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Effect of symmetry on ensemble-averaged level density

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The effect of a general additive symmetry on level density in finite spaces is investigated. It is found that with the introduction of symmetry, the level density moves closer towards a Gaussian distribution.

I. INTRODUCTION

The level density for a system of "p" particles distributed among "s" single-particle states and interacting via an r-body random interaction has been studied through shape parameters by Mon and French[1] for the case of fermions, and by Kota and Potbhare[2] for the case of bosons. Mon and French[1] have shown that the fermionic level density is a semicircle when all the particles interact simultaneously and there is a gradual transition from semicircle to Gaussian in the limit when "r" becomes much smaller than "p." In the case of a dilute system ($s\rightarrow\infty$, $p\rightarrow\infty$, $p/s\rightarrow 0$) all the shape parameters have been shown[1] to approach zero when $r \ll p$. Kota and Potbhare have shown that a dilute system of bosons also gives a Gaussian level density when $r \ll p$. For a finite system, only the low-order shape parameters have been evaluated[1,2] in both the cases. The evaluation of higher-order shape parameters, though straightforward, becomes cumbersome. In the case of bosons, for finite "s" and $p\rightarrow\infty$ (the dense limit), the level density is a Gaussian only when s is "sufficiently large."[2] Using a similar procedure, it can be shown that even for distinguishable particles, for finite "r" we get a Gaussian in the limit when $s\rightarrow\infty$, $p\rightarrow\infty$, and $p/s\rightarrow 0$ and $p \gg r$. In the dense limit $p\rightarrow\infty$, for finite "s" we again get a Gaussian.

Our aim is to investigate the effect of symmetry on the level density in finite spaces. In Sec. II, this is investigated through numerical examples. Section III gives a summary of the results.

II. SYMMETRY PROPERTIES AND LEVEL DENSITY

The effect of symmetry on level density is, in a sense, akin to that of a reduction in the rank of interaction, in that the number of nonvanishing matrix elements in "r" particle space gets reduced. To study the variation in the level density $\rho(E)$ we evaluate its shape parameters $S_p (p>2)$ which are given by

$$S_p = K_p/(K_2)^{p/2} \quad , \tag{1}$$

where K_p, the pth cumulant, is the coefficient of $(it)^p/p!$ in $\ln F(t)$, where $F(t)$ is the Fourier transform of the level density.

$$F(t)=\int_{-\infty}^{\infty} e^{itE}\rho(E)dE \quad . \tag{2}$$

K_2 is the variance. S_3 and S_4 are the standard measures of skewness and excess. For a Gaussian distribution, all S_p's vanish. In the event of higher shape parameters vanishing, a positive excess would correspond to a peaked distribution, while a negative excess to a flattened one.

Mon and French[1] evaluated the shape parameters of the level density of fermions using propagation technique. The Hamiltonian in r particle space is given by

$$H=\sum_{z_1 z_2} W_{z_1 z_2} Z_1(r) Z_2^{\dagger}(r) \quad . \tag{3}$$

The dimensionality of H in r particle space is given by ${}^s c_r$. In Eq. (3), Z_1 and $Z_2^{\dagger}$ are the r-fermion creation and annihilation operators, and W's are the r-body matrix elements (RBME). Since H is real symmetric, we have $Y(Y+1)/2$ RBME where $Y={}^s c_r$. The average of different powers of the Hamiltonian in "p" particle space is then evaluated using the relation

$$\langle O(r)\rangle^p = {}^p c_r \langle O(r)\rangle^r \quad , \tag{4}$$

where $O(r)$ is any r-body operator. Thus the traces of different powers of H in "p" particle space are evaluated without actually diagonalizing the matrix.

The values of S_4 for two-body random ensemble (TBRE) for different particle number "p" in (sd) shell calculated using this technique[1] are given in Table I. Since the number of single-particle states in sd shell is 24, the dimensionality of the two-particle space is ${}^{24}C_2 = 276$, and the number of independent two-body matrix elements is $(276\times 277/2)$ when no symmetry considerations are introduced.

Potbhare[3] has evaluated S_4 for the same configurations by introducing conservation of the total angular momentum (J) and isospin (T) quantum numbers in two-particle space. The two-particle matrix breaks down into block matrices along the diagonal and the total number of independent matrix elements gets reduced to 63. The values of S_4 for these cases (with symmetry) are also given in Table I. With symmetry, the values of S_4 are smaller compared to those without symmetry, and closer to that of Gaussian. The subspace of dimensionality 56 corresponding to $J=2$, $T=0$ for four particles in ds shell, gives a value of S_4 which is -0.54 as against the ${}^{24}C_4$ dimensional space, with $S_4=-0.85$ without any symmetry.

For a complete knowledge of the level density in both cases, one needs to evaluate all the shape parameters. Hence, to ascertain the faster approach to Gaussian with symmetry considerations, we resort to matrix diagonalization in "p" particle space. This is because the propagation technique becomes laborious in the evaluation of higher-order shape parameters. From the definition of the r-body Hamiltonian in Eq. (3), the matrix elements and the Hamiltonian in "p" particle space could be written as

TABLE I. Values of S_4 for different particle number "p" in sd shell for the two cases without and with symmetry.

p	S_4 (without symmetry)	S_4 (with symmetry)	S_4
4	−0.85	−0.58(0.14)	for Gaussian
8	−0.59	−0.32(0.11)	0.0
12	−0.56	−0.22(0.12)	for semicircle
	(Ref. 1)	(Ref. 3)	−1.0

$$H_{ij}=\sum_{z_1z_2} W_{z_1z_2}\sum_{x}\langle p_i|z_1(r)|(p-r)_x\rangle\langle(p-r)_x|z_2^\dagger(r)|p_j\rangle\ , \tag{5}$$

$$H=\sum_{z_1<z_2} W_{z_1z_2}[A(z_1)\tilde{A}(z_2)+A(z_2)\tilde{A}(z_1)]\ , \tag{6}$$

where x's are $p-r$ particle states and $A(z)$ the matrix that creates "p" particle states from $p-r$ particle states through "z." Since the number of $p-r$ and p particle states is given by ${}^sc_{p-r}$ (X_1, say) and ${}^sc_p$ (X_2, say), A is an $X_2\times X_1$ matrix; the rows and columns of A are labeled by p and $p-r$ particle states. Only those elements of A, where the row and column indices have $p-r$ states in common, are nonvanishing with an absolute value of 1. The determinantal basis states for fermions leads to the appropriate sign of the matrix elements in A for any permutation of the single-particle state indices in many-particle states. In Eq. (6) for a system of bosons, basis states should be symmetric, and for distinguishable particles the basis states are direct products of individual single-particle states. Now that the $A(Z)$'s are built in this fashion, the Hamiltonian in "p" particle space is known and an ensemble of H's can be generated for any rank of interaction.

To study the effect of symmetry on fermionic level density using the above scheme, we choose $p=4$, $s=8$, and $r=2$. The dimensionalities of the four-

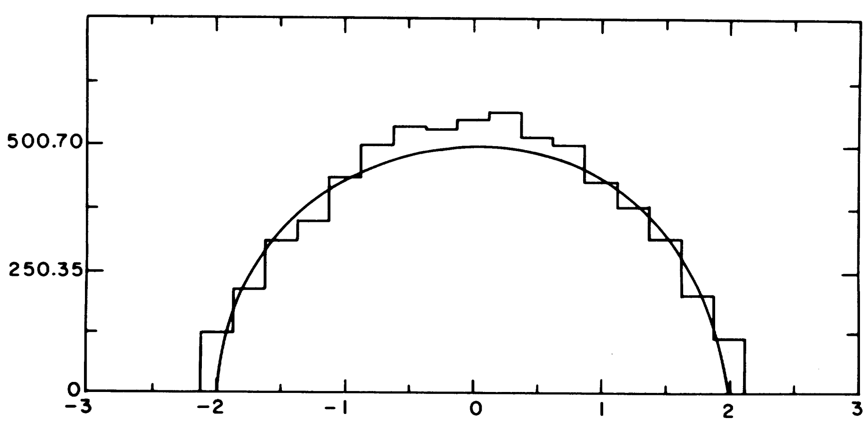

FIG. 1. The exact eigenvalue density (histogram) of 70-dimensional matrices without symmetry along with semicircle (continuous curve).

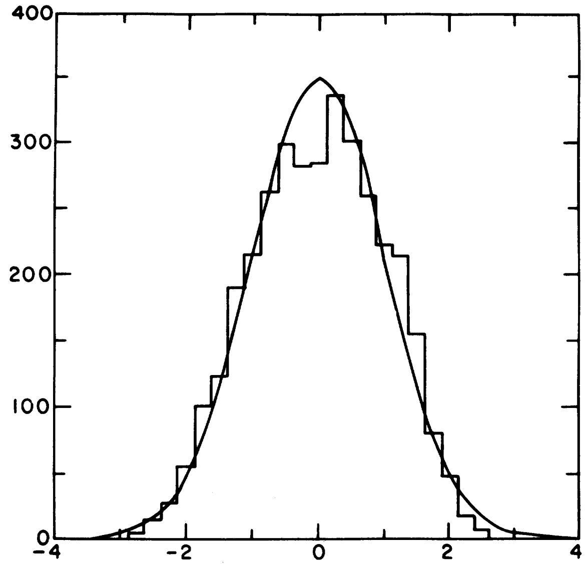

FIG. 2. The exact eigenvalue density (histogram) of 70-dimensional matrices with symmetry along with Gaussian (continuous curve).

and two-particle spaces are 70 and 28, respectively. Without symmetry considerations one has $(28\times 29)/2$ two-body matrix elements (TBME) and an ensemble of H's is generated and diagonalized. This is the first case without symmetry. In the second case, each of the single-particle states is assigned a quantum number "m" corresponding to any additive symmetry (say, J_z). With the conservation of total "m" in two-particle space, one has only 50 nonvanishing TBME. Again an ensemble of H's is generated and diagonalized. The level densities for the two cases without and with symmetry are shown in Figs. 1 and 2, respectively. In the latter case we find that the distribution is closer to Gaussian. The ensemble-averaged shape parameters S_3 to S_8 for the two cases are given in Table II along with the rms deviations. The values of S_p's for Gaussian and semicircular distributions are also given in the table. The even-shape parameters go down in magnitude when symmetry considerations are introduced, thus taking the eight-particle level density closer to Gaussian. With symmetry the distribution becomes more asymmetric, as is evident from Fig. 2 and S_7 (Table II). A closer look at Fig. 1 shows that the semicircular distribution itself is asymmetric. In fact, S_9 turns out to be -3.73 for this case. But in the second case even the lower odd-shape parameters are relatively higher. This cannot be attributed to inadequate statistics, since increasing or decreasing the ensemble dimensionality does not change these averages in either case. It may be that in higher dimensional matrices, this asymmetry would be reduced when no symmetry is introduced. This still does not guarantee that, in such cases, distributions with symmetry will be devoid of asymmetry. But the overall effect would still be that, with symmetry, the distribution would move closer to a Gaussian. The fluctuations in the shape parameters increase with the introduction of symmetry. This is due to the fact that the ensemble is not invariant under orthogonal transformation in "p" particle space.

TABLE II. Shape parameters S_3 to S_8 and the rms deviations for the level density of 70×70 matrices without and with symmetry.

Ensemble dimensionality	S_3 (S_3 rms)	S_4 (S_4 rms)	S_5 (S_5 rms)	S_6 (S_6 rms)	S_7 (S_7 rms)	S_8 (S_8 rms)
47 without symmetry	0.01 (0.06)	−0.84 (0.06)	−0.02 (0.28)	3.64 (0.49)	0.10 (2.82)	−35.79 (7.00)
50 with symmetry	−0.08 (0.18)	−0.57 (0.25)	0.29 (0.67)	1.78 (1.68)	−2.14 (6.12)	−13.90 (18.56)
Semicircle	0.0	−1.0	0.0	5.0	0.0	−56.00
Gaussian	0.0	0.0	0.0	0.0	0.0	0.0

III. CONCLUSION

With the introduction of an additive symmetry, the fermionic level density moves closer towards Gaussian. For other general symmetries, we have numerical examples of shell-model studies in literature.[4] Since a dilute system of bosons gives the same results as a dilute system of fermions, we conclude that the effect of symmetry on a system of bosons and distinguishable particles should also be to accelerate the rate of approach of the level density to normality.

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[2]V. K. B. Kota and V. Potbhare, Phys. Rev. C 21, 2637 (1980).

[3]V. Potbhare, Pramana 11, 205 (1978).

[4]V. K. B. Kota and V. Potbhare, Pramana 11, 209 (1978).