PHYSICAL REVIEW A VOLUME 27, NUMBER 2 FEBRUARY 1983

Direct pair production in heavy-ion—atom collisions

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(Received 14 September 1982)

Direct pair production in ~5-MeV/amu heavy-ion—atom collisions with uranium target atoms is calculated with the plane-wave Born approximation and the semiclassical approximation. Briggs's approximation is used to obtain the electron and positron wave functions. Since pair production involves high momentum transfer q from the moving projectile to the vacuum, use is made of a high-q approximation to greatly simplify the numerical computations. Coulomb deflection of the projectile, the effect of finite nuclear size on the electronic wave functions, and the energy loss by the projectile exciting the pair are all taken into account in these calculations.

I. INTRODUCTION

One of the first applications of the semiclassical approximation (SCA) to electronic excitation in ion-atom collisions included a calculation of electron-positron pair production in 1.5-MeV $H^+ + Ta$ collisions.[1] This calculation discouraged further investigations. While previous Born-approximation calculations gave pair-production cross sections of the order of 10^{-33} cm^2,[2] the semiclassical approximation gave $\sim 10^{-48}$ cm^2, far smaller than one can measure.[3,4] The pair-production cross section is small for two reasons. The momentum transferred to the pair $q > 1022$ keV$/v$, where v is the ion velocity, is more than ten times larger than the momentum transfer needed to ionize K electrons, less than 100 keV$/v$. Since excitation cross sections decrease with a large power of the momentum transfer,[5] we expect much smaller pair-production cross sections than ionization cross sections. Coulomb deflection of the projectile by the target nucleus also drastically affects the pair-production cross section[1,6]; it reduces ionization cross sections by one to two orders of magnitude when the product ξ of the momentum transfer q and one half of the internuclear distance of closest approach in head-on collisions, d, is greater than unity. Since q is over ten times larger for pair production, ξ will be ten times larger, and Coulomb deflection reduces the pair-production cross section by many orders of magnitude (by a factor of 10^{-10} in 1.5-MeV $p + Ta$ collisions[1]).

Bang and Hansteen's calculation[1] of pair production neglected electronic relativistic effects, however.[7,8] Pair production has recently been observed in $U + U$ collisions, where electronic relativistic effects play a major role.[9] Part of the pair production in $U + U$ collisions is due to pair conversion of Coulomb-excited γ rays. Part of it is due also to coherent pair production, possible only when the binding energy of $1s$ vacancies exceeds 1022 keV. However, direct excitation of pairs due to the transfer of the projectile momentum to the vacuum (a process which Soff *et al.*[10] call "shakeoff of the vacuum polarization") accounts for a substantial fraction of the observed $U + U$ electron-positron pair-production cross section.

Given the successful observation of positrons in $U + U$ collisions and the negative results in proton and α-particle collisions,[3,4] we undertook calculations of pair production in higher-energy collisions involving light ions. The maximum bombarding energy which one can use is near the nuclear Coulomb barrier, approximately 5-MeV/amu for most heavy ions.[11] If the particle energy exceeds the Coulomb barrier, positrons due to β^+ decay and γ-ray internal pair conversion may exceed the number created by direct pair excitation. Experience with K-shell ionization suggests that pair production would be more likely with higher-Z targets, due to electronic relativistic effects, and if α particles or heavy ions are used instead of protons.[7] Equal velocity particles with $Z/A \approx \frac{1}{2}$ have a smaller internuclear distance of closest approach than protons; thus Coulomb-deflection effects are less severe.

The *ab initio* evaluation of pair-production cross sections using the SCA (Ref. 1) requires the evaluation of quintuple integrals, integrals over the electron coordinate r, time t, impact parameter b, electron energy W_-, and positron energy W_+. Calculations are greatly simplified, however, if one initially neglects Coulomb-deflection effects, and does a plane-wave Born-approximation (PWBA) calculation.[5,7] Furthermore, we discovered a high–momentum-transfer approximation[12] which gives accurate electronic form factors needed for the PWBA, reducing the number of integrals to two. Section II of this paper presents the PWBA results.

Of course, the PWBA does not give realistic results for electron-positron pair production by slow heavy ions, mainly due to Coulomb-deflection effects. Rather than abandoning the PWBA, however, we calculated correction factors for Coulomb-deflection and finite–nuclear-size (FNS) effects, as presented in Sec. III. We discuss wave-function and projectile energy-loss effects in Sec. IV. In any slow-ion–atom collision one must take into account the adjustment of the electronic wave functions due to the presence of the projectile nucleus. Diatomic molecular orbitals are often used to evaluate inner-shell ionization.[13,7] For pair production, we used the Briggs approximation[14] to take into account these wave-function effects. In Sec. V it is shown that no further quantum-electrodynamical (QED) corrections to our results are to be expected. Numerical results are discussed in Sec. VI.

Relativistic units ($\hbar = m = c = 1$) are used throughout this paper, except for places where we have explicitly given the units for clarity.

II. PLANE-WAVE BORN APPROXIMATION

Momentum transfer from the projectile to the vacuum is mediated by the Coulomb interaction between the projectile nucleus and the electron or positron. The matrix element leading to pair formation is proportional to[1]

$$\left\langle \psi_+ \left| \frac{Z_1\alpha}{|\vec{r}-\vec{R}|} \right| \psi_- \right\rangle , \tag{1}$$

where ψ_+ and ψ_- are positron and electron wave functions, $\vec{r}$ is the electron coordinate, $\vec{R}$ is the internuclear coordinate, Z_1 is the projectile charge, and $\alpha^{-1} = 137.04$. The double-differential cross section is given by

$$\frac{d^2\sigma_{\text{pair}}}{dW_+ dW_-} = \frac{8\pi Z_1^2\alpha^2}{v^2} \sum_{\substack{j_+,j_- \\ m_+,m_- \\ P_+,P_-}} \int_q^\infty \frac{dq'}{(q')^3} \left| \langle \psi_{j_+m_+} | \exp(i\vec{q}\,'\cdot\vec{r}) | \psi_{j_-m_-} \rangle \right|^2 , \tag{2}$$

where W_+, j_+, m_+, and P_+ are the (positive-definite) positron energy (including the rest-mass energy), angular momentum, and parity quantum numbers; W_-, j_-, m_-, and P_- refer to the electron, v is the ion velocity, and q is the minimum momentum transfer given by $(W_+ + W_-)/v$.

The angular momentum algebra is easily carried out, and so we obtain[15,16]

$$\frac{d^2\sigma_{\text{pair}}}{dW_+ dW_-} = \frac{8\pi Z_1^2\alpha^2}{v^2} \sum_{\substack{j_+,j_-,l \\ P_+,P_-}} A_{j_+j_-} \int_q^\infty \frac{dq'}{(q')^3} |F_{j_+j_-l}(q')|^2 , \tag{3}$$

where

$$A_{j_+j_-} = \begin{pmatrix} j_+ & j_- & l \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 (2j_+ + 1)(2j_- + 1)(2l+1)\tfrac{1}{2}[1 + P_+P_-(-1)^l] ,$$

$$F_{j_+j_-l}(q) \equiv \langle R_+ | j_l(qr) | R_- \rangle = \int_0^\infty r^2 j_l(qr)(g_+g_- + f_+f_-)dr , \tag{4}$$

l is the angular momentum connecting j_+ and j_-, and g and f are the components of the Dirac electron and positron radial wave functions. We used the Dirac wave functions given in Ref. 17 to calculate the electronic form factor $F(q)$. The evaluation of $F(q)$ requires integrals such as

$$I_{11}=\int_0^\infty dr\, r^{2\gamma} j_l(qr) e^{ik_+r}\,{}_1F_1(\gamma_+ - i\nu_+, 2\gamma_+ + 1, -2ik_+r) e^{ik_-r}\,{}_1F_1(\gamma_- - i\nu_-, 2\gamma_- + 1, -2ik_-r)\,, \tag{5}$$

where $\gamma_\pm^2=\kappa_\pm^2-\alpha^2 Z_2^2$, Z_2 is the target atomic number, $\kappa_\pm$ is the Dirac quantum number, k_+ is the positron momentum, k_- is the electron momentum, $\nu_+=-\alpha Z_2 W_+/k_+$, $\nu_-=\alpha Z_2 W_-/k_-$, and $2\gamma=\gamma_+ + \gamma_-$. The large momentum transfer q needed to produce pairs makes the cross section small, but makes possible the application of a high-q approximation to evaluate I_{11}. Since $q \gg k_+$ and $q \gg k_-$, only the small-r part of the integral in Eq. (5) contributes; hence one can neglect the plane-wave and hypergeometric functions in Eq. (5). Thus we have

$$I_{11}\approx \int_0^\infty r^{2\gamma} j_l(qr)dr=\frac{1}{q^{2\gamma+1}}\int_0^\infty x^{2\gamma} j_l(x)dx=\frac{1}{q^{2\gamma+1}}\Gamma(2\gamma)\sin(\pi\gamma), \text{ for } l=0 \tag{6}$$

which is a formal result for arbitrary γ since the integral may diverge, but which agrees with the $q\to\infty$ expansion of the exact evaluation of (5).

The double-differential pair-production cross section for $l=0$ excitation is given by

$$\frac{d^2\sigma_{\text{pair}}}{dW_+dW_-}=\frac{8\pi Z_1^2\alpha^2}{v^2}\sum_{\substack{j_+,j_-,\\ P_+,P_-}} A_{j_+j_-} C^2(l,j_+,j_-,W_+,W_-,\gamma_+,\gamma_-)[(4\gamma+4)q^{4\gamma+4}]^{-1}\,, \tag{7}$$

where

$$C(l=0)=N_+N_-\{[(W_+-1)(W_-+1)]^{1/2}\sin\xi_+\sin\xi_- + [(W_- -1)(W_+ +1)]^{1/2}\cos\xi_+\cos\xi_-\}\Gamma(2\gamma)\sin(\pi\gamma)\,,$$

$$N_\pm=\frac{1}{\sqrt{\pi k_\pm}}e^{\pi\nu_\pm/2}\frac{|\Gamma(\gamma_\pm+1+i\nu_\pm)|}{\Gamma(2\gamma_\pm+1)}(2k_\pm)^{\gamma_\pm}\,, \tag{8}$$

and $\xi_\pm$ is calculated from the expression

$$e^{2i\xi_\pm}=\frac{\kappa_\pm - i|\nu_\pm/W_\pm|}{\gamma_\pm - i\nu_\pm}\,.$$

Much can be deduced about pair production by slow heavy ions by inspection of Eq. (7).

(i) Only transitions involving the $s_{1/2}$ and $p_{1/2}$ electron and positron wave functions are important. For an $s_{1/2}$-to-$p_{3/2}$ transition, e.g., either γ_+ or γ_- will be roughly one unit larger; hence the cross section will be smaller by a factor of q^{-2}. For 5-MeV/amu α + U collisions, the $s_{1/2}$-to-$p_{3/2}$ transition probabilities are approximately three orders of magnitude smaller; for $p_{3/2}$ to $p_{3/2}$, they are six to seven orders of magnitude smaller than $s_{1/2}$-to-$s_{1/2}$ transition probabilities.

(ii) Confining our considerations to just the $s_{1/2}$ and $p_{1/2}$ wave functions, we find that dipole transitions are also negligible. This is easily seen from the transition density $r^2(f_+f_- + g_+g_-)$ in Eq. (4). For small r, f and g vary as $r^{\gamma-1}$ and the ratio f/g is equal to $(\gamma+\kappa)/Z\alpha$.[18] Hence we obtain

$$f_+f_- + g_+g_- \sim 1+\frac{f_+f_-}{g_+g_-}
=\begin{cases} 1+\left[\dfrac{\gamma-1}{Z\alpha}\right]^2, & s_{1/2}-s_{1/2} \\ 1+\left[\dfrac{\gamma+1}{Z\alpha}\right]^2, & p_{1/2}-p_{1/2} \\ 1+\dfrac{(\gamma-1)(\gamma+1)}{(Z\alpha)^2}=0, & s_{1/2}-p_{1/2}\,. \end{cases} \tag{9}$$

In the high-q approximation, the $s_{1/2}$-to-$p_{1/2}$ dipole transition probability is identically zero. Of course, this is only true to first order. It is possible to expand the wave functions to higher powers in r, but each higher power in r gives a factor of q^{-2} in the cross section, so that finally the $s_{1/2}$-to-$p_{1/2}$ transition probability would have about the same small magnitude as the $s_{1/2}$-to-$p_{3/2}$ probability.

(iii) Since q is proportional to the inverse of the ion velocity, Eq. (7) implies that the cross section varies as $v^{4\gamma+2}$, i.e., as v^2 to v^6, depending on γ or Z.

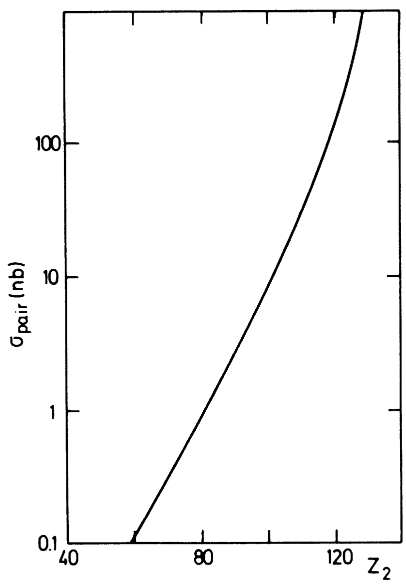

FIG. 1. Pair-production cross section for α particles with $v/c=0.1$ vs target atomic number calculated with the plane-wave Born approximation.

Coulomb-deflection effects will drastically reduce the pair-production cross section at small velocities, however, as we shall see.

(iv) As Z increases, γ decreases, so the enhancement due to the use of relativistic electronic wave functions is of the order $(q/k)^{4-4\gamma}$, where k is some characteristic electron or positron momentum. Since $q/k \gg 1$, pair production in U + U collisions where γ is effectively near zero, is fairly likely, but in p + Ta collisions where $\gamma=0.85$ (γ was unity in Bang and Hansteen's calculations), pair production is much less likely.

Figure 1 shows numerical calculations of pair-production cross sections by $\sim$5-MeV/amu α particles as a function of the target atomic number, calculated with the PWBA. This figure shows that even with the maximum-energy α particles (the energy being determined by the nuclear Coulomb barrier), the pair-production cross section (without Coulomb deflection) is not large, 4 nb for α + U. This cross section would be measurable, however, if no nuclear background effects were present.

Finally, we discuss the accuracy of the high-q approximation. We found that for $Z_2=50$, 92, and 130 for the $s_{1/2}$-$s_{1/2}$ transitions the exact and high-q approximation pair-production cross sections agree to within $<4\%$ for $v/c=0.1$ and $<20\%$ for $v/c=0.3$. By retaining one further term in the q^{-1} expansion of the exact form factor, one obtains agreement within 1% (for $v/c=0.1$). However, the first-order high-q approximation also greatly simplifies the evaluation of the Coulomb-deflection correction, so to be consistent, we shall neglect higher powers of q^{-1}.

III. COULOMB DEFLECTION AND FINITE NUCLEAR SIZE

To evaluate Coulomb-deflection effects, we use the semiclassical theory which in the straight-line case is equivalent to the plane-wave Born approximation.[19] For an arbitrary internuclear trajectory $\vec{R}(t)$, we have instead of (3),[15]

$$\frac{d^2\sigma_{\rm pair}}{dW_+dW_-}=32Z_1^2\alpha^2\sum_{\substack{j_+,j_-,\\P_+,P_-}}\sum_{l,m}\frac{1}{2l+1}A_{j_+j_-}\int_0^\infty b\,db\,|a_p^l|^2\,, \tag{10}$$

$$a_p^l=\int_0^\infty dq' F_{j_+j_-l}(q')\int_{-\infty}^{\infty}dt\,e^{i\omega t}j_l(q'R)Y_{lm}^*(\hat{R})\,,$$

where $\omega=W_++W_-$, and b is the impact parameter. Inserting the high-q approximation (6) for the form factor, a_p^l reduces in the case $l=0$ to

$$a_p^0=K^0\int_{-\infty}^{\infty}dt\,e^{i\omega t}R^{2\gamma},\quad K^0=-\frac{\sqrt{\pi}}{8\gamma(2\gamma+1)}\frac{C(l=0)}{\Gamma(2\gamma)\sin(\pi\gamma)}\,. \tag{11}$$

Thus the dependence on R factors out and we define the Coulomb-deflection correction factor C_γ as

$$C_\gamma(R^{2\gamma})=\frac{\displaystyle\int_0^\infty b\,db\left|\int_{-\infty}^{\infty}dt\,e^{i\omega t}R^{2\gamma}\right|^2}{\displaystyle\int_0^\infty b\,db\left|\frac{1}{v}\int_{-\infty}^{\infty}dz\,e^{iqz}(b^2+z^2)^\gamma\right|^2} \tag{12}$$

with $q=\omega/v$. In the numerator, the time integral is performed along a Coulomb trajectory. Although the denominator of this expression vanishes as $\gamma\to 0$, C_γ is well defined; indeed $C_0=1$. For $\gamma\to 1$, however, C_γ diverges. This is because the coefficient of the leading term in q^{-1} vanishes in this limit [cf. (6)], and higher-order terms must be retained to obtain a meaningful result. However, in this limit, σ_{pair} becomes negligible anyhow.

The integral in the denominator of (12) can easily be evaluated by equating (10) together with (11) for a straight line with the straight-line result (7) (or directly). We get

$$\frac{1}{D_0} \equiv \int_0^\infty b\,db \left| \frac{1}{v} \int_{-\infty}^{\infty} dz\, e^{iqz}(b^2+z^2)^\gamma \right|^2 = \frac{\Gamma^2(2\gamma+2)\sin^2(\pi\gamma)}{(\gamma+1)v^2 q^{4\gamma+4}} . \tag{13}$$

For the evaluation of the numerator of (12), we introduce the parametrization of the Coulomb trajectory

$$R = d(\epsilon \cosh w + 1), \quad t = \frac{d}{v}(\epsilon \sinh w + w) , \tag{14}$$

where

$$\epsilon = [\sin(\theta/2)]^{-1} = [(b/d)^2+1]^{1/2} , \tag{15}$$

θ is the center-of-mass scattering angle, and d is one-half of the distance of closest approach in a head-on collision. Insertion into (12) leads to

$$C_\gamma = D_0 \frac{d^{4\gamma+4}}{v^2} \int_1^\infty d\epsilon\, \epsilon \left| \int_{-\infty}^{\infty} dw (\epsilon \cosh w + 1)^{2\gamma+1} \exp[i\xi(\epsilon \sinh w + w)] \right|^2 . \tag{16}$$

Equation (16) with (13) shows that C_γ depends only on γ and $\xi = qd$ and is independent of the electron and positron momenta and energy and angular momenta κ, except that q depends on $W_+ + W_-$. Although the exact evaluation of the pair-production cross section using the SCA would involve a quadruple integral (over W_+, W_-, b, and t, after the r integral is evaluated), the use of the Coulomb correction factor decouples the quadruple integral into two double integrals. C_γ can be evaluated as a function of q for several q, then can be interpolated at desired q values when the integrations over W_+ and W_- are done.

Until now we have used point-nucleus (PN) electron and positron wave functions, which vary as $r^{\gamma-1}$ at small r. This behavior leads to the γ dependence of C_γ. With a uniformly charged sphere target nucleus of radius R_N, the wave functions will instead approach a constant for $r \ll R_N$ for all Z_2.[18] The same is true for more realistic nuclear shapes. Thus the Coulomb-deflection effects and the finite—nuclear-size (FNS) effects cannot be separated. The small-r behavior does not affect the FNS wave functions for $r \gtrsim 10R_N$, however. For W_+ $(W_-) \lesssim 10$, the asymptotically normalized PN and FNS wave functions are nearly identical in this region.

Figure 2 shows the ratio of the transition densities,

$$R_\rho(r) = \frac{(f_+ f_- + g_+ g_-)_{\text{FNS}}}{(f_+ f_- + g_+ g_-)_{\text{PN}}} , \tag{17}$$

for the special case of $k_+ = k_- = 0.25$, $Z = 110$, and $s_{1/2}$-$s_{1/2}$ transitions. Here, R_N is approximately 7 fm. It was calculated using $R_N = 1.07 A^{1/3}$ fm with $A = 278$, appropriate, for reasons discussed in the next section, for $^{40}\text{Ar} + {}^{238}\text{U}$ collisions.[20] At $r \gtrsim 100$ fm, the density ratio is nearly unity.

In order to evaluate the influence of the FNS on the transition matrix elements it is practical to go over to the coordinate representation by performing

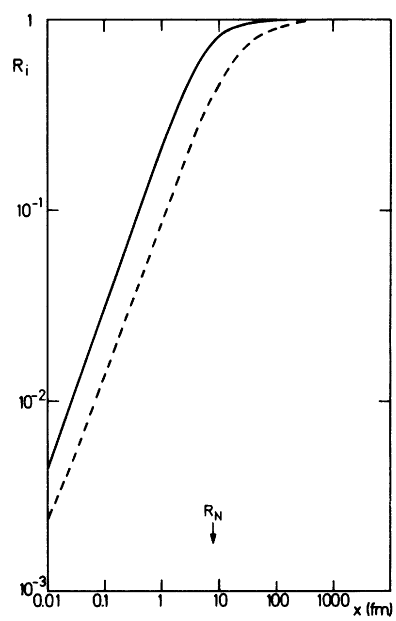

FIG. 2. Ratios of transition densities $R_\rho(r)$ defined by Eq. (17) (solid line), and transition matrix elements $R_M(R)$ defined by Eq. (19) (dashed line) versus electron r or internuclear R coordinate for $Z = 110$ and $k_+ = k_- = 0.25mc$. Inside the nucleus ($r < R_N$), the transition density goes to zero with r faster than $r^{2\gamma-2}$, obtained for point nuclei. For $r > R_N$, however, both the FNS and PNS densities increase as $r^{2\gamma-2}$; so the ratio approaches unity.

the q' integral in (10) together with the definition (4) of $F(q)$ analytically. For the relevant $l=0$ transition one then finds

$$a_p^0=\frac{\sqrt{\pi}}{4}\int_{-\infty}^{\infty}dt\,e^{i\omega t}\left\langle R_+\left|\frac{1}{r_>}-\frac{1}{r}\right|R_-\right\rangle^{\mathrm{FNS}}$$

$$=K^0\int_{-\infty}^{\infty}dt\,e^{i\omega t}R_M(R)R^{2\gamma}\,, \qquad (18)$$

where $r_>=\max(r,R)$ and

$$R_M(R)=\frac{\left\langle R_+\left|\frac{1}{r_>}-\frac{1}{r}\right|R_-\right\rangle^{\mathrm{FNS}}}{\left\langle R_+\left|\frac{1}{r_>}-\frac{1}{r}\right|R_-\right\rangle^{\mathrm{PN}}}\,, \qquad (19)$$

and we have used that in the high-q approximation we have effectively [cf. Eq. (11)]

$$\left\langle R_+\left|\frac{1}{r_>}-\frac{1}{r}\right|R_-\right\rangle^{\mathrm{PN}}\sim R^{2\gamma}\,. \qquad (20)$$

The reason for subtracting the $1/r$ term—which actually does not contribute to the first-order amplitude for inelastic processes—in the matrix elements will become clear in the next section. Figure 2 also shows R_M as a function of R for the $^{40}Ar + {}^{238}U$ collision system. Both $R_p(x=r)$ and $R_M(x=R)$ vary as $x^{2-2\gamma}$ when $x \ll R_N$.

The above considerations mandate that the factor $R^{2\gamma}$ in the numerator of Eq. (12) should be weighed by $R_M(R)$ in order to take the FNS influence on the Coulomb deflection into account. The denominator is unchanged, though, as long as hydrogenic wave functions are used in evaluating the PWBA results. The resultant C_γ therefore depends on R_N, in addition to its dependence on γ and ξ. When necessary, we shall write the functional dependence of C_γ on R as $C_\gamma(R_M R^{2\gamma})$. Since R_N and d tend to be of similar magnitude, Coulomb deflection and FNS effects are *not* separable. The projectile must almost always penetrate to distances where the transition matrix element is reduced by FNS if positrons are to be produced. If d were always much greater than R_N, the FNS effects could be neglected entirely.

We have also investigated to what extent the FNS effects depend on electron and positron momenta, energy, and angular momenta and on the details of the nuclear wave functions. This was done by evaluating the monopole transition form factor Eq. (4) numerically and comparing with point-nucleus results. We found the ratio $F^{\mathrm{FNS}}(q)/F^{\mathrm{PN}}(q)$ to be (i) independent of positron (electron) momentum k_+ (k_-) for k_+ $(k_-)<3$, (ii) identical for $s_{1/2}$-$s_{1/2}$ and $p_{1/2}$-$p_{1/2}$ transitions for $Z \le 120$, and (iii) insensitive within 10% if a Fermi distribution was taken to describe the nuclear charge distribution instead of a homogeneous charged sphere. In conclusion, we infer that C_γ depends just on γ, ξ, and R_N, and since for a given Z one can assume R_N is essentially fixed, C_γ can be taken to depend only on Z and ξ.

Figure 3 shows numerically evaluated correction factors for several atomic numbers as a function of the excitation energy $W=W_+ + W_-$ for $v/c=0.1$. Neglecting FNS, $C_\gamma(W)$ is nearly independent of Z for $\gamma<0.75$. For $\gamma>0.75$, however, the correction factors vary significantly with γ. FNS reduces C_γ more at higher Z than at lower Z. One might be tempted to use the Bang-Hansteen Coulomb-deflection correction factor for pair production, which has been used widely to calculate K-shell ionization cross sections.[1,6] As Fig. 3 shows, however, this factor would overestimate the pair-production cross section considerably.

IV. WAVE-FUNCTION AND ENERGY-LOSS EFFECTS

In principle, in the slow-ion–atom collisions under consideration, one should use diatomic molecular electron and positron wave functions.[7,13] The relevant electron and positron velocities are always

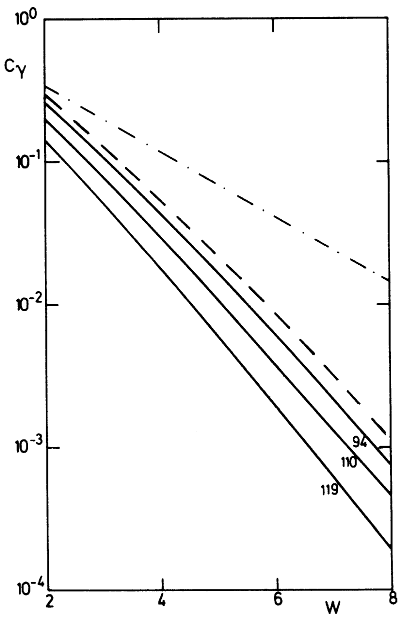

Fig. 3. Dashed line: Coulomb-deflection correction factor for 5-MeV/amu $\alpha+U$ collisions versus the energy transferred to the electron-positron pair $W=W_+ + W_-$ (in units of mc^2). Solid lines include the FNS effects for $\alpha+U$ $(Z=94)$, $Ar+U$ $(Z=110)$, and $Co+U$ $(Z=119)$. The chain curve gives the Bang-Hansteen Coulomb-deflection factor $\exp(-\pi\xi)$ for $\alpha+U$ collisions.

much greater than the ion velocity, so the electrons and positrons have time to adjust their motion to the presence of both the target and projectile nuclei. However, diatomic molecular orbitals have rarely been used in calculations of cross sections for processes involving high momentum transfer. Various approximations have been used instead.[14,21,22] The approximation most simply applied to pair production is the Briggs approximation,[14] which uses united-atom wave functions centered on the center of charge of the two nuclei. We find that only impact parameters of the order of d contribute to the pair-production cross section. Since d is much smaller than the distance over which the separated-atom wave functions evolve into molecular orbitals, and finally into united-atom wave functions, the use of united-atom wave functions for all R should be valid.

In the Briggs approximation, the transition matrix element in Eq. (1) is modified to [the inclusion of the $(Z_1+Z_2)/r$ term is numerically convenient since it makes the perturbation vanish in the united-atom limit]

$$\left\langle \psi_+(Z_u) \left| \frac{Z_1\alpha}{|\vec{r}-\alpha_0\vec{R}|} + \frac{Z_2\alpha}{|\vec{r}+\beta_0\vec{R}|} - \frac{Z_1+Z_2}{r} \right| \psi_-(Z_u) \right\rangle , \tag{21}$$

where $Z_u = Z_1+Z_2$, $\alpha_0 = Z_2/Z_u$, and $\beta_0 = Z_1/Z_u$. The resulting pair-production amplitude from (10) is then replaced by

$$a_p^0(\gamma) \to \frac{Z_B}{Z_1} a_p^0(\gamma_u) , \tag{22}$$

where

$$Z_B = Z_1(Z_2/Z_u)^{2\gamma_u} + Z_2(Z_1/Z_u)^{2\gamma_u}$$

with $\gamma_u = (1-\alpha^2 Z_u^2)^{1/2}$. From these expressions we prescribe the following modifications to Eqs. (7) and (12) to account for wave-function effects: (i) replace $\gamma(Z_2)$ with $\gamma(Z_u)$ everywhere; (ii) replace Z_1^2 with Z_B^2; and (iii) use the sum of the projectile and the target mass to calculate R_N. The latter prescription may actually easily be improved upon by calculating the FNS effects for the monopole part of the two-center potential from projectile and target, but as long as one considers reasonably asymmetric collisions, the projectile contribution will only represent a small correction to the target term; and how it is incorporated is not critical, since the FNS effects are fairly insensitive to the adapted nuclear charge distribution.

Since the minimum energy to create an e^+e^- pair is non-negligible compared with the proton or α-particle total collision energy of a few MeV/amu, the usual SCA assumption of an unchanged velocity is not valid. However, it is well known that the SCA closely approximates the Coulomb distorted-wave Born approximation if the projectile velocity is symmetrized between the incoming and outgoing channels.[1,23,24] This can be implemented by changing the projectile velocity from v to $v' = v - (W_+ + W_-)/4vM$, where M is the reduced mass.[23]

Insertion of this modified velocity into Eq. (7) reduces the double-differential pair-production cross section by a factor of $(v'/v)^{4\gamma+2} = 0.9$ for 5-MeV/amu α + U collisions for $W_+ + W_- = 4mc^2$. Since the Coulomb-deflection—FNS correction factor also depends on v [because $q = (W_+ + W_-)/v$], the net pair-production cross section is further reduced by a factor of 0.6 for $W_+ + W_- = 4mc^2$, giving a net energy-loss correction of 0.54. A more accurate evaluation of the energy-loss effect may be desirable if pair production is observed in low-energy collisions with very light ions.

After inclusion of all corrections to the PWBA result (7) the differential cross section for positron production finally becomes

$$\frac{d^2\sigma_{\text{pair}}}{dW_+ dW_-} = \left[\frac{Z_B}{Z_1}\right]^2 C_{\gamma_u}(R^{2\gamma_u} R_M(R)) \times \frac{d^2\sigma_{\text{pair}}^{\text{PWBA}}(\gamma_u; v')}{dW_+ dW_-} . \tag{23}$$

It should be stressed that because the high-q approximation in this case is a good approximation for the form factor, this result is expected to be rather accurate.

V. QED CORRECTIONS

Since pair production is a QED process, one might question if there are sizable QED corrections to the simple picture of "ionization of the Dirac sea" that we have used. As judged from the known corrections to the (united-atom) binding energies due to vacuum polarization[25] and electron self-energy[26] ($\ll 1\%$), this may appear unlikely. However, the binding energies probe the average properties of the wave functions, while the ionization process probes the wave functions at large momentum transfer, and the invariant momentum transfer (squared),

$$-y = \omega^2 - \vec{q}^{\,2} \approx (W_+ + W_-)^2 \left[\frac{1}{c^2} - \frac{1}{v^2}\right] \approx -q^2 \le -4/v^2 , \tag{24}$$

is indeed large when $v/c \leq 0.1$.

In order to investigate these effects in somewhat more detail, we note that all the relevant potentials under consideration in this paper have Fourier transforms of the type ($x_0=t$, $q_0=\omega$, $xq=x_0q_0-\vec{x}\cdot\vec{q}$)

$$V(q_0,\vec{q})=(2\pi)^{-2}\int d^4x\, V(\vec{r}-\lambda\vec{R})e^{ixq}$$
$$=4\pi\frac{\rho(\vec{q})}{q^2}B(\omega,\lambda\vec{q}) , \quad (25)$$

where $\rho(\vec{q})$ is the (effective) charge form factor giving rise to the potential, and

$$B(\omega,\vec{q})=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dt\, e^{-i\vec{q}\cdot\vec{R}(t)}e^{i\omega t} . \quad (26)$$

This factor is easily evaluated for a straight-line path,

$$B^{\mathrm{SL}}(\omega_0,\vec{q})=\frac{\sqrt{2\pi}}{v}\delta\left[\frac{\omega_0}{v}-q_z\right]e^{-i\vec{q}\cdot\vec{b}} , \quad (27)$$

if $\vec{R}(t)=\vec{b}+vt\vec{e}_z$. Also for a hyperbolic path, with the parametrization of Eq. (14), it can readily be evaluated. We shall not do this since the explicit representation is not needed. The important point is that B is peaked at $q_z=q_0/v$, so that it is indeed the large spacelike components of the momentum transfer that dominate the pair production process.

The QED corrections may now easily be implemented by noting that the vacuum polarization induces an effective dielectric function $\epsilon(y)$ for the vacuum, while the electron self-energy diagrams give the electron an electric form factor $F_e(y)$. The electron, of course, also acquires a magnetic form factor, but this represents a small correction to the magnetic transition amplitude, which is itself negligible for $v \ll c$.

Thus, both the vacuum polarization and the self energy can be taken into account by modifying the potentials as

$$V(q_0,\vec{q})\rightarrow V(q_0,\vec{q})F_e(y)/\epsilon(y) . \quad (28)$$

To the lowest order in Z_α the QED corrections are well known,[27]

$$\epsilon(y)\approx 1-\frac{\alpha}{3\pi}\left[\ln y+5+O\left[\frac{\ln y}{y}\right]\right] ,$$
$$F_e(y)\approx 1+\frac{\alpha}{\pi}\left[(\ln\mu+1)(\ln y-1)-\tfrac{1}{4}(\ln y)^2-\tfrac{1}{4}\ln y+\frac{\pi^2}{12}+O\left[\frac{1}{y},\frac{\ln y}{y}\right]\right] . \quad (29)$$

Here μ is an infrared cutoff, which by the usual arguments is of the order of Z_α, where Z is the effective charge corresponding to the potential. The potentials enter our discussion both in determining the wave functions [with $\lambda=0$ in Eq. (25), $B^{\mathrm{SL}}(q_0,\vec{0})=\sqrt{2\pi}\delta(q_0)$], and in the transition matrix elements. For the wave functions, we can approximately estimate the effects of the QED corrections by noting that the corrections are slowly varying functions of y. Since the relevant portions of the wave functions are situated around $q^{-1}\approx v/2$, and the effective nuclear potential is proportional to Z_u, we can make the following replacement [$\ln\mu\approx\ln(Z\alpha)\approx 0$]:

$$Z_u\rightarrow\frac{Z_uF_e(4/v^2)}{\epsilon(4/v^2)}\approx Z_u\frac{1+(\alpha/\pi)\{\frac{3}{4}\ln(4/v^2)-\frac{1}{4}[\ln(4/v^2)]^2+\pi^2/12-1\}}{1-(\alpha/3\pi)[\ln(4/v^2)+5]} . \quad (30)$$

For $v/c=0.1$ this yields $Z_u\rightarrow 0.998Z_u$—thus a negligible effect. The effects on the transition matrix elements can be evaluated in the same manner, and also give negligible corrections. The influence of vacuum polarization and self-energy separately amounts to about 1% but they cancel each other almost completely, as they tend to do for the binding energies. Thus we conclude that although QED corrections are larger for the pair-production process than estimated from the effect of the united-atom binding energies, they are still negligibly small in any realizable experimental situation. This picture is not changed by higher-order QED effects, which only become significant if $\ln y\sim\alpha^{-1}$.[27]

VI. NUMERICAL RESULTS

Figure 4 shows pair-production cross sections for 5-MeV/amu heavy ions colliding with U targets. The PWBA pair-production cross section increases as Z_1^2, from 4 nb for α+U to 1400 nb for Sr + U ($Z_1=38$). With wave-function effects, σ_{pair} increases from 6 nb (α+U) to 1.4 mb (Sr + U). This is mainly due to the rapid increase of σ_{pair} with Z_u, as was seen in Fig. 1. The Coulomb-deflection,

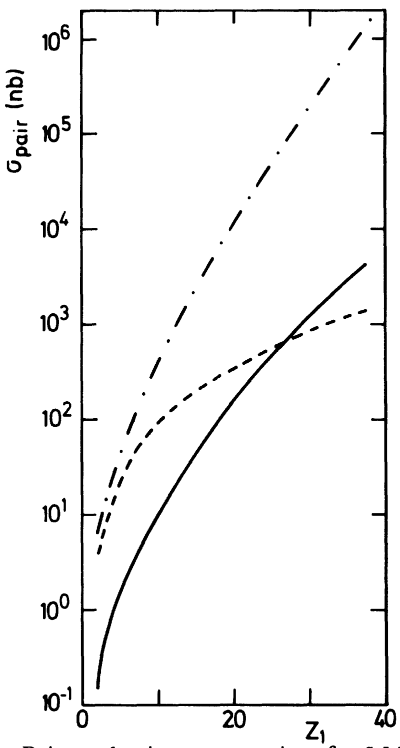

FIG. 4. Pair-production cross sections for 5-MeV/amu heavy-ion collisions with U. Dashed curve: plane-wave Born approximation, Eq. (7). Chain curve: plane-wave Born approximation, including wave-function effects. Solid line: including Coulomb-deflection, finite—nuclear-size, wave-function, and energy-loss effects.

FNS, and energy-loss effects reduce the pair-production cross section by two to three orders of magnitude, however. For $\alpha+$U, the new pair-production cross section is only 0.14 nb.

Figure 5 shows the $\alpha+$U electron-positron pair-production cross section for several projectile energies. The PWBA cross section and the cross section including wave-function effects both increase as $v^{4\gamma+2}$, the difference between γ and γ_u being negligible here. When one includes Coulomb-deflection, FNS, and energy-loss effects, σ_{pair} decreases very rapidly with lower projectile energy. It is clear from Fig. 5 why pairs have not been seen in 2.5-MeV $p+$W and 2.5-MeV (0.625-MeV/amu) $\alpha+$Ta collisions. Heykant *et al.*[4] find that σ_{pair} is less than 3 nb for 2.5-MeV $\alpha+$Ta collisions (roughly converting their thick-target yield limit to a cross-section limit). We predict that σ_{pair} for 0.625-MeV/amu $\alpha+$Ta collisions is much smaller than for 3-MeV/amu $\alpha+$U collisions, which is ~ 1 pb. Owing to the proton's anomalous mass ($Z_1=A$), d is larger for protons, therefore the Coulomb-deflection factor is much smaller, giving small (<1 nb) cross sections, even for 5-MeV protons.

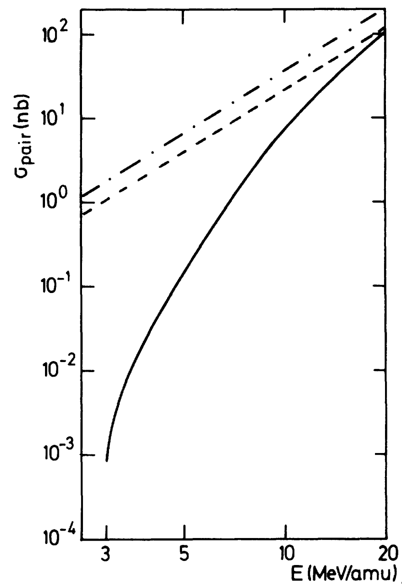

FIG. 5. Pair-production cross sections in $\alpha+$U collisions versus projectile energy. Curves have same meaning as Fig. 4.

Finally, we show the differential pair-production cross section in Fig. 6. With increasing projectile energy, positrons can be excited to higher kinetic energy. As is usual for positron spectra, the differential cross section goes to zero as W_+ goes to zero, due to Coulomb effects on the positron's wave function.

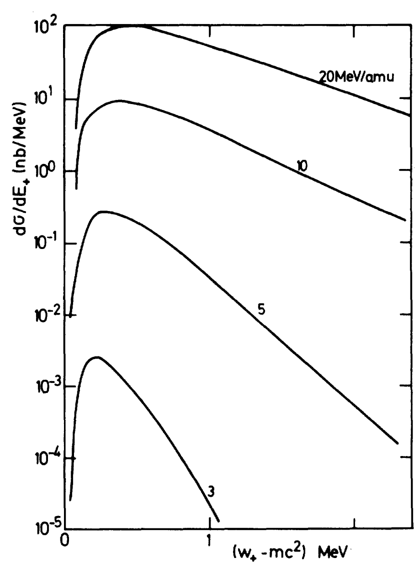

FIG. 6. Differential pair-production cross section in $\alpha+$U collisions for different collision energies E, calculated including all effects.

VII. CONCLUSIONS

Pair-production cross sections in heavy-ion–atom collisions have been calculated using a high–momentum-transfer approximation, and by making corrections to the plane-wave Born-approximation cross sections for wave-function, Coulomb-deflection, and other effects. The high–momentum-transfer approximation is well suited to this problem, and gives accurate cross sections even for the highest energies considered, 5 MeV/amu. It may be objected that the use of correction factors, which affect the cross sections by many orders of magnitude, may be inaccurate. However, we have used the correction factor as a device for simplifying the numerical problem of evaluating the quadruple integrals needed to obtain the pair-production cross sections. Using correction factors, the quadruple integral decouples into two double integrals. We have carefully examined the separability of these integrals. The major approximations used are the Briggs approximation to account for wave-function effects and the high–momentum-transfer approximation. Within this framework, our evaluation of pair-production cross sections is nearly exact, equivalent to performing the quadruple integral over time, impact parameter, positron energy, and electron energy. Calculations similar to these, but using a different wave-function approximation, have been made by Soff *et al.*[10] to calculate pair production in U + U collisions. The only improvements which one might make is to replace the semiclassical evaluation of the amplitude with an evaluation using quantum-mechanical Coulomb waves for the projectile motion. This would give a better energy-loss correction, which is important at low projectile energies with light projectiles.

The experience of Stephens and Staub[3] and Heykants and Niecke[4] indicates that pair-production cross sections smaller than 10 nb are exceedingly difficult to measure, even when nuclear-reaction backgrounds are not present. This suggests that at 5 MeV/amu, direct pair production may be observable only with projectiles with $Z_1 > 10$. Positrons from nuclear processes may give apparent pair-production cross sections exceeding 10 nb, however. Even with projectile energies below the Coulomb barrier, γ rays can be Coulomb excited which convert into electron-positron pairs. Heavy-ion nuclear reactions with the target nucleus or target impurities can occur with a small probability at energies below the Coulomb barrier. Products from such reactions may β_+ decay or γ-ray decay by pair conversion, giving apparent pairs. In conclusion, therefore, direct pair production can only be observed in very careful measurements in heavy-ion–atom collisions.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation Grant No. PHY-80-15348 and NORDITA (R.A.), by the Gesellschaft für Schwerionenforschung, Darmstadt (D.H.J.), by the Norwegian Research Council for Science and the Humanities (K.Aa.), and by the Alexander von Humboldt Stiftung and NORDITA (P.A.A.).

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$$E_{1\mathrm{LAB}}(\mathrm{MeV/amu}) = 1.44 Z_1 Z_2 \frac{(A_1 + A_2)}{A_1 A_2 r_0 (A_1^{1/3} + A_2^{1/3})} ,$$

where A_1 and A_2 are projectile and target masses. For heavy ions one can use $r_0 \approx 1.44 \pm 0.2$ fm: J.R. Birkelund, J. R. Huizenga, H. Freiesleben, K. L. Wolf, J. P. Unik, and V. E. Viola, Jr., Phys. Rev. C 13, 133 (1976).

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