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Technique for Lamb-shift measurements on high-Z ions produced by a hot plasma

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A new technique is proposed for the measurement of high-Z Lamb shifts using ions created by a hot tokamak plasma. By using the laser resonance technique to measure the $2S_{1/2}$-$2P_{1/2\text{ or }3/2}$ energy splittings, it is possible to obtain a fractional accuracy of 10^{-4}–10^{-3}. This would be a considerable improvement on existing measurements and would allow a clear test of current calculations.

The measurement of the Lamb shift in high-Z hydrogenlike ions tests quantum electrodynamics (QED) at short distances and high-field strengths. These high-Z experiments are sensitive to higher-order corrections to the binding energy and are important for assessing QED calculations[1,2] and interpretation of hydrogen measurements.[3] This paper proposes a new technique for high-Z Lamb-shift measurements which involves laser resonance quenching and x-ray detection of metastable hydrogenic ions produced by a hot plasma. The ability to achieve large metastable formation rates ($\sim 10^{12}$ $\text{cm}^{-3}\,\text{sec}^{-1}$) at moderate ion energies (~ 1 keV) yields relatively high laser-induced count rates. The predicted fractional accuracy (statistical and systematic) utilizing presently available lasers would be 10^{-3}–10^{-4} which is an order-of-magnitude improvement on existing results.

Previous experiments have measured high-Z Lamb shifts of hydrogenic ions up to $Z = 18$ using fast (multi-MeV) ion beams. Most of these measurements have been obtained using the indirect method of Stark quenching of the metastable $2S_{1/2}$ state with large electric fields. The other technique is a direct resonance measurement by using a laser to drive the $2S_{1/2}$-$2P_{1/2,3/2}$ transitions. The highest-Z Lamb-shift measurements and corresponding accuracies obtained with the respective methods are $Z = 18$ (1.6%) (Ref. 4) and $Z = 17$ (0.7%) (Ref. 5) (the best $Z > 3$ measurement has an accuracy of $\sim 0.5\%$, $Z = 8$).[6,7] Improvements in the electric field quenching method appear difficult[4,8] while the laser resonance method appears to be limited by a small signal size due to the limited laser-ion interaction time.[5] The proposed plasma experiment is similar to the above beam laser resonance experiment except that the ions are relatively slow so that interaction time could be increased from v/d (v = ion velocity, d = laser diameter) to τ_{2s} which is an increase of up to $\sim 10^3$. For equivalent metastable formation rates and detection efficiencies, the time required to obtain a similar accuracy would be reduced by $\sim 10^6$.

The $n = 1$ and 2 levels of a hydrogenic ion are shown schematically in Fig. 1. The atomic parameters are like those of hydrogen with the difference being that the Lamb-shift (S) and fine-structure (ΔE) energies $\sim Z^4$, $E_i \sim Z^2$, the lifetimes $\tau_{2p} \sim Z^{-4}$, $\tau_{2s} \sim Z^{-6}$, and the matrix element $\langle 2S|r|2P\rangle \sim Z^{-1}$. These atomic parameters are shown in Fig. 2 as a function of Z. The $2S_{1/2}$ level decays mainly via two-photon ($2E1$) emission ($Z < 42$) while the $2P$ levels decay via a single x-ray photon ($E1$) of energy $E_i >> S$, ΔE. The measurement of the Lamb-shift interval would be similar to hydrogen measurements. A laser is used to drive either the $2S_{1/2}$-$2P_{1/2}$ (S) or $2S_{1/2}$-$2P_{3/2}$ ($\Delta E - S$) transitions which can be detected as a change in ei-

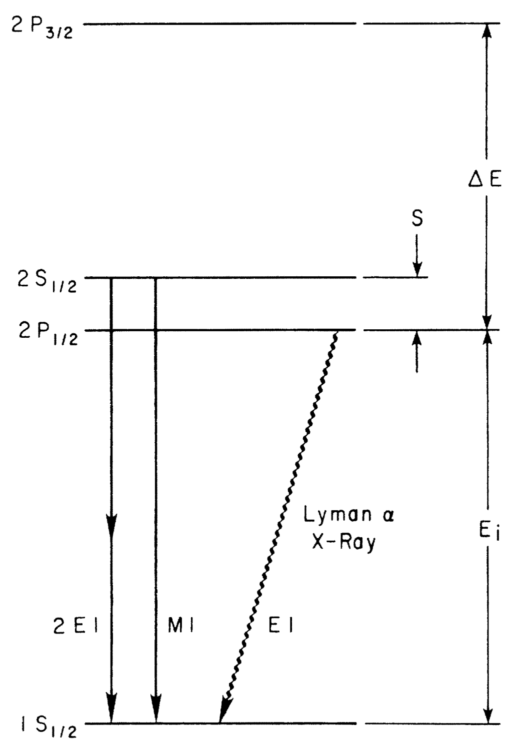

FIG. 1. Energy-level diagram of hydrogenlike ion showing the $n = 1$ and $n = 2$ manifolds.

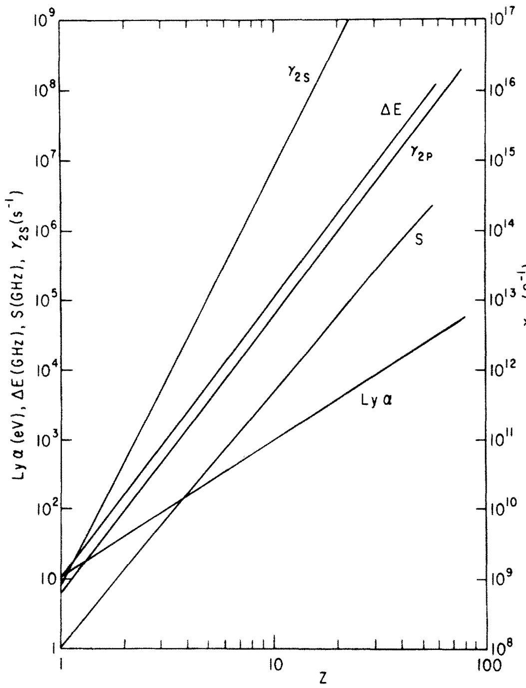

FIG. 2. Scaling of atomic parameters with Z.

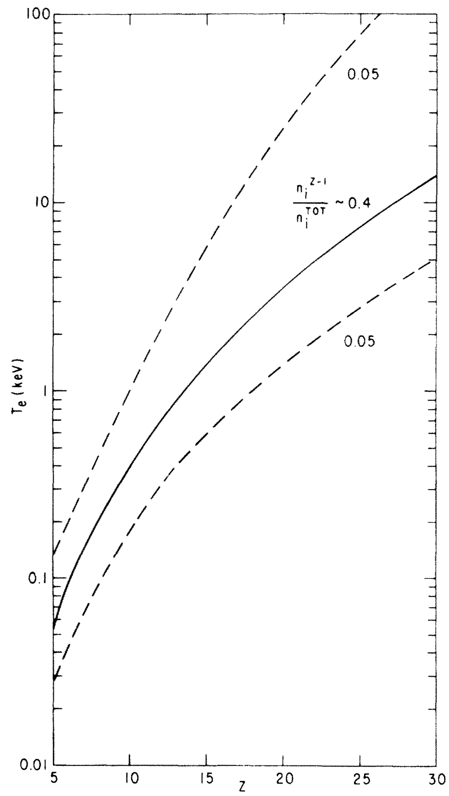

FIG. 3. Relative fraction of hydrogenlike ions as a function of electron temperature and nuclear charge.

ther the $2E1$ or $E1$ emission rates.

Present tokamaks achieve hot plasmas ($T_e \lesssim 3$ keV) and moderate electron densities ($n_e \sim 1-15 \times 10^{13}$ cm^{-3}) which can be maintained for periods up to a few seconds. The energetic electrons will ionize atoms to various charge states. The dominant charge state of an ion will be dependent on the reaction rates for ionization and recombination. In general, hydrogenic ions will dominate when $T_e \sim Z^2 E_H$, where $E_H = 13.6$ eV is the ionization potential of hydrogen. Figure 3 shows the temperature region where a significant fraction (> 5%) of the atoms are ionized to a hydrogenlike charge state.[9] For a large temperature range, the fraction hydrogenic ions can be greater than 5% with $n_{i\,\max}^{Z-1} \sim 0.4 n_i^{\rm tot}$. The typical impurity concentration $n_i^{\rm tot}$ is $\sim 1\%$ of the electron density though pure neon ($Z = 10$) plasmas have been produced.[10]

Excited-state ions will be formed due to electron excitation impact of hydrogenic ions,

$$A^{Z-1}(1S) + e^- \rightarrow A^{Z-1}(nl) + e^- \ , \tag{1}$$

and radiative electron recombination of fully stripped ions,

$$A^{Z} + e^- \rightarrow A^{Z-1}(nl) + h\nu \ . \tag{2}$$

The metastable formation rate R_{2s} can be calculated from the two processes[11,12] with the result

$$R_{2s} = (5.8 \times 10^{-9} Z^{-3} + 1.2 \times 10^{-14} Z) n_e (\mathrm{s}^{-1}) \ . \tag{3}$$

The excitation rate of $2P$ levels is about $4R_{2s}$ and the excitation of higher levels scale as $\sim n^{-3}$. The metastable state will decay radiatively ($2E1$ or $M1$ photon emission) or due to collisions with electrons, protons, or other ions. The largest collisional destruction rate is due to proton- and ion-induced collisions,[13] $2S_{1/2} \rightarrow 2P_{1/2}$,

$$A^{\rm collision} \simeq 3 \times 10^{-4} Z^{-3} n_e (\mathrm{s}^{-1}) \ . \tag{4}$$

For a typical density ($n_e = 10^{13}$ cm^{-3}), the natural decay will be the dominant decay mode for $Z > 8$. Thus a tokamak with even a small fraction of impurities (for $Z = 10$, $n_i/n_e \sim 10^{-2}$, $n_e \sim 10^{13}$ cm^{-3}) can produce a large metastable formation rate ($\sim 2 \times 10^{12}$ cm^{-3} sec^{-1}) of slow (keV) ions which will decay mainly radiatively.

A laser can be used to affect the metastable population by driving electric dipole transitions between the $2S$ and $2P$ levels. For modest laser power, the fractional change F in the $2S$ population can be written as[5]

$$F = \frac{2\pi e^2 I \gamma_{2p} |r_{ab}|^2 \tau}{c\hbar^2 [(\omega - \omega_0)^2 + (\gamma_p/2)^2]} \ , \tag{5}$$

where I is the applied laser intensity, ω the laser angular frequency, ω_0 the transition frequency, and τ is the interaction time which will be either τ_{2s} or d/v. For slow high-Z ions, $\tau = \tau_{2s}$, and the fractional change at resonance becomes

$$F(\omega = \omega_0) = \alpha \frac{3.2 \times 10^7}{Z^{12}} I(\mathrm{W/cm^2}) \ , \tag{6}$$

where $\alpha = 1$ for the $S_{1/2}$-$P_{1/2}$ transition and $\alpha = 2$ for the $S_{1/2}$-$P_{3/2}$ transition.

The signal S is defined as the laser-induced change in the $2S$ or $2P$ emission rate, $S = R_{2s} F \epsilon_{\rm det} d^2 l$, where $\epsilon_{\rm det}$ is the detection efficiency and $d^2 l$ is the effective interaction volume. The noise will be assumed to be shot noise arising from excited-state decay and the bremsstrahlung continuum. The signal-to-noise ratio for 1 sec, S/N, can be written as

$$S/N = \frac{F R_{2s} \epsilon_{\rm det} d^2 l \, n_i^{Z-1}}{[R_{2s} \epsilon_{\rm det} d^2 l (1+C) n_i^{Z-1} + B \delta \epsilon_{\rm det} d^2 L]^{1/2}} , \quad (7)$$

where C is the relative excited-state photon emission from other states such as the $2P$ levels and cascades ($C \sim 4$), B is the bremsstrahlung emission, δ the detector bandwidth, and L an effective plasma diameter. The bremsstrahlung continuum in a tokamak plasma is typically

$$B \sim 10^{-14} \exp(-E_i/kT_e) n_e^2 \text{ photons cm}^{-3} \text{ keV}^{-1} . \quad (8)$$

Assuming that the filter and detector efficiency is 30% with a bandwidth of 15%,[14] $l \sim 10$ cm, $L \sim 60$ cm, and the detector is at $R = 60$ cm with an area comparable to the laser beam, then the signal-to-noise ratio can be approximated by

$$S/N \simeq 7 \alpha P n_e f^{1/2} Z^{-13.5} , \quad (9)$$

where f is the fraction of heavy atoms. One promising experiment is the measurement of $\Delta E - S$ interval for $Z = 10$ (neon) for which it is possible to obtain high laser power (100-W, HF laser) and pure neon plasmas ($n_e \sim 2 \times 10^{13}$). This system would provide a signal-to-noise ratio of 280 in 1 sec. The statistical uncertainity in $\Delta f / f_0$ will depend on S/N and $Q = f / \Delta f_{\rm natural}$, which for data obtained at the steepest slope $f / \Delta f_{\rm natural}$ is

$$\left(\frac{\Delta f}{f_0} \right)_{\rm stat.} \simeq \frac{\sqrt{2}}{Q(S/N)} .$$

The above example would yield $(\Delta f / f_0)_{\rm stat.} \sim 1.2 \times 10^{-3}$ in 1 sec.

The accuracy of the measurement will be dependent on the above statistical uncertainity and the affects of systematic shifts. In general, the sizes of the systematic shifts are small due to the scaling of atomic parameters with Z (e.g., $r_{ab} \sim Z^{-1}$). The following systematic effects were investigated: first- and second-order Doppler shift, $v \times B$, Zeeman, hyperfine splittings, plasma broadening, laser power slope, stray $\vec{E}$ and $\vec{B}$ fields, and plasma-wave interactions. For a typical tokamak plasma, only the effect of plasma broadening and laser power slope will affect the line-center determination greater than $10^{-6} f_0$ for $z \geq 10$. The effect of electron and ion shifts can be calculated using the impact approximation[15] with the resultant fractional shift d/S,

$$\frac{d}{S} \sim \frac{8 \times 10^{-13} n_e}{Z^{19/3}} , \quad (10)$$

which would be 7×10^{-6} for the neon example. An unknown slope in the relative laser power, $\Delta P / P_0$, will create an apparent shift,

$$\frac{d}{f_0} \sim \frac{(\Delta P / P_0)}{8Q} ,$$

which would produce an uncertainity in S of 7×10^{-5} if $\Delta P / P_0$ is known to 0.25%. These systematics would limit the accuracy to $\sim 10^{-4}$. Assuming 12 tokamak pulses per hour, the statistical uncertainty would be at this level in ~ 24 h of data accumulation.

This method of measurement of the Lamb shift of plasma ions would improve existing measurements by about an order of magnitude. This would allow a clear test of current calculations[1,2] of the Lamb shift which differ by 6×10^{-3} at $Z = 10$. It is possible that such a precision high-Z Lamb-shift measurement would allow the possibility of observing new and unexpected phenomena arising from QED interactions in the high-field regime.[16]

ACKNOWLEDGMENTS

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