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Electron capture for fast highly charged ions in gas targets: An empirical scaling rule

A. S. Schlachter, J. W. Stearns, W. G. Graham,[*]
K. H. Berkner, R. V. Pyle, and J. A. Tanis[†]
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

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A universal empirical scaling rule for electron-capture cross sections is reported for fast, highly charged ions in atomic and molecular targets. Projectiles range in energy from 0.3 to 8.5 MeV/amu, with charge states as high as 59+. This rule permits prediction of electron-capture cross sections for a wide variety of projectile-target combinations.

Charge-transfer cross sections for fast, highly stripped projectiles in gas targets are useful for testing theoretical calculations as well as for a variety of applications including accelerator design, beam transport, and fusion plasmas. Given the vast number of combinations of projectile species, energy, and charge state, and the large number of possible gas and vapor targets, scaling rules are useful, if they exist, for predicting the magnitude of unmeasured cross sections. We have empirically found a universal scaling rule which permits prediction of electron-capture cross sections for a wide range of fast, highly charged projectiles in gas targets. This result is, to our best knowledge, the first *empirical* scaling rule for electron capture which includes the target dependence.

The present work is a generalization of our previous scaling rule[1] for electron capture by fast highly charged iron ions in an H_2 target, in which we found that the electron-capture cross section σ can be described by

$$\sigma = 1.2\times 10^{-8} q^{3.15} E^{-4.48}\ \mathrm{cm}^2 \tag{1}$$

for projectile energies greater than 275 keV/amu, where E is the energy of the iron projectile in keV/amu and q is the projectile charge state. Alonso and Gould[2] have found, for fast lead and xenon ions in N_2, a q dependence of approximately 2.9–3.3; in addition, these authors have found a velocity dependence of about $v^{-5.8}$ or $E^{-2.9}$, based on data over a limited range of velocities. Knudsen, Haugen, and Hvelplund[3] found that electron-capture cross sections scale as σ/q and $E/q^{4/7}$ in a given target, and, based on the Lenz-Jensen atomic model, cross sections could be described in reduced coordinates $\sigma Z_2^{2/3}/q$ and $E/(q^{4/7} Z_2^{16/21})$, where Z_2 is the target atomic number. Ryufuku[4] has found that cross sections in atomic hydrogen could be scaled in the reduced coordinates $\sigma/q^{1.07}$ and $E/q^{0.35}$, while Janev, Presnyakov, and Shevelko[5] found $E/q^{0.5}$ to be a suitable scaling parameter for electron capture from inner shells of Ar atom targets.

We have taken a generalized empirical approach to scaling electron-capture cross sections for a large variety of projectiles and targets: We chose generalized reduced coordinates

$$\tilde{\sigma} = \sigma Z_2^{c_1}/q^{c_2}\ ,\quad \tilde{E} = E/(Z_2^{c_3} q^{c_4})\ , \tag{2}$$

where σ is the electron-capture cross section, q is the projectile charge state, E is the projectile energy per nucleon, Z_2 is the atomic number of the target, and c_1–c_4 are adjustable scaling parameters. A nonlinear least-squares fitting routine was used to adjust parameters c_1–c_4 to selected electron-capture cross-section data. We began by fitting our experimental cross sections,[1,6] the success of which encouraged us to test the result with other cross-section data available in the literature.[7-15] The best values for the reduced parameters were found to be

$$\tilde{\sigma} = \sigma Z_2^{1.8}/q^{0.5}\ ,\quad \tilde{E} = E/(Z_2^{1.25} q^{0.7})\ , \tag{3}$$

with σ in cm^2 and E in keV/amu. The parameters c_1–c_4 are not uniquely determined; many sets of parameters will fit the data nearly as well, with any one parameter varying by as much as 25%. We do not attach physical significance to the exact value of each parameter, but rather to the fact that all cross-section values reduce in this representation.

We fit an analytic expression of the form

$$\tilde{\sigma} = P_1 \left[\frac{1-\exp(-P_2 \tilde{E}^{P_4})}{P_2 \tilde{E}^{P_4}}\right]\left[\frac{1-\exp(-P_3 \tilde{E}^{P_5})}{P_3 \tilde{E}^{P_5}}\right] \tag{4}$$

to the data to obtain the coefficients c_1–c_4 and to obtain the values of the fitting parameters P_1–P_5. This form was chosen to match the general shape of the scaled data, e.g., to have a high-energy asymptotic form of $\tilde{E}^{-x}$ (x determined by the fit), and to approach a constant value P_1, independent of $\tilde{E}$, for low values of $\tilde{E}$. The other parameters determine the location of the two bends and slopes of the analytic function. The best fit to the data is

$$\tilde{\sigma} = \frac{1.1\times 10^{-8}}{\tilde{E}^{4.8}}[1-\exp(-0.037\tilde{E}^{2.2})] \times [1-\exp(-2.44\times 10^{-5}\tilde{E}^{2.6})] \ . \qquad (5)$$

Cross-section data for a wide variety of targets, from measurements by the present authors,[1,6] are shown in Fig. 1. Figure 2 shows representative data obtained by others.[7-15] All data in both figures are shown in the reduced coordinates of Eq. (3), and the curve in all figures is the representation of Eq. (5); approximately 70% of the data lie within a factor of 2 of the curve. Equation (5) was tested with much more data than are shown in the figures; all data tested fit as well as those shown, subject to several restrictions:

$$10 \leqslant \tilde{E} \ , \qquad (6a)$$

where $\tilde{E}$ is defined in Eq. (3), with energy E in keV/amu; and

$$q \geqslant 3 \ , \qquad (6b)$$

i.e., the reduction method does not scale the data well for singly and doubly charged ions, nor for low reduced energies. We have observed that the charge-state restriction (6b) can be removed if q is replaced by $q+0.4$.

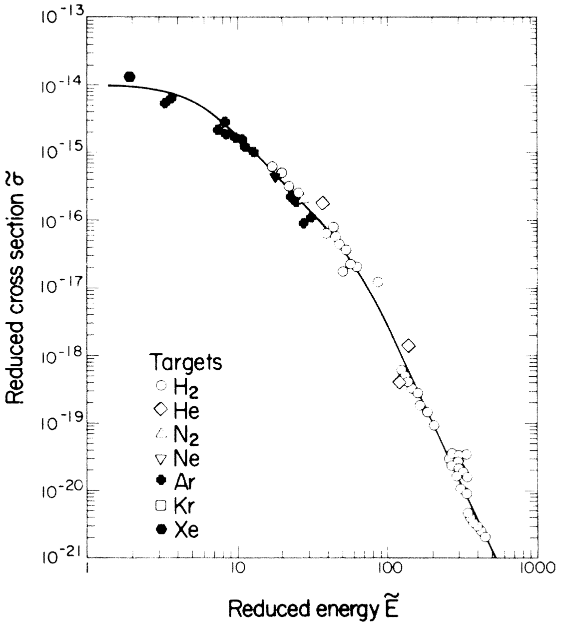

FIG. 1. Reduced plot of single-electron-capture cross sections[1,6] for fast highly charged ions with charge state q incident on gas targets with atomic number Z_2. Cross sections in molecules are divided by 2 and plotted with the atomic Z_2. The line is an empirical fit [Eq. (5)] to the cross sections. The symbols used to represent the targets are shown in the figure. $\tilde{E} = E/(Z_2^{1.25}q^{0.7})$ and $\tilde{\sigma} = \sigma Z_2^{1.8}/q^{0.5}$.

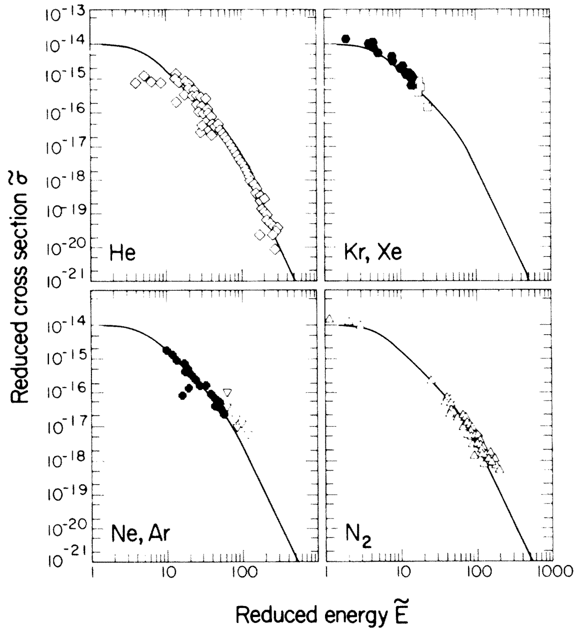

FIG. 2. Same as Fig. 1, with electron-capture cross-section data from Refs. 7–15. The same curve, Eq. (5), is shown in all figures.

The scaling rule has only been tested for

$$\tilde{E} < 1000 \ , \qquad (6c)$$

and its use should be restricted to smaller values since it is possible that the curve will continue to bend with increasing steepness for higher values of $\tilde{E}$.

At high reduced energies, σ [Eqs. (3) and (5)] asymptotically approaches

$$\sigma = 1.1\times 10^{-8}q^{3.9}Z_2^{4.2}/E^{4.8} \ . \qquad (7)$$

The energy and charge-state dependences are similar to, but somewhat greater than, our previous result [Eq. (1)] for iron ions in an H_2 target. The new feature of this result is the target atomic-number dependence, allowing prediction of electron-capture cross sections for many targets.

Crothers and Todd[16] have shown that semiclassical theory accounts for q^3 scaling in electron capture in atomic hydrogen at intermediate energies, and that semiclassical Oppenheimer-Brinkman-Kramers (OBK), eikonal, continuum intermediate-state, and continuum distorted-wave theories all lead to q^3 scaling. In the limit of high velocities, OBK and more modern theories[17] predict a q^5 dependence. The asymptotic charge-state dependence observed in the present result is $q^{3.9}$.

Eichler and Narumi[18] have compared the Born approximation with the classical-trajectory eikonal approximation: the second Born approximation gives rise to a v^{-11} term ($E^{-5.5}$) which will dominate the first Born term, which varies as v^{-12} (E^{-6}), while the classical-trajectory eikonal approximation only predicts the v^{-12} (E^{-6}) velocity dependence. We observe a v^{-9} dependence on velocity for large velocities, rather than the theoretically expected v^{-11} or v^{-12}.

It seems likely that, were data to exist for much higher reduced energies, the asymptotic behavior we observe [Eq. (7)] would have to be modified. The q dependence may approach q^5 and the velocity dependence v^{-12}.

We note that electron-capture cross sections in H_2 were treated here by dividing the measured cross section by 2 and then using $Z_2 = 1$. Knudsen, Haugen, and Hvelplund[19] find a ratio of about 3.8 for the electron-capture cross section for fast highly charged projectiles in H_2 and H targets. The factor of 2 we used in our analysis does not imply a constant ratio for electron-capture cross sections in H_2 and H targets. Measured cross sections for N_2 in the present work were also compared with the empirical scaling rule by dividing the cross section by 2 and then using $Z_2 = 7$.

In summary, an empirical scaling rule has been found for single-electron capture by fast highly charged projectiles in gas targets, which can be used to predict cross sections for many projectile-target systems in the reduced energy range $10 < \tilde{E} < 1000$. To our knowledge, this is the first empirically determined target Z_2 dependence which has been included in a scaling rule for electron capture. We also find projectile charge state, projectile energy, and target atomic-number dependences for electron-capture cross sections in the limit of high reduced energy: $q^{3.9}$, $E^{-4.8}$, and $Z_2^{4.2}$.

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[*] Present address: The New University of Ulster, Coleraine, BT52 1SA, Northern Ireland.

[†] Present address: Western Michigan University, Kalamazoo, MI 49008.

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