PHYSICAL REVIEW A VOLUME 27, NUMBER 5 MAY 1983

Relativistic quantum states of a particle in an electromagnetic plane wave and a homogeneous magnetic field

J. Bergou
Central Research Institute for Physics, Post Office Box 49, H-1525 Budapest, Hungary

F. Ehlotzky
Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria
(Received 19 January 1983)

The exact solutions of the Klein-Gordon and Dirac equations are found by purely algebraic procedures for a charged particle embedded simultaneously in a plane-wave radiation field and in a uniform magnetic field which is directed parallel to the direction of propagation of the plane wave. Two cases of the solutions are considered: (i) the radiation field is a classical external plane wave and (ii) it is a quantized field. The connection with less explicit or more specific solutions of this problem known previously is established and possible applications are discussed.

I. INTRODUCTION

Magnetic and laser fields are frequently used in the study of fundamental processes. These processes include, in particular, different types of scattering of charged particles in the presence of strong radiation and/or magnetic fields. The recent interest in the theoretical as well as experimental studies of charged-particle scattering has been motivated by the rapid developments in the production of strong radiation fields by high-power lasers and of strong magnetic fields for thermonuclear confinement. Moreover, there are interesting applications to astrophysical problems.

For the theoretical description of the basic phenomena there essentially exist two approaches which have been widely used in the past. For not-too-strong fields, the initial and final states of the scattering process are considered to be free-particle states, and the influence of the fields is taken into account by perturbation methods. For strong fields a different approach has been found more appropriate. The initial and final states are taken as those of a charged particle embedded in the strong fields (i.e., the exact quantum states of a charged particle interacting with these fields). However, with a few exceptions, this method was restricted to applications in the nonrelativistic domain mainly due to the complexity of the existing relativistic solutions. Therefore the main aim of our paper is to present the exact relativistic solutions in a relatively simple algebraic form particularly suited for practical calculations.

The study of exact solutions of relativistic wave equations in the presence of external fields started many years ago. The stationary states of an electron in a homogeneous magnetic field were obtained by Rabi.[1] This solution was rederived later and discussed in detail by Johnson and Lippmann.[2] A particular feature of these states was observed quite recently by Bloch.[3] The solution for an electron in a plane electromagnetic wave was first given by Volkov[4] and, since then, it has been rederived several times in connection with different applications (bremsstrahlung, Compton scattering, etc.). The combined action of homogeneous magnetic and plane-wave radiation fields has been investigated by Redmond[5] for the particular case in which the magnetic field is directed parallel to the direction of propagation of the plane wave (so-called Redmond configuration). Seely[6] has derived the corresponding nonrelativistic solution together with an important application to plasma heating by inverse bremsstrahlung. The difficulties with the solutions presented in Refs. 5 and 6 rest in the fact that the expressions for the wave functions are given in a form which is not very transparent for their physical interpretation. Moreover, for the spinor part of the Redmond solution a highly implicit form was chosen. We found it convenient to put the solution of Ref. 6 into a slightly different, purely algebraic form.[7] This facilitated the calculation of the nonrelativistic bremsstrahlung cross section.[8]

In order to achieve similar simplifications for the relativistic theory of induced and inverse bremsstrahlung and other related phenomena, we shall in the following outline derivations of the exact solutions of the Klein-Gordon and Dirac equations containing a plane-wave radiation field and a uniform magnetic field, choosing the Redmond configuration for convenience. These solutions will be derived in a purely algebraic manner, as a result of which they can be written in a compact and transparent form.

We first investigate the case in which both fields are classical. In Sec. II we present the solution for the Klein-Gordon equation. It will be shown how the problem can be put into a form which is identical with the nonrelativistic problem, the solution of which has been presented in detail in Ref. 7. In Sec. III the corresponding solution for the Dirac equation is dealt with. By analogy with the Volkov solution[4] we introduce a transformation which reduces the problem to the previous Klein-Gordon equation plus the problem for a free bispinor. In Sec. IV we briefly discuss how the foregoing results can be generalized to the case in which the radiation field is a quantized plane wave. In Sec. V we summarize the main results, discuss their connections with previous calculations as well as their physical interpretation, and also point out some possible applications.

II. KLEIN-GORDON EQUATION WITH EXTERNAL FIELDS

We consider a particle of charge e and mass m which is interacting simultaneously with a magnetic and a laser field. The fields are accounted for in the external field approximation, and we therefore describe them by classical electromagnetic potentials in the Coulomb gauge. The magnetic field $\vec{B}=(0,0,B)$ is homogeneous and constant and is directed along the z axis, which is also the direction of propagation of the plane-wave radiation field. The latter is consequently polarized in the x-y plane.

The Klein-Gordon equation for the charged particle embedded in the two fields reads

$$\{(i\partial-\epsilon A)^2-\kappa^2\}\psi_{\rm KG}=0 \ . \tag{2.1}$$

Here $i\partial=\{i\partial_\mu\}=\{i\partial/\partial x^\mu\}$; $\epsilon=e/\hbar c$ and $\kappa=mc/\hbar$. The scalar product of two four-vectors a and b is defined as $a\cdot b=a_\mu b^\mu=g_{\mu\nu}a^\nu b^\mu$. The metric tensor is chosen to have the metric $1,-1,-1,-1$, and all the other conventions also agree with those of Bjorken and Drell.[9] The vector potential A is composed of two parts,

$$A(x)=A^R(u)+A^B(\vec{x}_\perp) \ , \tag{2.2a}$$

where A^R describes the radiation field

$$A^R(u)=(0,a_1(u),a_2(u),0)=(0,\vec{A}(u)) \ , \tag{2.2b}$$

and A^B is the vector potential of the magnetic field

$$A^B(\vec{x}_\perp)=(0,Bx^2/2,Bx^1/2,0)\equiv(0,\vec{A}(\vec{x}_\perp)) \ . \tag{2.2c}$$

It is convenient at this stage to introduce lightlike coordinates[10] u,v instead of x^0 and x^3, and we introduce the following definitions:

$$u=(x^0-x^3)/\sqrt{2},\quad v=(x^0+x^3)/\sqrt{2} \ . \tag{2.3}$$

Then we rewrite (2.1) in lightlike variables

$$\{2(i\partial_u)(i\partial_v)-[\hat{p}_\perp-\epsilon\vec{A}(u)-\epsilon\vec{A}(\vec{x}_\perp)]^2-\kappa^2\}\psi_{\rm KG}=0 \ ,$$
$$\hat{p}_\perp=(-i\partial_x,-i\partial_y,0) \ . \tag{2.4}$$

Since the Hamiltonian does not contain the variable v explicitly, we can look for solutions in the form $\psi=\exp(-ip_v v)\psi_{p_v}(u,\vec{x}_\perp)$. Furthermore, the term with $-\kappa^2$ can be eliminated if we put $\psi_{p_v}(u,\vec{x}_\perp)=\exp(-ip_u u)\Phi_p(u,\vec{x}_\perp)$ with $2p_up_v=\kappa^2$. Thus p_u is uniquely defined by p_v and κ. The equation for Φ_p then reads as

$$\{i\partial_u-(1/2p_v)[\hat{p}_\perp-\epsilon\vec{A}(u)-\epsilon\vec{A}(x_\perp)]^2\times\Phi_p(u,\vec{x}_\perp)=0 \ . \tag{2.5}$$

If we now make the following replacements: $u\to t$ and $p_v\to m$, then (2.5) is easily recognized to go over into the (nonrelativistic) Schrödinger equation of a particle in a uniform magnetic field and in a radiation field where the latter is taken into account in dipole approximation. The relativistic problem has thus been reduced to the much simpler nonrelativistic problem which has been investigated in all details in Ref. 7. In particular, (2.5) has to be compared with Eq. (1) of Ref. 7, and with the above substitutions both expressions are identical. Thus instead of solving (2.5), we merely need to quote the result. Let us denote the solution of the Schrödinger equation [Eq. (11) of Ref. 7] by $\Phi_s(t,x_\perp)$. Then the solution for $\psi_{\rm KG}$ of Eq. (2.1) reads as

$$\psi_{\rm KG}=e^{-ipx}\Phi_s(t\equiv u,\vec{x}_\perp)_{m=p_v} \ ,$$
$$p=(p_u,p_v,0,0),\quad p^2=\kappa^2 \ . \tag{2.6}$$

For the sake of completeness we write the result for Φ_s,

$$\Phi_s=C_\theta D_\sigma|n_a\rangle|\phi_b\rangle\exp\left[-i\int_0^t[\omega_c(n_a+\tfrac{1}{2})+{\rm Re}(\sigma\alpha)+(\epsilon^2/2m)\vec{A}^2(\tau)]d\tau\right] \ , \tag{2.7}$$

where instead of $(x,\hat{p}_x)$ and $(y,\hat{p}_y)$ the creation and annihilation operators $(a^\dagger,a)$ and $(b^\dagger,b)$ have been introduced by the usual definitions and

$$C_\theta=\exp[i\theta(a^\dagger b+ab^\dagger)],\quad \theta=-\pi/4 \tag{2.8a}$$

is the generator of a Bogoliubov transformation, whereas

$$D_\sigma=\exp(\sigma a^\dagger-\sigma^* a);\dot{\sigma}+i\sigma\omega_c=-i\alpha^* \tag{2.8b}$$

denotes the generator of a displacement transformation. Moreover, $|n_a\rangle$ is a Fock state (number state) of the a oscillator, while $|\phi_b\rangle$ is an arbitrary state of the b oscillator. Finally, $\omega_c=eB/mc$ is the cyclotron frequency and

$$\alpha=\epsilon(\hbar\omega_c/2m)^{1/2}[a_1(u)+ia_2(u)] \ . \tag{2.8c}$$

Thus we have found that the desired solution of the Klein-Gordon equation has a very similar structure as the one of the nonrelativistic Schrödinger equation. We may summarize the above correspondence in the following:

	Relativistic	Nonrelativistic
(i)	The solution has free-particle plane-wave structure in the v direction.	The solution has free-particle plane-wave structure in the v direction.
(ii)	The four-momentum of this free-particle motion is on the mass shell $(2p_u p_v=\kappa^2)$.	The energy-momentum relationship is that of a free-particle motion $(E=p_z^2/2m)$.
(iii)	The solution describes two coupled and displaced harmonic oscillators in the x^1,x^2 plane; the displacement is a function of u only.	The solution describes two coupled and displaced harmonic oscillators in the x,y plane; the displacement is a function of t only.

The analogy between the two cases is thus complete. Moreover, ψ_{KG} is the solution which in the nonrelativistic limit exactly reduces to the solution of the Schrödinger equation.

III. DIRAC EQUATION WITH EXTERNAL FIELDS

The Dirac equation for a charged particle embedded in a plane-wave radiation field and in a homogeneous magnetic field can be written as

$$\{i\partial\!\!\!/-\epsilon A\!\!\!/-\kappa\}\psi=0 \ . \tag{3.1}$$

The γ matrices satisfy the usual anticommutation relations, and we introduced Feynman's dagger notation, i.e., $a\!\!\!/=a\cdot\gamma$. The other notations and conventions are the same as in Sec. II. In particular, A is given by (2.2).

The equation (3.1) can be rewritten in light-cone variables to yield

$$[i\,\partial_u\gamma_v+i\,\partial_v\gamma_u+(\hat{p}_x-\epsilon A_x)e\!\!\!/_1 +(\hat{p}_y-\epsilon A_y)e\!\!\!/_2-\kappa]\psi_D=0 \ ,$$

$$\hat{p}_x=-i\,\partial_x,\quad \hat{p}_y=-i\,\partial_y \ . \tag{3.2}$$

This equation does not explicitly depend on v and therefore possesses solutions in the form $\psi_D=\exp(-ip_v x)\psi_{p_v}(u,\vec{x}_\perp)$. If we recall the expression for the Volkov solution, we can make for the solution of (3.2) the ansatz

$$\psi_{p_v}(u,\vec{x}_\perp)=\{1-[k\!\!\!/ /2(kp)][(\hat{p}_x-\epsilon A_x)e\!\!\!/_1+(\hat{p}_y-\epsilon A_y)e\!\!\!/_2]\}\Phi \ . \tag{3.3}$$

With some elementary algebra we find that the equation satisfied by Φ is

$$(p\!\!\!/-\kappa+\gamma_v\{i\partial_u-(1/2p_v)[(\hat{p}_x-\epsilon A_x)^2+(\hat{p}_y-\epsilon A_y)^2]+(\epsilon B/2p_v)\sigma^3\})\Phi=0 \ . \tag{3.4}$$

This equation has the enormous advantage that Φ can be chosen to be an eigenfunction of the spin component σ^3 along the direction of the magnetic field with the eigenvalues $s=\pm 1$. Then, apart from an additional constant $\epsilon Bs/2p_v$, we realize that the coefficient of γ_v on the left-hand side (lhs) of (3.4) is the Hamiltonian (2.5) of the transformed Klein-Gordon equation in Sec. II. Therefore the solution of (3.4) can be obtained with the further ansatz

$$\Phi=\Phi_{p_v}(u,\vec{x}_\perp)u_s \quad . \tag{3.5}$$

This means we can look for the solution of (3.4) in a product form, where the first term is spin independent and is a solution of the Klein-Gordon equation (2.5), whereas the second term is independent of the variables $(u,\vec{x}_\perp)$ and is thus a constant bispinor which satisfies the following equation:

$$\{\not p+\not k[\epsilon B/2(kp)]s-\kappa\}u_s=0 \quad . \tag{3.6}$$

For solving this equation we define a new four-momentum q with nonvanishing components q_u and q_v by setting

$$p=q-k[\epsilon B/2(kp)]s=q-k[\epsilon B/2(kq)]s \quad , \tag{3.7}$$

since from $k^2=0$ follows $p\cdot k=q\cdot k$. Then (3.6) reads as

$$(\not q-\kappa)u_{q,s}=0 \quad . \tag{3.8}$$

So q is on the mass shell, i.e., $q^2=\kappa^2$. Consequently the four-momentum p is the sum of an on-shell free-particle four-momentum and a lightlike vector which arises from the interaction of the spin with the magnetic field. We should like to mention at this point that the term $eBs/2(kp)$ can be eliminated from (3.4) if we use a slightly different ansatz for Φ, viz.,

$$\Phi=\exp[-iu(eBs/2p_v)]\Phi_p u_s \quad . \tag{3.5$'$}$$

This immediately leads to (3.8), but the physical meaning does not change. We again get an additional four-momentum directed along k.

Reversing our steps we can consequently put the solution of (3.1) into the explicit form

$$\psi_D=\exp[-iu(\epsilon Bs/2p_v)] \times\{1-[\not k/2(kp)](\not p_\perp-\epsilon\not A)\}\psi_{\rm KG}u_{p,s} \quad . \tag{3.9}$$

In other words, by means of the transformation (3.3) and the ansatz (3.5) the solution of the problem has been separated into two problems. The first agrees with the solution of the Klein-Gordon (and by analogy the Schrödinger) equation. The second one is identical with the equation for a constant free-particle bispinor and thus can be easily solved.

IV. KLEIN-GORDON AND DIRAC EQUATIONS WITH A QUANTIZED RADIATION MODE AND A MAGNETIC FIELD

The problem of the particle motion in a quantized radiation field is an immediate generalization of the situation analyzed by Volkov.[4] It has been addressed first by Berson[11] who later on generalized it to the case in which a constant magnetic field is also present. The latter problem has then been reinvestigated by Abakarov and Oleinik[12] with special emphasis on the evaluation of the spectrum and the cyclotron-type resonance contributions to it. On account of the nonalgebraic representation of the absorption and emission operators, however, the expression derived for the wave functions are much too complicated and not sufficiently explicit to permit direct applications to any physical problem. Therefore we considered it necessary to derive these solutions in a relatively simple algebraic manner.

Our starting point is again Eq. (3.1). The magnetic part of the vector potential continues to be given by (2.2c), whereas the radiation term differs from (2.2b). For the sake of simplicity we consider a circularly polarized mode so that

$$A^R=\alpha(eae^{-ikx}+e^*a^\dagger e^{ikx}) \quad . \tag{4.1}$$

a and $a^\dagger$ are the usual absorption and emission operators, obeying $[a,a^\dagger]=1$. e defines a complex vector of polarization with

$$e=(1/\sqrt{2})(e^1-ie^2) \quad , \qquad e^2=e^{*2}=e\cdot k=0, \quad e\cdot e^*=-1 \quad . \tag{4.2}$$

The particular expression for the constant α depends on the choice of the system of units, e.g., in the Gaussian system $\alpha=(2\pi\hbar c^2/\omega V)^{-1/2}$. Given Eq. (3.1) with the above vector potentials, we look for solutions in the form $\psi=\exp[ikx(a^\dagger a+\frac{1}{2})]\Phi$. If we insert this into (3.1) we find the lhs to be independent of u and v. Therefore we take Φ to have the form $\Phi=\exp(-ipx)\Phi_p$ with $p=(p_u,p_v,0,0)$. The structure of the equation satisfied by Φ_p is most easily revealed if we introduce the creation and annihilation operators b_1, $b_1^\dagger$ and b_2, $b_2^\dagger$ instead of x, $\hat{p}_x$ and y, $\hat{p}_y$ by means of the usual definitions for a harmonic oscillator of frequency $\epsilon B/2$. Then Φ_p obeys the equation

$$(\not p-\kappa-\not k(a^\dagger a+\tfrac{1}{2})-\{[ga+i(\epsilon B/2)^{1/2}(b_1+ib_2)]\not e+[ga^\dagger-i(\epsilon B/2)^{1/2}(b_1^\dagger-b_2^\dagger)]\not e^*\})\Phi_p=0 \quad , \tag{4.3}$$

where $g=\epsilon\alpha=(2\pi e^2/\hbar\omega V)^{-1/2}$. Φ_p we now choose to have the form

$$\phi_p=C_\theta^{-1}\Phi_p\ ,\quad C_\theta\equiv\exp[i\theta(b_1^\dagger b_2+b_1 b_2^\dagger)]\ . \tag{4.4}$$

The operator C_θ has the property

$$C_\theta^{-1}b_1C_\theta=b_1\cos\theta+ib_2\sin\theta\ . \tag{4.5}$$

By means of (4.4) and (4.5) with the particular choice $\theta=(3\pi/4)$ we obtain for ϕ_p

$$\{\not p-\kappa-\not k(a^\dagger a+\tfrac{1}{2})-[(ga+\sqrt{\epsilon B}\,b)\not e+(ga^\dagger+\sqrt{\epsilon B}\,b^\dagger)\not e^*]\}\phi_p=0 \tag{4.6}$$

with $b\equiv b_2$. The solution of (4.6) can again be found with the Volkov ansatz

$$\phi_p=\{1+(\not k/2kp)[(ga+\sqrt{\epsilon B}\,)\not e+(ga^\dagger+\sqrt{\epsilon B}\,b^\dagger)\not e^*]\}\chi_p \tag{4.7}$$

and the equation for χ_p reads as

$$\{\not p-\kappa-\not k[\omega_1(a^\dagger a+\tfrac{1}{2})+\omega_2(b^\dagger b+\tfrac{1}{2})+g'(ab^\dagger+a^\dagger b)+B_0\sigma^3]\}\chi_p=0\ , \tag{4.8}$$

where $B_0=(g^2+\epsilon B)/2kp$, $\omega_1=(kp+g^2)/(kp)$, $\omega_2=\epsilon B/(kp)$, and $g'=g\sqrt{\epsilon B}\,/(kp)$. The advantage of the above transformation comes from the fact that all the terms containing the operators a and b have the same common matrix coefficient $\not k$. Furthermore, the expression to be diagonalized is just the same as in the corresponding Klein-Gordon problem.

The expression in (4.8) representing the coefficient of $\not k$ can be easily diagonalized with the help of the transformation

$$\chi_p'=V^{-1}\chi_p,\quad V=\exp[-\phi(ab^\dagger-a^\dagger b)]; \tag{4.9}$$

for if we choose ϕ such that

$$\tan 2\phi=-2g'/(\omega_1-\omega_2)\ , \tag{4.10}$$

then we get from (4.8)

$$\{\not p-\kappa-\not k[\omega_1'(a^\dagger a+\tfrac{1}{2})+\omega_2'(b^\dagger b+\tfrac{1}{2})+B_0\sigma^3]\}\chi_p'=0 \tag{4.11}$$

with $\omega_{1,2}'=(\omega_1+\omega_2)/2\pm[(\omega_1-\omega_2)^2/4+g'^2]^{1/2}$. χ_p' may again be represented as a product of a bispinor and a spin-independent part. The spin-independent part reads as

$$|n_a\rangle\,|n_b\rangle\ , \tag{4.12}$$

where $a^\dagger a\,|n_a\rangle=n_a\,|n_a\rangle$, $b^\dagger b\,|n_b\rangle=n_b\,|n_b\rangle$. The spinor part $u_{p\lambda}$ is determined by

$$(\not p-\kappa-\not k\lambda)u_{p,\lambda_s}=0 \tag{4.13}$$

with $\lambda_s=\omega_1'(n_a+\tfrac{1}{2})+\omega_2'(n_b+\tfrac{1}{2})+B_0 s$. If we again introduce $p-\lambda k=q$ as a new four-vector, then

$$(\not q-\kappa)u_q=0\ , \tag{4.14}$$

i.e., q is on the mass shell and since as before $p\cdot k=q\cdot k$, p can be entirely expressed in terms of q, viz.,

$$p=q+\lambda_{q,s}k\ . \tag{4.15}$$

Finally, repeating the steps which have led to (4.14) in opposite order we may thus write the solution ψ of Eq. (3.1) with the radiation field (4.1) in the form

$$\psi=\exp[-ipx+ikx(a^\dagger a+\tfrac{1}{2})]C_\theta\{1+(\not k/2kp)[(ga+\sqrt{\epsilon B}\,b)\not e+(ga^\dagger+\sqrt{\epsilon B}\,b^\dagger)\not e^*]\}V_\phi\,|n_a\rangle\,|n_b\rangle u_q\ . \tag{4.16}$$

This comparatively simple formula for the wave function together with the expression (4.15) for p allows for a particularly transparent interpretation according to which the four-momentum of the total system is composed of four parts, viz., (i) a free motion of the particle on the mass shell in the v direction ($q^2=\kappa^2$), (ii) a free propagation of the mode (nk), (iii) the interaction of the electromagnetic mode with the motion in the magnetic field $k[\omega_1(n_a+\tfrac{1}{2})+\omega_2(n_b+\tfrac{1}{2})]$, and (iv) the interaction of the particle spin with the magnetic field (kBs).

V. DISCUSSION

In Secs. II and III we have derived the solutions of relativistic wave equations for a charged particle embedded in a plane-wave radiation field and in a homogeneous magnetic field. The solutions were derived by purely algebraic procedures and were presented as very compact algebraic expressions in closed form. In previous investigations these solutions were found in less transparent form, and consequently their application in the investigation of particular physical problems turned out to be very complicated. We expect our solutions to be given in sufficiently tractable form, in order to permit explicit applications, mainly to scattering problems in intense fields.

We have presented, in particular, the solution of the Klein-Gordon equation. By introducing lightlike coordinates the problem has been essentially reduced to the solution of the corresponding nonrelativistic problem, which has recently been derived in its algebraic form in Ref. 7. In the case of the Dirac equation we have introduced a suitably chosen transformation of the wave function which allowed for a separation of the solution into a bispinor and an amplitude function. The equation for the amplitude emerged as the previously treated Klein-Gordon equation while the bispinor satisfied a free-particle equation. In Sec. IV we finally demonstrated that the solution can still be found if the classical radiation field is replaced by a quantized radiation mode. While the solution of the resulting equation is not very useful for practical applications, it permits a particularly simple interpretation of the different factors of the resulting expressions in terms of free-photon momenta and free-particle momenta.

In closing, we mention that the method of solution outlined in Sec. IV can be generalized to the treatment of many radiation modes propagating in the same direction. In forthcoming work the solutions presented here will be applied, e.g., to the study of charged-particle scattering in the presence of intense fields and to the investigation of photon statistics of a free-electron laser.

ACKNOWLEDGMENTS

This work was completed during a short visit of one of the authors (J.B.) at Innsbruck. The partial financial support by the Österreichische Fonds zur Förderung der Wissenschaftlichen Forschung under Contract No. 3852 and by the Exchange Program of the Austrian and Hungarian Academies is gratefully acknowledged.

---

[1]I. I. Rabi, Z. Phys. 49, 507 (1928).

[2]M. H. Johnson and B. A. Lippmann, Phys. Rev. 75, 828 (1949); 77, 702 (1950).

[3]F. Bloch, Phys. Rev. A 25, 102 (1982).

[4]D. M. Volkov, Z. Phys. 94, 250 (1935).

[5]P. J. Redmond, J. Math. Phys. 6, 1163 (1965).

[6]J. F. Seely, Am. J. Phys. 42, 326 (1974), presented the solution of the nonrelativistic problem and applied it to charged-particle scattering in J. F. Seely, Phys. Rev. A 10, 1863 (1974).

[7]J. Bergou and S. Varró, J. Phys. B 15, L179 (1982).

[8]J. Bergou, F. Ehlotzky, and S. Varró, Phys. Rev. A 26, 470 (1982).

[9]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[10]R. A. Neville and F. Rohrlich, Nuovo Cimento 1A, 625 (1971); Phys. Rev. D 3, 1692 (1971).

[11]I. Berson, Zh. Eksp. Teor. Fiz. 56, 1627 (1969) [Sov. Phys.—JETP 29, 871 (1969)], investigated the interaction with a quantized mode. Later on he included a constant magnetic field in the Redmond configuration in I. Berson, J. Acad. Sci. Latv. Sov. Soc. Repub. Ser. Math. Phys. Sci., No. 5, 3 (1969).

[12]D. I. Abakarov and V. P. Oleinik, Theor. Math. Phys. (USSR) 12, 78 (1972), solved the problem of the spectrum for a single mode, and M. V. Federov and A. E. Kazakov, Z. Phys. 261, 191 (1973), generalized it to many modes, all propagating in the same direction.