Squeezed states and intensity fluctuations in degenerate parametric oscillation

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The relationship between the squeezing and photon-number fluctuations in the output of a degenerate parametric oscillator is investigated. The addition of a second driving field at the idler frequency allows the direction of the squeezing to be changed. The squeezing may appear in the amplified quadrature. Photon antibunching or bunching may occur depending on whether the quadrature carrying the coherent excitation is squeezed.

I. INTRODUCTION

In a recent paper[1] we investigated the amount of squeezing that could be obtained in the output of a degenerate parametric oscillator. In this paper we wish to investigate the relationship between the photon-number fluctuations and the squeezing in the output of the device. By including an additional driving field at the idler frequency in a model which includes depletion of the pump mode we are able to change the sign of the squeeze parameter and hence transfer the squeezing from one quadrature to the other. The output field may show photon antibunching or photon bunching depending on whether the quadrature carrying the coherent excitation is squeezed or not. The addition of a second driving field allows the amplified quadrature to be squeezed.

II. PHOTON-NUMBER FLUCTUATIONS IN SQUEEZED STATES

A large number of references to squeezed states and their properties are given in Ref. 1. We shall briefly describe a few of the properties relevant to this paper. The squeezed states of a single mode are defined by

$$|\alpha,\epsilon\rangle=\exp(\alpha a^\dagger-\alpha^* a)\times\exp[\tfrac{1}{2}\epsilon^* a^2-\tfrac{1}{2}\epsilon(a^\dagger)^2]\,|0\rangle\;, \tag{1}$$

where $\epsilon=re^{i\theta}$ is the complex squeeze parameter and α is the complex amplitude of the state. The variances in the quadratures of the complex field $[X_1=\frac{1}{2}(a+a^\dagger), X_2=(1/2i)(a-a^\dagger)]$ are

$$\Delta X_1^2=\tfrac{1}{4}e^{-2r}\;,\qquad \Delta X_2^2=\tfrac{1}{4}e^{2r}\;, \tag{2}$$

for the squeeze parameter chosen real ($\theta=0$).

The photon-number correlations are characterized by the second-order correlation function

$$g^{(2)}(0)=\frac{\langle a^\dagger a^\dagger a a\rangle}{\langle a^\dagger a\rangle^2}\;. \tag{3}$$

For a squeezed state with $|\alpha|^2 \gg \sinh^2 r$ and α real (the coherent excitation in X_1) $g^{(2)}(0)$ is approximately given by

$$g^{(2)}(0)\simeq 1+\frac{1}{\alpha^2}(e^{-2r}-1)\;. \tag{4}$$

For the case of α pure imaginary (i.e., the coherent excitation in X_2) we find

$$g^{(2)}(0)\simeq 1+\frac{1}{|\alpha|^2}(e^{2r}-1)\;. \tag{5}$$

Equations (4) and (5) together with Eqs. (2) show that photon antibunching $[g^{(2)}(0)<1]$ occurs whenever the quadrature component carrying the coherent excitation is squeezed. Changing the sign of r (i.e., the "direction" of squeezing) takes us from a region of photon antibunching to a region of photon bunching $[g^{(2)}(0)>1]$.

The photon-number distribution for a squeezed state is given by Yuen[2]:

$$P(n)=\frac{1}{n!\cosh r}\left[\frac{\beta}{2\cosh r}\right]^{2n} Y^{-2n}H_n^2(Y)\times\exp[-\beta^2(1-\tanh r)]\;, \tag{6}$$

where

$$Y=\frac{\beta}{\sqrt{2\cosh r\sinh r}}\;,\qquad \beta=\alpha e^{r}\;.$$

$H_n(Y)$ is the Hermite polynomial.

In Fig. 1 we have plotted $P(n)$ for $r>0$, clearly showing the reduced number fluctuations obtained in this case in comparison with the Poisson distribution of a coherent state.

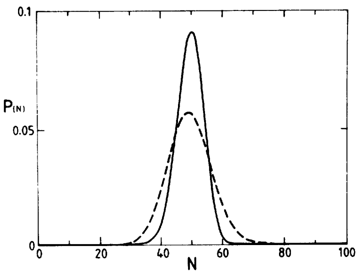

FIG. 1. Photon-number distribution $P(N)$ vs photon number for a coherent state $|\alpha\rangle$ (— — —) and a squeezed state $|\alpha,r\rangle$ (——). $\alpha=7$ and $r=0.5$.

III. QUANTUM STATISTICS OF DEGENERATE PARAMETRIC OSCILLATION

We consider a degenerate parametric oscillator where a pump mode at frequency 2ω interacts with an idler mode at frequency ω via a nonlinear crystal with a second-order optical susceptibility. The nonlinear crystal is placed within a Fabry-Perot interferometer and both modes are driven externally with coherent fields. The modes suffer losses due to cavity damping. This system is described by the following Hamiltonian:

$$H=\hbar\omega a_1^\dagger a_1+2\hbar\omega a_2^\dagger a_2+i\hbar\frac{\kappa}{2}(a_1^{\dagger 2}a_2-a_1^2a_2^\dagger)+i\hbar(\epsilon_1 a_1^\dagger e^{-i\omega t}-\epsilon_1^* a_1 e^{i\omega t})+i\hbar(\epsilon_2 a_2^\dagger e^{-2i\omega t}-\epsilon_2^* a_2 e^{2i\omega t})+(a_1\Gamma_1^\dagger+a_1^\dagger\Gamma_1)+(a_2\Gamma_2^\dagger+a_2^\dagger\Gamma_2)\,, \tag{7}$$

where a_1 and a_2 are the boson operators for the idler and pump modes, respectively, κ is the coupling for the interaction and is chosen real. Γ_1 and Γ_2 are heat-bath operators which represent cavity losses and ϵ_1 and ϵ_2 are proportional to coherent driving-field amplitudes. This model includes depletion of the pump mode and has been analyzed in detail in Ref. 3.

The following Fokker-Planck equation for the complex P distribution[4] $P(\alpha,\alpha^\dagger,t)$ of the idler mode may be derived in the limit where the pump mode has high cavity losses and may be eliminated adiabatically:

$$\frac{\partial P(\alpha,\alpha^\dagger,t)}{\partial t}=\left\{\frac{\partial}{\partial\alpha}\left[\gamma_1\alpha-\epsilon_1-\frac{\kappa}{\gamma_2}\left[\epsilon_2-\frac{\kappa}{2}\alpha^2\right]\alpha^\dagger\right]+\frac{\partial}{\partial\alpha^\dagger}\left[\gamma_1\alpha^\dagger-\epsilon_1^*-\frac{\kappa}{\gamma_2}\left[\epsilon_2^*-\frac{\kappa}{2}\alpha^{\dagger 2}\right]\alpha\right]+\frac{1}{2}\left[\frac{\partial^2}{\partial\alpha^2}\frac{\kappa}{\gamma_2}\left[\epsilon_2-\frac{\kappa}{2}\alpha^2\right]+\frac{\partial^2}{\partial\alpha^{\dagger 2}}\frac{\kappa}{\gamma_2}\left[\epsilon_2^*-\frac{\kappa}{2}\alpha^{\dagger 2}\right]\right]\right\}P(\alpha,\alpha^\dagger,t)\,. \tag{8}$$

γ_1 and γ_2 are the damping constants for the idler and pump modes, respectively.

Linearizing Eq. (8) about the deterministic steady-state α_0 we find the approximate steady-state solution

$$P(\alpha,\alpha^\dagger)\simeq N\exp\{2(\alpha-\alpha_0)(\alpha^\dagger-\alpha_0^*)-a[(\alpha-\alpha_0)^2+(\alpha^\dagger-\alpha_0^*)^2]\}\,, \tag{9}$$

where

$$a=\frac{\gamma_1\gamma_2+\kappa^2|\alpha_0|^2}{\kappa\left[\epsilon_2-\frac{\kappa}{2}|\alpha_0|^2\right]}\,.$$

This distribution has the same form as the complex P representation for a pure squeezed state[5] with the squeezing given by the parameter a.

The variances in X_1 and X_2 follow directly,

$$\Delta X_1^2=\frac{1}{4}\left[1+\frac{1}{a-1}\right],\qquad \Delta X_2^2=\frac{1}{4}\left[1-\frac{1}{a+1}\right]. \tag{10}$$

We shall hold the pump field amplitude ϵ_2 fixed at

$$\epsilon_2=\epsilon_2^c=\frac{\gamma_1\gamma_2}{\kappa}\,,$$

the threshold for parametric oscillation, and vary the idler amplitude ϵ_1. Initially the quadrature X_2 is squeezed. As ϵ_1 is increased the sign of a will change when

$$\epsilon_1=\hat{\epsilon}_1=\left[\frac{2\gamma_1^3\gamma_2}{\kappa^2}\right]^{1/2}.$$

At this point the variances $\Delta X_1^2=\Delta X_2^2=\frac{1}{4}$. For ϵ_1 greater than $\hat{\epsilon}_1$ the quadrature X_2 is squeezed.

The results of the linearized analysis are confirmed by an exact analysis. The exact steady-state solution to Eq. (9) together with all the moments of the distribution are given in Ref. 3. In Fig. 2 we

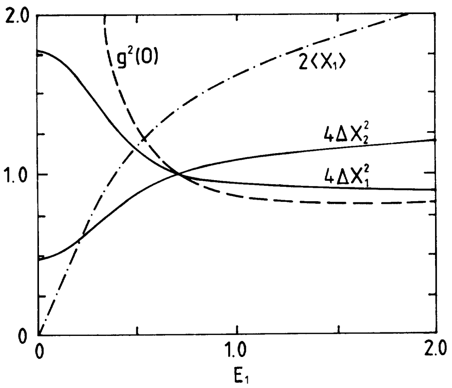

FIG. 2. $\langle X_1 \rangle$, ΔX_1^2, ΔX_2^2, and $g^2(0)$ vs ϵ_1 for subsecond harmonic generation $\epsilon_2 = \epsilon_2^c = 5.0$, $\gamma_1 = 1.0$, $\gamma_2 = 100$, $\kappa = 20.0$.

plot the results for ΔX_1^2, ΔX_2^2, and $g^{(2)}(0)$ with ϵ_2 held fixed equal to ϵ_2^c. For the parameters chosen here $\langle \alpha \rangle$ is real. At $\epsilon_1 = 0$ the component carrying the excitation (X_1) is not squeezed, X_2, however, is squeezed. The photon-correlation function $g^{(2)}(0)$ is greater than unity reflecting the enhanced amplitude fluctuations. As ϵ_1 is increased beyond $\hat{\epsilon}_1$ the squeezing changes from the imaginary to the real components of the complex amplitude and $g^{(2)}0$ becomes less than unity (photon antibunching). Thus the output of the degenerate parametric oscillator shows behavior qualitatively similar to the ideal squeezed state discussed in Sec. II.

The linearized analysis predicts that the change in the sign of the squeeze parameter should occur at $\epsilon_1 = 1/\sqrt{2}$ for the parameters chosen here and that at this point $\Delta X_1^2 = \Delta X_2^2 = \frac{1}{4}$. These predictions agree well with the exact result.

We also see that the quadrature carrying the squeezing, is amplified when $\epsilon_1 > 1/\sqrt{2}$. This contrasts with the result obtained for pure subharmonic generation (1) where the squeezed quadrature amplitude was not amplified. The addition of the extra field thus permits squeezing in a quadrature with significant amplitude.

ACKNOWLEDGMENTS

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