ , \tag{16}$$

$$q = \frac{I}{\hbar v_1} \ . \tag{17}$$

Functions $K_0(qx)$ and $K_1(qx)$ are the modified Bessel functions, m is the electron mass, and I is the mean ionization energy of stopping material. The density of the atomic electrons $\rho(r)$ is given by the Molière formula[7]

$$\rho(r)=\frac{Z_2}{4\pi r}\left[\frac{0.3}{a_{\rm TF}}\right]^2(0.35e^{-0.3r/a_{\rm TF}}+8.8e^{-1.2r/a_{\rm TF}}+40e^{-6r/a_{\rm TF}})\ . \quad (18)$$

Since the target material is assumed to be random medium, the impact parameter is considered to have a uniform distribution over all target atoms and determined by pseudorandom numbers from a computer. Once an impact parameter is determined, we can calculate the scattering angle and the energy loss from Eqs. (13)—(18). By following a number of successive collisions we can find the relation between the emergence angle and the energy loss of emitted protons in the target.

IV. RESULTS AND DISCUSSION

Experimental and calculated results for the energy loss of the Cu target of 3.743 mg/cm^2 for the incident protons of 7.020 MeV are shown in Fig. 4. The angular distribution due to multiple scattering is shown on the upper half. The radius R, which is the radius of the target atoms, is used to estimate the mean free path of protons in the target material and is treated as a free parameter. The value of $R=2.7a_{\rm TF}$ was found to give a good fit to the experimental angular distribution. On the lower half, the relative values of energy loss are shown as a function of emergence angles. In the present calculation we used the value of 323.5 eV (Ref. 10) as the mean ionization energy of copper. A solid line shows the results of computation, which gives a fairly good agreement with experimental results.

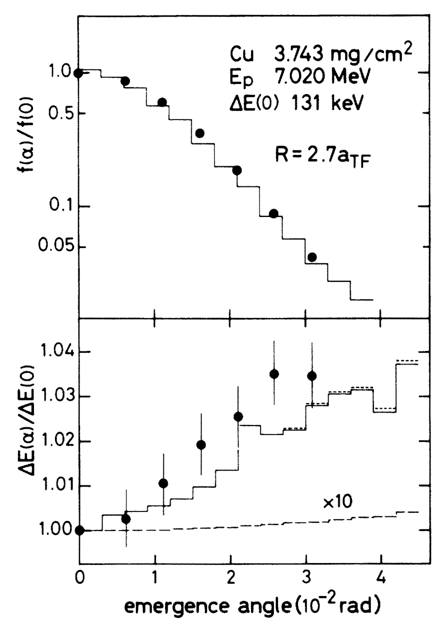

FIG. 4. Calculated and measured angular distribution due to multiple scattering and energy loss as a function of emergence angle α for 7.020-MeV protons transmitted through a 3.743-mg/cm^2 Cu target. Solid circles indicate the experimental results. A dotted line shows the variation of energy loss which includes the contribution from the energy transfer to recoil atoms due to elastic scattering. A dashed line represents the relative length of the actual path which protons passed in the target.

At this point let us examine the contribution of (i) the effective increase of the target thickness due to multiple scattering and (ii) the energy transfer to recoil atoms caused by the elastic scattering. A dashed line in Fig. 4 shows a little increase of the actual path length with increasing emergence angle. This effect is too small to explain the experimental results. Under the small-angle approximation the energy transfer to a recoil atom due to a single collision is described by

$$\frac{M_1}{M_2}E_0\theta^2\ , \quad (19)$$

where M_2 is the mass of target atoms and E_0 is the kinetic energy of protons. Then the total energy E_t transferred to recoil atoms in multiple scattering process is given by

$$E_t=\frac{M_1}{M_2}\bar{E}\sum_{i=1}^{n}\theta_i^2\ , \quad (20)$$

where $\bar{E}$ means the average energy of protons within the target. Considering the contribution of this effect in the calculation of energy loss, we obtain slightly larger energy losses, which are shown by a dotted line in Fig. 4. At small emergence angles this dotted line is not shown because this line cannot be resolved from the solid line. As seen in the figure the contribution of this effect is also too small to explain the experimental results. Therefore we can conclude that the angle-dependent energy loss of protons is not due to the increase of the actual path length and the energy transfer to recoil atoms in multiple scattering process, but due to the dependence of energy loss on the impact parameter in individual collision with atomic nucleus which protons undergo within the target.

In Fig. 5 average values of the number of collisions, of the scattering angles, and of the impact parameters are shown as a function of emergence angles. Although fluctuations are fairly large, the number of collisions increases with increasing emergence angle. This fact indicates that a larger angle emergence needs, on the average, more collisions.

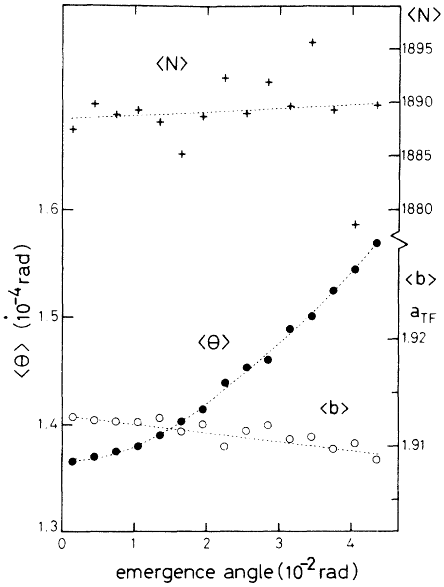

FIG. 5. Average values of the number of collisions $\langle N \rangle$, of the scattering angles $\langle \theta \rangle$, and of the impact parameters $\langle b \rangle$ as a function of emergence angle.

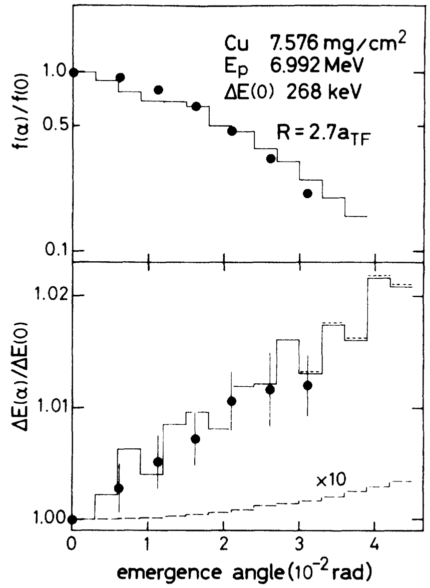

FIG. 6. Calculated and measured angular distribution due to multiple scattering and energy loss as a function of emergence angle α for 6.992-MeV protons transmitted through a 7.576-mg/cm^2 Cu target. Solid circles indicate the experimental results. A dotted line shows the variation of energy loss which includes the contribution from the energy transfer to recoil atoms due to elastic scattering. A dashed line represents the relative length of the actual path which protons passed in the target.

As the emergence angle increases, the impact parameter shows a slight decrease and the scattering angle shows a definite increase. This effect causes the dependence of energy loss on the emergence angle.

The result of computation for the Cu target of 7.576 mg/cm^2, which is about twice as thick as the former target, is shown in Fig. 6 together with the experimental results. This calculation is performed in order to investigate whether the computer simulation is successful or not for the target of different thickness with the same parameter R. Although the statistics of the calculation are not so good, the computer simulation is found to give a good agreement with the experimental results. In this case it is also proved that the contribution of the effective increase of the target thickness due to multiple scattering and the increase of energy loss due to the energy transfer to the recoil atoms is too small to explain the experimental results.

V. CONCLUSION

Energy losses of 7-MeV protons in Cu foils have been measured with very high angular resolution as a function of emergence angles. Measured energy losses show a common trend that the energy loss increases with increasing emergence angle. Computer simulation has been performed in order to investigate the origin of this phenomenon. Individual collision which protons undergo within the target has turned out to be restricted by the determination of the emergence angle. Hence if energy loss of protons in a single collision depends on the impact parameter, this effect will cause the dependence of energy loss upon the emergence angle. Using the relation between the impact parameter and the scattering angle of Lindhard *et al.*[8] and the energy-loss formula of Kitagawa and Ohtsuki,[9] we can reproduce the angular distributions due to multiple scattering and the relative energy losses using one parameter R. This fact indicates that the computer simulation can predict the target-thickness dependence of the effect correctly. The effects of the increase of the actual path length and the energy transfer to recoil atoms have also proved to be too small to explain the experimental results. More systematic study of this phenomenon will bring us more precise information about individual collision which protons undergo within the target material.

Finally, we will mention the relation between the usual stopping-power value and the energy loss measured at zero emergence angle. In usual stopping-power measurement, all particles that pass through the sample foil are detected. If we measure only particles that emerge from the foil at zero angle and deduce the stopping-power value, we will obtain smaller stopping-power value than that of usual definition. For example, the deviation is about 1.5% in the case of the thinner Cu target (3.743 mg/cm^2). This deviation is supposed to be larger for a thinner target. Therefore if the stopping-power measurement is performed under the condition that it is difficult to detect all particles that pass through the sample foil, which is the case when time-of-flight technique is used, the resultant stopping-power value will be systematically small. The amount of deviation will strongly depend on the target thickness. In order to decrease this deviation, the sample foil should be as thick as possible to the extent that average energy of particles in the target has good physical meaning.

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