Nonlinear propagation of kinetic Alfvén waves in a plasma

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(Received 16 April 1982)

A finite-amplitude long- (short-) wavelength kinetic Alfvén wave is found to be modulationally stable (unstable) in a plasma with β (ratio of particle pressure to magnetic pressure) much smaller than electron-to-ion mass ratio.

Recently, Alfvén waves, propagating at an arbitrary angle to an external magnetic field, have received a great deal of attention.[1–3] The dynamics of the shear Alfvén wave cannot be described with the aid of the classical magnetohydrodynamic equations.[1,2] The reason is that the shear Alfvén wave accompanies an electrostatic electric field component, and that the mode is a mixture of the electromagnetic and electrostatic fields.

The linear dispersion relation of the shear Alfvén wave is given by[3]

$$\omega^2 = k_\parallel^2 v_A^2 \left[1 + \tfrac{3}{4} k_\perp^2 \rho_i^2 \frac{T_e}{T_i} [W(y)]^{-1}\right] , \qquad (1)$$

where $y = \omega / k_\parallel v_{te}$, ω is the wave frequency, $\omega << \Omega_i$, Ω_i is the ion gyrofrequency. In Eq. (1), $k_\parallel$ and $k_\perp$ are the wave numbers parallel and perpendicular to the external magnetic field $\hat{z} B_0$, v_A is the Alfvén speed, v_{te} is the electron thermal velocity, T_e (T_i) is the electron (ion) temperature, $W(y)$ is the plasma dispersion function as defined by Ichimaru.[4] Furthermore, in deriving (1), $k_\perp^2 \rho_i^2 << 1$ is assumed, where ρ_i is the ion Larmor radius.

Equation (1) consists of two branches. When the parallel phase velocity of the wave is much smaller than the electron thermal velocity, but is of the order of the Alfvén speed, then we have $m_e/m_i << \beta << 1$ [$\beta = 8\pi n_0 T / B_0^2$ is the ratio of the plasma to magnetic pressure, m_e (m_i) is the electron (ion) mass]. In this case, Eq. (1) yields the linear dispersion relation of the kinetic Alfvén wave[1,2]:

$$\omega^2 = k_\parallel^2 v_A^2 \left[1 + \left(\frac{3}{4} + \frac{T_e}{T_i}\right) k_\perp^2 \rho_i^2\right] . \qquad (2)$$

The kinetic Alfvén wave is employed for the heating of the fusion plasma with $m_e/m_i << \beta << 1$. Nonlinear effects associated with the kinetic Alfvén wave have been investigated in detail.[2,5,6]

On the other hand, for $y >> 1$, the electron inertia dominates over the electron pressure. When $\omega \lesssim k_\parallel v_A$, then we have $\beta << m_e/m_i$, and Eq. (1) reduces to the linear dispersion relation of the kinetic Alfvén wave[3] at a different range of β. We have

$$\omega^2 = \frac{k_\parallel^2 v_A^2}{1 + k_\perp^2 \lambda^2} , \qquad (3)$$

where $\lambda = c/\omega_p$ is the electron inertial length, and ω_p is the electron plasma frequency. The second kind [given by Eq. (3)] of kinetic Alfvén waves play a vital role in space plasmas,[3] where $\beta << m_e/m_i$. Therefore, it is of interest to inquire the stability of a finite-amplitude kinetic Alfvén wave. In this Brief Report, we consider the quasistatic slow plasma response to the latter. The problem of modulational instability is analyzed.

Consider the propagation of a large-amplitude kinetic Alfvén wave in the presence of an external magnetic field $\hat{z} B_0$. We consider a plasma with $\beta << m_e/m_i$. Nonlinear interaction of the kinetic Alfvén wave with the slow plasma motion gives rise to envelope of waves. The dynamics of the latter can be described within the framework of the WKB approximation. That means that the Alfvén wave magnetic field varies on a time scale associated with the slow plasma motion. Thus, following Karpman and Krushkal,[7] we introduce two time and space scales. Accordingly, an evolution equation can be written as

$$i \frac{\partial}{\partial t} B_\perp + \tfrac{1}{2} v_g^1 \frac{\partial^2 B_\perp}{\partial \xi^2} - \frac{\Omega^2}{2\omega_0}(2b - N) B_\perp = 0 , \qquad (4)$$

where we used the relation $\omega_0^2 (1 + k_0^2 \lambda^2) = \Omega^2 \equiv k_{0\parallel}^2 v_{A0}^2$. In Eq. (1),

$$v_g = \partial\omega_0/\partial k_0 = -\omega_0 k_0 \lambda^2 / (1 + k_0^2 \lambda^2) ,$$

$$v_g^1 = \partial v_g / \partial k_0 = -\omega_0 \lambda^2 (1 - 2k_0^2 \lambda^2)/(1 + k_0^2 \lambda^2)^2 ,$$

$$b = \delta B_z / B_0 << 1, \quad N = \delta n / n_0 << 1 ,$$

$$v_{A0}^2 = B_0^2 / 4\pi n_0 m_i ,$$

n_0 is the average plasma density, δn (δB_z) is the slowly varying density (magnetic field variation parallel to $\vec{B}_0$) perturbations due to nonlinear interaction, $B_\perp$ is the perpendicular component of the kinetic Alfvén wave magnetic field. In deriving (4), we let

$\omega=\omega_0+i\partial/\partial t$, $k_\perp=k_0+i\partial/\partial\xi$, $\xi=x-v_g t$, $n=n_0+\delta n$, $B=B_0+\delta B_z$ in Eq. (3), and assumed $\omega_0^{-1}|\partial/\partial t| << 1$, and $k_0^{-1}|\partial/\partial\xi| << 1$. We also retained the lowest-order leading terms.

For the slow plasma motion, we use the notion of "frozen in field lines." Thus

$$N=b \quad . \tag{5}$$

We obtain b from the pressure balance equation

$$\frac{\delta P}{B_0^2}+\frac{\langle H^2\rangle}{8\pi}+\frac{b}{4\pi}=0 \quad , \tag{6}$$

where $H=B_\perp/B_0$, δP is the perturbed pressure, and the angular bracket denotes averaging over the period of the kinetic Alfvén waves. Since $\delta P=2\gamma T\delta n$ ($\gamma=\frac{5}{3}$ for a collisional plasma, and $\gamma=2$ for a collisionless plasma), we readily obtain from (6)

$$b=-\frac{|H|^2}{2(1+\gamma\beta)}\simeq-\frac{|H|^2}{2} \quad . \tag{7}$$

Combining (4), (5), and (7), we get

$$i\frac{\partial H}{\partial t}+P\frac{\partial^2 H}{\partial\xi^2}+Q|H|^2H=0 \quad , \tag{8}$$

where the time and space variables are normalized by ω_0^{-1} and λ, respectively. In Eq. (8), we have defined $Q=\Omega^2/4\omega_0^2$ and

$$P=(2k_0^2\lambda^2-1)/2(1+k_0^2\lambda^2)^2 \quad .$$

Since the second and third terms on the left-hand side of (8) would have opposite sign for $k_0^2\lambda^2<\frac{1}{2}$, a finite-amplitude long-wavelength kinetic Alfvén wave is modulationally stable.[8,9] However, the nonlinearity due to the ponderomotive force produces the enhancement and a spreading of the kinetic Alfvén wave accompanied by a flattening of the density profile.

On the other hand, for $k_0^2\lambda^2>\frac{1}{2}$, Eq. (8) exhibits a modulational instability.[8,9] The maximum growth rate ensues for a modulational wave number $K=K_m=|Q/P|^{1/2}B_{\perp 0}/\lambda B_0$, and is given by $\gamma_m=\omega_0 QB_{\perp 0}^2/B_0^2$, where $B_{\perp 0}$ ($<< B_0$) is the amplitude of the kinetic Alfvén wave pump. Possible final state of this instability can lead to a bell shaped Alfvén wave magnetic field profile, together with a self-consistent inverted bell shaped density perturbations.

In conclusion, we have demonstrated that a finite-amplitude long- (short-) wavelength kinetic Alfvén wave is modulationally stable (unstable). Our results can be useful to the understanding of nonlinear kinetic Alfvén wave propagation in space plasmas where $\beta << m_e/m_i$. Finally, we mention that we have not yet investigated the possibility of three-wave decay interaction. Here, a kinetic Alfvén wave can parametrically decay into a daughter and an ion-acoustic wave. This work is in progress.

This research was performed under the auspices of the Sonderforschungsbereich Plasmaphysik Bochum/Jülich, and the Deutsche Forschungs- und Versuchsanstalt für Luft- und Raumfahrt/Indian Space Research Organization exchange program.

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