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# Dominance of ion motion over electron motion in some intensity-induced wave processes in a magnetized plasma

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Intensity-induced frequency shift and precessional frequency of two strong circularly polarized waves are derived in a magnetized plasma. An unexpected feature in the low-frequency limit of a magnetohydrodynamic Alfvén wave is that the ion velocity is comparable to the velocity of light in the presence of a static magnetic field. This process may be useful in the astrophysical energy-transfer studies and in the experiment with ion–laser-induced plasma.

## I. INTRODUCTION

In this paper we report the investigation of the nonlinear interactions of two circularly polarized waves propagating parallel to the direction of a uniform and static magnetic field in a homogeneous and cold plasma, taking into consideration the motion of both electrons and ions. We have evaluated (i) the intensity-dependent frequency shift and precessional frequency of the resultant waves as nonlinear corrections, and (ii) the values of these nonlinear interactions in the limit of low-frequency Alfvén waves. Stenflo and Tsintsadze[1] have recently shown that for small-amplitude electromagnetic ion waves, in the presence of an external magnetic field, the motion of ions are most important and its relativistic nonlinear effects dominate over those of electrons. This fact should be important in the study of continuous impinging on a plasma by ion-laser beam, in the energy-transfer processes which occur in many astrophysical bodies,[2,3] for large amplitude wave propagation in plasmas, and for laboratory experiments where strong laser radiation interacts with a high-density target. Our findings also support the contention of Stenflo and Tsintsadze[1] and, moreover, bring a new dimension to those studies by the simultaneous evaluation of the complementary effect of precessional rotation and its consequences on low-frequency nonlinear effects.

The major intent of this Brief Report is not only the demonstration that relativistic ion effects can be important in the propagation of strong waves, but also the demonstration that the magnitude of the intensity-induced precessional rotation of the waves from ion motion is about $1.26\times10^{13}$ times greater than that from electron motion. The precessional rotation[4–6] of the waves modifies the synchrotron radiation[4,6] from them, and also the induced magnetization (the inverse Faraday effect).[7–10] This effect should be thoroughly worked out in the near future in the low-frequency limit to see properly the dependence of the magnitude of the modifications on the ion motion. Stenflo and Tsintsadze[1] or others do not initiate anything about the dependence of the nonlinearly induced precessional rotation of the waves on the motion. Certainly ours is the first report on the dependence on ion motion of self-generating rotations from strong low-frequency waves in a plasma.

## II. BASIC ASSUMPTIONS AND FIELD EQUATIONS

In general, all the self-action effects occur simultaneously. But which one will dominate depends on the experimental conditions (e.g., long pulse or short excitation) and power of the laser beam. So, following Whitmar and Barrett[11] and others,[4–6] we have introduced some physically possible conditions: (i) Initially the fluids exist in the Lorentz frame, and then perturbed by the strong applied electromagnetic radiation which is sinusoidal, i.e., the perturbation of the field variables are harmonic in nature. (ii) The incident electromagnetic radiation is strong so that the motion of electrons and ions becomes relativistic; but the power of the radiation does not exceed the threshold power limit for the appearance of self-focusing and the self-trapping mechanism[12]; also, the difference between the velocities of electrons and ions is much less than the velocity of light, so that instabilities of the system can be minimized. (iii) There is no first-harmonic density fluctuation due to the interaction of waves with plasma, but nonlinearly excited second-harmonic density fluctuation exists and its effect on stimulated Brillouin and Raman scattering will be visible in orders of approximation higher than three and so are neglected in our consideration[13]; the self-action effects arising from pondermotive forces and thermal instabilities are neglected because pressure variation and thermal velocity in plasmas are insignificant.[4] (iv) The problem of nonlinear propagation of an intense wave has to be solved in a closed form where the wave processes remain quasimonochromatic.[4,13]

Under the above set of assumptions, the fluid motion is given by

$$\left[\frac{\partial}{\partial t}+(\vec{v}_e\cdot\vec{\nabla})\right]\vec{p}_e=-e\vec{E}-\frac{e(\vec{v}_e\times\vec{H})}{c} \quad , \tag{1}$$

$$\left[\frac{\partial}{\partial t}+(\vec{v}_i\cdot\vec{\nabla})\right]\vec{p}_i=e\vec{E}+\frac{e(\vec{v}_i\times\vec{H})}{c} \quad , \tag{2}$$

and the field equations of Maxwell reduce to

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}=-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}+\frac{4\pi e}{c^2}\frac{\partial}{\partial t}(N_e\vec{v}_e-N_i\vec{v}_i) \quad , \tag{3}$$

$$\nabla^2\vec{H}=\frac{1}{c^2}\frac{\partial^2\vec{H}}{\partial t^2}+\frac{4\pi e}{c}\vec{\nabla}\times(N_e\vec{v}_e-N_i\vec{v}_i) \quad , \tag{4}$$

where the subscripts $e$ and $i$ represent electrons and ions, respectively; $\vec{p}$, $\vec{v}$, $N$, $m$, $\vec{E}$, $\vec{H}$, and $c$ stand for relativistic momentum, velocity, density, mass, electric field, magnetic field, and velocity of light, respectively. For work relativistic effects $v^2 << c^2$ and so $\vec{p}\simeq m\vec{v}(1-v^2/2c^2)$.

Let the resultant of two circularly polarized waves of electric field be elliptically polarized, so it has the form [Krall and Trivelpiece,[14] Eq. (4.10.6)]

$$\vec{E}=(m_e+m_i)c\omega \times\frac{(\hat{y}+i\hat{z})(\alpha e^{i\theta_R}+\beta e^{-i\theta_L})+(\hat{y}-i\hat{z})(\alpha e^{-i\theta_R}+\beta e^{i\theta_L})}{2e} \quad , \tag{5}$$

where $\hat{x}$, $\hat{y}$, $\hat{z}$ are the unit vectors; $\theta_{R,L}=K_{R,L}x-\omega t$; $i=\sqrt{-1}$, $\alpha$, $\beta$ are the dimensionless amplitudes of the two waves; $R$, $L$ represent the right- and left-handed circularly polarized waves propagating in the $x$ direction with dispersion relations

$$n_R^2=1-(x_e+x_i)/(1-y_e)(1+y_i) \quad , \tag{6}$$

and

$$n_L^2=1-(x_e+x_i)/(1+y_e)(1-y_i) \quad , \tag{7}$$

where

$$x_{e,i}=\omega_{pe,i}^2/\omega^2;\quad y_{e,i}=\Omega_{e,i}/\omega \quad ;$$

$$n_{R,L}=K_{R,L}c/\omega \quad .$$

## III. SECULAR-FREE SOLUTIONS

For evaluating the secular-free closed-form nonlinear solutions of the field equations correct up to the third order of approximation,[4,6,13] the field variables are written in the series form

$$u=u_0+\epsilon u_1+\epsilon^2u_2+\epsilon^3u_3+\cdots \quad , \tag{8}$$

and a process of successive approximation[15(a)–15(c),16] is adopted; where $u_0$ is the equilibrium state value of $u$, $u_1$ is the linearized first harmonic approximation, $u_n$ is the $n$th ordered approximation and is given by a linear combination of the $n$th harmonics of $u$, and the expansion parameter $\epsilon$ is only a mathematical artifice, allowing us to group terms of comparable degree of approximation in a mathematical and convenient fashion. Evidently, the following values for the frequency shift and the precessional rate of rotation are obtained:

$$\left[\frac{\partial\omega}{\omega}\right]=\frac{(\Gamma_1D-\tau_1B)+(\Gamma_2D-\tau_2B)}{AD-BC} \quad , \tag{9}$$

$$\left[\frac{\dot{\varphi}}{\omega}\right]=\frac{(\tau_1A-\Gamma_1C)+(\tau_2A-\Gamma_2C)}{AD-BC} \quad , \tag{10}$$

provided $AD\neq BC$, where

$$[A,C]=-[2(x_e+x_i)+2\epsilon_{e_\pm}\epsilon_{i_\mp}+(n_{R,L}^2-1)(\epsilon_{i_\pm}+\epsilon_{e_\mp})] \quad ,$$

$$[B,D]=\pm[\epsilon_{e_\mp}\epsilon_{i_\pm}(n_{R,L}^2-1)\mp\epsilon_{i_\pm}P_{R,L;i_\pm}\pm\epsilon_{e_\pm}P_{R,L;e_\mp}] \quad ,$$

$$[\Gamma_1,\tau_1]=[\epsilon_{e_\pm}\{2x_in_{R,L}\mp P_{R,L;e_\mp}\epsilon_{i_\pm}n_{R,L}\pm P_{R,L;e_\mp}\epsilon_{i_\mp}n_{R,L}\} +\epsilon_{i_\mp}\{-2x_en_{R,L}\mp P_{R,L;i_\pm}\epsilon_{e_\mp}n_{R,L}\pm P_{R,L;i_\pm}\epsilon_{e_\pm}n_{L,R}\}](\beta^2,\alpha^2)/2 \quad ,$$

$$[\tau_2,\Gamma_2]=\pm[\epsilon_{e_\pm}P_{L,R;e_\pm}\{P_{L,R;e_\pm}(\beta^2,\alpha^2)+2P_{R,L;e_\mp}(\alpha^2,\beta^2)\}\mp\epsilon_{i_\mp}P_{L,R;i_\mp}\{P_{L,R;i_\mp}(\beta^2,\alpha^2)+2P_{R,L;i_\pm}(\alpha^2,\beta^2)\}] \quad ,$$

where also

$$\epsilon_{i_\pm}=1\pm y_i;\quad \epsilon_{e_\pm}=1\pm y_e \quad ,$$

$$P_{R,L;i_\pm}=P_{R,L}\pm y_i(n_{R,L}^2-1);\quad P_{R,L;e_\pm}=P_{R,L}\pm y_e(n_{R,L}^2-1),$$

$$P_{R,L}=n_{R,L}^2-1+x_e+x_i;\quad n_{R,L}(\alpha^2,\beta^2)=n_R\alpha^2 \text{ and } n_L\beta^2 \quad ;$$

and also the square-bracket notation implies different sets of values, viz., if $[\tau,\Gamma]=\pm\Phi_\mp$ (say), then $\tau=+\Phi_-$ and $\Gamma=-\Phi_+$, etc.

The first term of (9) and (10) are the sum of the drift forces, Lorentz forces, and plasma currents of electrons and ions, and the last term of the same equations are the resultant contribution of relativistic masses of electrons and ions.

Numerically, $\dot{\varphi}$ and $\partial\omega$, for the interaction of nonresonant strong radiation {power flux $(P) \simeq 10^{23}$ ergs cm$^{-2}$ sec$^{-1}$, wavelength $(\lambda) \simeq 1.06$ $\mu$m, frequency $(\omega) \simeq 1.78 \times 10^{15}$ sec$^{-1}$, with magnetized plasma [magnetic field $(H_0)$] $\simeq 1.58 \times 10^7$ G, number density $(N_0) \simeq 1.5 \times 10^{12}$ cm$^{-3}$} are not very large. But these effects would be enhanced, even detectable, in laboratory experiments and would have significant impact on nonlinear distortion of atoms[10] in the case of resonant or near-resonant interaction, or even phase-matching interaction of waves with plasmas.[5,6,17]

## IV. RESULTS FOR LOW-FREQUENCY WAVES

In the low-frequency limit the right and left circular polarizations have the same dispersion rates and an electromagnetic wave reduces to an Alfvén wave (Alfvén and Falthammar)[18]; then Eqs. (6) and (7) reduce to

$$n^2 = 1 + (x_e + x_i)/y_e y_i = 1 + c^2/c_A^2 \ , \qquad (11)$$

where $c_A^2 = 4\pi\rho/H_0^2$; and $\rho = N_0/(m_e + m_i)$. Since $c_A^2 << c^2$, we get

$$n_A (= k_A c/\omega) = 1 \ . \qquad (12)$$

This is the well-known dispersion relation of Alfvén waves. For this limit

$$\left[\frac{\partial\omega}{\omega}\right] = -\frac{y_i c^4}{4 x_e x_i c_A^4}\left[\left(\frac{y_i^3}{y_e^3} - \frac{y_e}{y_i}\right)(\alpha^2 - \beta^2) + x_e y_i \left(\frac{y_i}{y_e} + 1\right)(\alpha^2 + \beta^2)\right] \ , \qquad (13)$$

and

$$\left[\frac{\dot{\varphi}}{\omega}\right] = -\frac{c^4}{4 x_e x_i c_A^4}\left[\left(\frac{y_L^3}{y_e^3} - \frac{y_e}{y_L}\right)(\alpha^2 - \beta^2) + x_e y_i \left(\frac{y_i}{y_e} + 1\right)(\alpha^2 + \beta^2)\right] \ . \qquad (14)$$

From these two equations, the following properties can be observed. (1) The magnitude of the frequency shift $(\partial\omega/\omega)$ is $y_i$ times greater than the magnitude of the precessional frequency $(\dot{\varphi}/\omega)$. (2) The last two terms of the square bracket come from the nonlinear nonrelativistic effects of charged particles (electrons and ions) which are mainly from the plasma currents of them because the other nonrelativistic effects of these particles are seen to be comparatively small. In the nonrelativistic contributions, the nonlinear correction for ions is approximately $1.8 \times 10^3$ times greater than that of electrons. (3) The first two terms come from the nonlinear relativistic effects of electrons and ions for the limit leading to Alfvén waves. It is evident that the magnitude of the nonlinear corrections of ions is approximately $1.26 \times 10^{13}$ times greater than that of electrons, in spite of the fact that electrons are much lighter than ions; this fact supports the contention of Stenflo and Tsintsadze[1] that the particle (ion) velocity is comparable to the velocity of light in the presence of static magnetic field for low-frequency waves. Moreover, the nonlinear corrections of $(\partial\omega/\omega)$ and $(\dot{\varphi}/\omega)$ have two terms, one of which exists in the nonlinear magnetohydrodynamic (MHD) equations, but the other term comes from the relativistic masses of electrons and ions, and is exclusively an electromagnetic source which filters out of the MHD equations because of the imposition of the MHD approximation. (4) If only the ions are not relativistic, but mobile, then (i) shift is approaching towards red, (ii) rotational direction will be changed, (iii) nonrelativistic contributions of charged particles will dominate over the relativistic contribution because $x_e y_e^3$ is greater than $y_i^2$, in general.

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