Nonlinear asymmetric self-phase modulation and self-steepening of pulses in long optical waveguides

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For nonlinear short-pulse propagation in long optical fibers, the conventional static approximation of the nonlinear terms in the wave equation must be extended to include the derivative of the pulse envelope. As a result, an initially symmetric pulse will develop an asymmetric self-phase modulation and a self-steepening, which ultimately lead to shock formation unless balanced by dispersion. This effect may be responsible for the pulse asymmetries observed in recent experiments.

I. INTRODUCTION

In several recent experiments[1–3] on nonlinear pulse propagation in optical fibers, the output pulse spectrum has been found to be asymmetric. This asymmetry seems to be an inherent property of propagation rather than due to asymmetric input spectra.[2–5]

In an effort to explain the observed asymmetry, the conventional theory of nonlinear self-phase modulation has been extended to include a nonlinear correction term involving the time derivative of the pulse envelope.[4] This correction term becomes important for long propagation paths and could play a significant role for short-pulse propagation in long optical fibers or waveguides. Using a perturbative procedure it was shown in Ref. 4 that indeed the effect of the new nonlinear term was to make the output spectrum of an initially symmetric pulse asymmetric. However, the perturbative approach restricts the applicability of the results to small changes of the initial pulse form.

In the present paper we consider in some more detail the nonlinear Schrödinger equation for the wave envelope including the correction term. In particular, we derive in Sec. III the exact general solution for the case when linear dispersion can be neglected. The obtained solution clearly conveys the asymmetry and the self-phase modulation caused by the nonlinear correction term. The results obtained in Ref. 4 are recovered in the limit of small deviations from the original pulse form. However, the exact solution also demonstrates that the asymmetric deformations of the envelope as found in Ref. 4 only are the first steps towards self-steepening and ultimate shock creation.

In Sec. IV we analyze the effects of wave damping on the self-steepening process. It is found that damping tends to suppress the creation of a shock. The effects of dispersion in balancing the self-steepening is discussed qualitatively in Sec. V, and exact soliton solutions are presented for the full generalized nonlinear Schrödinger equation.

Finally, in Sec. VI we compare the predictions from the self-steepening equation with several recent experimental results involving asymmetric pulse distortion. At least in one of these experiments, the observed pulse asymmetry has the characteristic features of the nonlinearly induced pulse distortion studied in the present work.

II. THE GENERALIZED NONLINEAR SCHRÖDINGER EQUATION

The one-dimensional wave equation for a linearly polarized optical wave pulse is

$$\frac{\partial^2 E}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 D_L}{\partial t^2}=\frac{2n_2 n_0}{c^2}\frac{\partial^2}{\partial t^2}(|E|^2 E)\ , \qquad (1)$$

where E and D are the electric field and the displacement field, respectively. n_2 characterizes the intensity-dependent part of the refractive index n, which is written as

$$n(\omega,|E|^2)=n(\omega)+n_2|E|^2\ . \qquad (2)$$

The electric field is assumed in the form

$$E(x,t)=A(x,t)\exp[i(k_0x-\omega_0t)]\ , \qquad (3)$$

where $A(x,t)$ is a slowly varying amplitude, $k_0=\omega_0 n_0/c$ is the wave number, ω_0 is the frequency, and $n_0\equiv n(\omega_0)$. Transforming to a coordinate system moving with the pulse group velocity and making use of the fact that the envelope $A(x,t)$ is slowly varying, one obtains[4]

$$i\frac{\partial A}{\partial \zeta}+\alpha\frac{\partial^2 A}{\partial \tau^2}+\beta|A|^2A+i\gamma\frac{\partial}{\partial \tau}(|A|^2A)=0 , \quad (4)$$

where $\zeta=x$, $\tau=t-x/v_g$, v_g is the group velocity at $\omega=\omega_0$, and the coefficients α, β, γ are defined by

$$\alpha=-\frac{1}{2}\frac{\partial^2 k}{\partial \omega^2}(\omega_0),\quad \beta=\frac{n_2\omega_0}{c},\quad \gamma=\frac{2n_2}{c} . \quad (5)$$

Equation (4) differs from the conventional form of the nonlinear Schrödinger equation by the presence of the last term proportional to γ, which turns out to be important for short-pulse propagation over long distances.

We proceed the analysis of Eq. (4) by separating $A(\zeta,\tau)$ into real amplitude $\rho(\zeta,\tau)$ and phase $\phi(\zeta,\tau)$ according to $A(\zeta,\tau)=\rho\exp(i\phi)$. The corresponding real and imaginary parts of Eq. (4) yield

$$\begin{aligned}\rho\phi_\zeta&=\alpha\rho_{\tau\tau}-\alpha\rho\phi_\tau^2+\beta\rho^3-\gamma\rho^3\phi_\tau ,\\ \rho_\zeta&=-2\alpha\rho_\tau\phi_\tau-\alpha\rho\phi_{\tau\tau}-3\gamma\rho^2\rho_\tau .\end{aligned} \quad (6)$$

Equation (6) was solved perturbatively in Ref. 4 as a power series in the parameters α and γ. Since the asymmetry was found to be caused by the term proportional to γ, a more detailed analysis was made for the case of vanishing group velocity dispersion, i.e., $\alpha=0$.

In the following sections we will consider Eqs. (4) and (6) in some detail and give exact solutions for several different situations.

III. EXACT SOLUTION FOR THE DISPERSIONLESS CASE

For the case when group velocity dispersion is negligible ($\alpha=0$), the general nonlinear solution of Eq. (4) can be obtained (see also Refs. 5 and 6). When $\alpha=0$ Eq. (4) reduces to the first-order partial differential equations

$$\phi_\zeta+\gamma\rho^2\phi_\tau=\beta\rho^2 , \quad (7a)$$

$$\rho_\zeta+3\gamma\rho^2\rho_\tau=0 . \quad (7b)$$

The amplitude equation (7b) is decoupled from the phase equation (7a), and we obtain directly the general solution for ρ as

$$\rho^2=f(\tau-3\gamma\zeta\rho^2) , \quad (8)$$

where f is an arbitrary function determined by the initial form of the pulse envelope. The solution (8) can be rewritten to yield τ as an explicit function of ρ and ζ, viz.,

$$\tau=3\gamma\zeta\rho^2+g(\rho^2), \quad (9)$$

where g is arbitrary but the inverse of f. We note that a similar solution for ρ has been obtained previously in Ref. 6 for propagation of pulses with intensity-dependent phase velocity.

The phase equation is more intricate. We introduce $y=\rho^2$ and rewrite Eq. (7a) as

$$\frac{1}{y}\phi_\zeta+\gamma\phi_\tau=\beta . \quad (10)$$

From the characteristic system

$$y\,d\zeta=\frac{d\tau}{\gamma}=\frac{d\phi}{\beta} , \quad (11)$$

a first constant of integration is obtained directly as

$$\phi-\frac{\beta}{\gamma}\tau=c_1 , \quad (12)$$

and for the characteristic $\tau=\tau(\zeta)$ we have the equation

$$\frac{d\tau}{d\zeta}=\gamma y(\tau,\zeta) , \quad (13)$$

where $y(\tau,\zeta)$ is determined by Eqs. (8) or (9).

We eliminate τ in favor of y by means of Eq. (9) and consider y as the dependent variable. Equation (13) then becomes

$$\frac{d\zeta}{dy}+\frac{3}{2y}\zeta=-\frac{g'(y)}{2\gamma y} , \quad (14)$$

which is easily integrated to yield the second constant of integration

$$\zeta y^{3/2}+\frac{1}{2\gamma}\int^y y^{1/2}g'(y)dy=c_2 . \quad (15)$$

The general solution of Eq. (10) can now be written as $F(c_1,c_2)=0$, where F is arbitrary. This implies the following solution for ϕ:

$$\phi=\frac{\beta}{\gamma}\tau+h(3\gamma\zeta\rho+\rho g'(\rho^2)) , \quad (16)$$

where h is an arbitrary function determined by the initial phase variation.

Equations (8) and (16) constitute the general solution of Eq. (7). The nonlinearly induced asymmetry in amplitude and frequency is manifested by the terms proportional to ζ. We consider the evolution of the asymmetry in some detail for two cases of initial wave form: a Gaussian pulse and a sech-shaped pulse.

Gaussian pulse. Assuming that

$$\rho^2(0,t)=A^2\exp(-t^2/T^2) , \quad (17)$$

we identify from Eq. (8)

$$f(t)=A^2\exp(-t^2/T^2)$$

and

$$g(\rho^2)=\pm T[\ln(A^2/\rho^2)]^{1/2} ,$$

which implies that the amplitude is given by

$$\rho^2(\zeta,\tau)=A^2\exp[-(\tau-3\gamma\zeta\rho^2)^2T^2] \quad (18)$$

or

$$\frac{\tau}{T}=\frac{3\gamma\zeta A^2}{T}\frac{\rho^2}{A^2}\pm\left[\ln\left[\frac{A^2}{\rho^2}\right]\right]^{1/2} , \quad (19)$$

where plus and minus refer to the trailing and leading edges of the pulse, respectively. Equation (18) can be expanded to first order in ζ to yield

$$\rho^2(\zeta,\tau)\simeq A^2e^{-\tau^2/T^2}\left[1+\frac{6\gamma\zeta\tau A^2}{T^2}e^{-\tau^2/T^2}\right] , \quad (20)$$

which is the expression obtained in Ref. 4 using the perturbative approach.

From the solution given by Eq. (18) [and Eq. (8)] we infer that the low-intensity part of the pulse is essentially unaffected (and still Gaussian), but the high-intensity part of the pulse is tilted towards larger τ, i.e., the peak of the pulse is propagating at a velocity $v_{\rm peak}$, which is less than v_g [$v_{\rm peak}=v_g/(1+3\gamma v_g A^2)$].

This causes a self-steepening of the trailing part of the pulse which ultimately leads to shock creation when ρ_τ becomes infinite. From Eq. (19) we obtain

$$\frac{\partial\tilde\rho}{\partial(\tau/T)}=2\tilde\rho[\ln(1/\tilde\rho)]^{1/2} \times\{2\mu\tilde\rho[\ln(1/\tilde\rho)]^{1/2}\mp 1\}^{-1} , \quad (21)$$

where $\tilde\rho=\rho^2/A^2$ and $\mu=3\gamma\zeta A^2/T$. A shock is formed on the trailing edge of the pulse (upper sign) when

$$2\mu\tilde\rho[\ln(1/\tilde\rho)]^{1/2}-1=0 . \quad (22)$$

Define $F(x)=x[\ln(1/x)]^{1/2}$. We find that

$$\max F(x)=(2e)^{-1/2}$$

for $x\epsilon(0,1)$, which implies that the shock develops when

$$\mu=\mu_{\rm cr}=(e/2)^{1/2} .$$

The corresponding critical distance of propagation, $\zeta_{\rm cr}$ is given by

$$\zeta_{\rm cr}=\frac{\mu_{\rm cr}}{3}\frac{T}{\gamma A^2} . \quad (23)$$

This approximately corresponds to the distance of propagation at which the peak of the pulse has been displaced a distance of the order of the pulse width from the center of the pulse. In Fig. 1 we picture the successive evolution of an initially Gaussian pulse as expressed by Eq. (18).

Sech-*shaped pulse.* For later comparison we also give the corresponding exact solution for an initially sech-shaped pulse, i.e.,

$$\rho^2(0,t)=A^2{\rm sech}^2(t/T) . \quad (24)$$

The general solution for $\zeta>0$ is then

$$\rho^2(\zeta,\tau)=A^2{\rm sech}^2[(\tau-3\gamma\zeta\rho^2)/T] \quad (25)$$

or

$$\frac{\tau}{T}=\frac{3\gamma\zeta A^2}{T}\frac{\rho^2}{A^2}\pm\ln\left[\frac{A}{\rho}+\left[\frac{A^2}{\rho^2}-1\right]^{1/2}\right] . \quad (26)$$

The critical distance of propagation $\zeta_{\rm cr}$ at which the shock develops is again given by Eq. (23) but with $\zeta_{\rm cr}=3\sqrt{3}/4$.

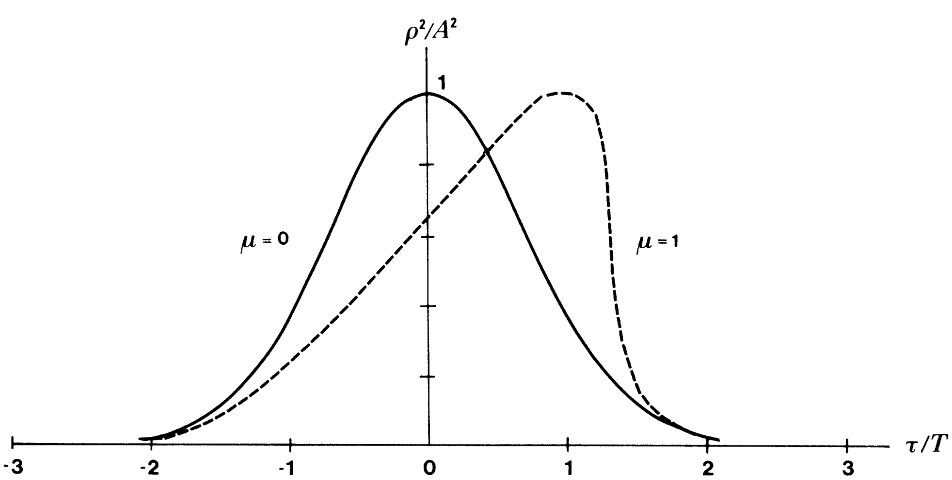

FIG. 1. Self-steepening of a Gaussian pulse.

IV. EXACT SOLUTION INCLUDING DAMPING

Although damping of wave pulses in optical fibers is weak, it will play a role for long-path propagation. If we still neglect dispersion but introduce a linear damping ν into Eq. (4), the amplitude equation (7b) changes to

$$\rho_\zeta+3\gamma\rho^2\rho_\tau+\nu\rho=0 \tag{27}$$

for which the general solution can be found as

$$\rho^2=e^{-2\nu\zeta}f\left[\tau-\frac{3\gamma}{2\nu}\rho^2e^{2\nu\zeta}(1-e^{-2\nu\zeta})\right], \tag{28}$$

where again f is arbitrary and determined by the initial pulse form.

For an initially Gaussian pulse we obtain the solution

$$\rho^2=A^2\exp\left[-2\nu\zeta+\left[\tau-\frac{3\gamma}{2\nu}\rho^2e^{+2\nu\zeta}(1-e^{-2\nu\zeta})\right]^2/T^2\right] \tag{29}$$

from which we infer that the shock will develop at a distance of propagation $\zeta_{\rm cr}$ given by

$$\zeta_{\rm cr}=-\frac{1}{2\nu}\ln\left[1-\frac{\mu_{\rm cr}}{\mu}\right], \tag{30}$$

where $\mu=3\gamma A^2(2\nu T)$ and $\mu_{\rm cr}=(e/2)^{1/2}$ as before. In the limit $\nu\to 0$ we regain the shock distance given by Eq. (23). Thus the effect of the damping is to delay the creation of the shock, and if

$$\nu>\nu_{\rm cr}\equiv 3\gamma A^2/(2\mu_{\rm cr}T),$$

the shock will not develop at all. The condition $\nu>\nu_{\rm cr}$ can be written as

$$\frac{1}{2\nu}<\frac{\mu_{\rm cr}T}{3\gamma A^2}, \tag{31}$$

and the physical interpretation is clear: The damping length is shorter than the nonlinear shock distance.

V. EXACT SOLITON SOLUTIONS FOR THE GENERALIZED NONLINEAR SCHRÖDINGER EQUATION

In neglecting group velocity dispersion, we have treated a simplified situation which clarifies the effects of the nonlinear self-modulation terms. Although dispersive effects are negligible at certain operating frequencies as demonstrated in Ref. 7, in most situations dispersion does play a non-negligible role.

Qualitatively, we understand that dispersion will tend to counteract the nonlinear self-steepening, and a dynamical balance is established, which should result in a more or less asymmetric pulse with a slightly decreased total pulse velocity. As the pulse steepens, the increase spectral width of the pulse makes dispersion more important and finally the dispersive velocity spread Δv_d, which tends to dissipate the shock, balances the nonlinear velocity change, $\Delta v_{\rm NL}$, which steepens the pulse. This provides a qualitative measure of the width $\Delta\tau_s$ of the shock as follows (cf. Ref. 6):

$$\Delta v_d\sim\frac{dv_g}{d\omega}\Delta\omega\sim\Delta v_{\rm NL}\sim 3\gamma A^2v_g^2, \tag{32}$$

implying that

$$\Delta\tau_s\sim\frac{v_g'}{3\gamma A^2v_g^2}. \tag{33}$$

However, since the general solution of the full nonlinear pulse equation [Eq. (6)] does not seem possible to find, a more quantitative analysis of the interplay between dispersion and nonlinearity can only be made analytically for certain special situations. In particular, we will demonstrate in the present section that exact, but particular, solutions of the modified nonlinear Schrödinger equation, [Eq. (4)] can be found in the form of solitary wave pulses, where the dispersive spreading is exactly balanced by nonlinear compressional effects. The solutions turn out to be modifications of the well-known soliton solutions of the conventional nonlinear Schrödinger equation.[8]

In order to obtain soliton solutions of Eq. (6) we look for solutions in the form

$$\begin{aligned}&\rho=\rho(z),\\&\phi=\phi(z,\zeta),\end{aligned} \tag{34}$$

where $z=\tau-M\zeta$, and the ζ dependence of ϕ is restricted by the conditions

$$\begin{aligned}&\phi_\zeta=\text{const}=k,\\&\phi_z' \text{ is independent of } \zeta.\end{aligned} \tag{35}$$

The constants M and k correspond to the inverse soliton velocity shift and wave-number shift, respectively.

Inserting the ansatz (34) and (35) into Eq. (6) we obtain

$$\begin{aligned}&\rho(k-M\phi_z')=\alpha\rho''-\alpha\rho\phi_z'^2+\beta\rho^3-\gamma\rho^3\phi_z',\\&M\rho'=2\alpha\rho'\phi_z'+\alpha\rho\phi_{zz}''+3\gamma\rho^2\rho',\end{aligned} \tag{36}$$

where prime denotes differentiation with respect to

z. After multiplication with ρ, Eq. (36) can be integrated once to yield

$$\tfrac{1}{2}M\rho^2-\alpha\rho^2\phi_z'-\frac{3\gamma}{4}\rho^4=\text{const.} \quad (37)$$

In the present work we consider only the case of a single-humped "bright" soliton solution where $\rho\to 0$ as $z\to\pm\infty$. This implies that the constant in Eq. (37) is equal to zero and we obtain ϕ_z' as an explicit function of ρ, viz.,

$$\phi_z'=\frac{1}{2}\frac{M}{\alpha}-\frac{3\gamma}{4\alpha}\rho^2 . \quad (38)$$

We insert this into Eq. (36) and obtain a second-order equation for ρ;

$$\rho''+\frac{3\gamma^2}{16\alpha^2}\rho^5+\left[\frac{\beta}{\alpha}-\frac{\gamma M}{2\alpha^2}\right]\rho^3-\left[\frac{k}{\alpha}-\frac{M^2}{4\alpha^2}\right]\rho=0 . \quad (39)$$

Equation (39) can be integrated once and put into a form analogous to the equation of motion of a particle in a one-dimensional potential field

$$\tfrac{1}{2}(\rho')^2+\pi(\rho)=0 , \quad (40)$$

where the potential field $\pi(\rho)$ is given by

$$\pi(\rho)=\frac{\gamma^2}{32\alpha^2}\rho^6+\frac{1}{4}\left[\frac{\beta}{\alpha}-\frac{\gamma M}{2\alpha^2}\right]\rho^4 -\frac{1}{2}\left[\frac{k}{\alpha}-\frac{M^2}{4\alpha^2}\right]\rho^2+C . \quad (41)$$

The constant C is determined by the condition that ρ and ρ' vanish as $z\to\pm\infty$, i.e., $C=0$. Since without loss of generality we can assume that the peak of the pulse is located at $z=0$, i.e., $\rho(0)=\rho_0$ and $\rho'(0)=0$, we also have $\pi(\rho_0)=0$, which specifies the wave-number shift k in terms of the soliton peak amplitude as follows:

$$\frac{k}{\alpha}=\frac{M^2}{4\alpha^2}+\frac{1}{2}\left[\frac{\beta}{\alpha}-\frac{\gamma M}{2\alpha^2}\right]\rho_0^2+\frac{\gamma^2\rho_0^4}{16\alpha^2} . \quad (42)$$

For a potential well to exist between $\rho=0$ and $\rho=\rho_0$, the coefficient of ρ^2 in $\pi(\rho)$ must be negative, i.e.,

$$k/\alpha-M^2/(4\alpha^2)>0 .$$

The formal solution of Eq. (40) is obtained as

$$\int_{\rho_0}^{\rho}\frac{d\rho}{[-2\pi(\rho)]^{1/2}}=\pm z , \quad (43)$$

which after some algebraic manipulations is found to be expressable as

$$\rho^2=\frac{\rho_0^2}{2-\nu}\left[\cosh^2(\mu z)+\frac{\nu-1}{2-\nu}\right]^{-1} , \quad (44)$$

where

$$\mu^2=\frac{1}{2}\left[\frac{\beta}{\alpha}-\frac{\gamma M}{2\alpha^2}\right]\rho_0^2+\frac{\gamma^2\rho_0^4}{16\alpha^2} ,$$

$$\nu=\frac{1}{2\mu^2}\left[\frac{\beta}{\alpha}-\frac{\gamma M}{2\alpha^2}\right] . \quad (45)$$

In the limit when $\gamma\to 0$, we obtain $\mu^2=\frac{1}{2}(\beta/\alpha)\rho_0^2$, $\nu=1$, and the soliton solution given by Eq. (44) reduces to the conventional sech solution.

Some features of the obtained soliton solution should be emphasized. From Eq. (38) we conclude that the frequency shift of the soliton ϕ_z' is nonlinearly modulated during the pulse, as opposed to the ordinary soliton solution where it is constant. For the conventional nonlinear Schrödinger equation "bright" and "dark" solitons exist depending on whether $\beta/\alpha>0$ or $\beta/\alpha<0$, respectively.[7] In the present case the condition for the existence of a bright soliton becomes

$$\frac{\beta}{\alpha}-\frac{\gamma M}{2\alpha^2}+\frac{\gamma^2\rho_0^2}{8\alpha^2}>0 , \quad (46)$$

implying that media with $\beta/\alpha<0$ may still support bright solitons provided Eq. (46) is fulfilled. This is reminiscent of the influence of finite fiber diameter on solitons, where transverse waveguide effects have been shown to modify the borderline dispersion between bright and dark soliton solutions.[9]

When this work was completed it was brought to our knowledge[10] that Eq. (4) can be transformed into the so-called derivative nonlinear Schrödinger equation, which determines the evolution of finite amplitude Alfvén waves propagating parallel to a magnetic field in a plasma. In particular, the soliton properties of the corresponding equation have been studied in several works.[11,12] The application of these results to short-pulse propagation in long optical fibers should be an interesting task but must be deferred to a later paper.

VI. COMPARISON WITH EXPERIMENTS

The importance of the nonlinear self-steepening effect in a given experiment can be qualitatively assessed by comparing the characteristic shock distance ζ_{cr} with the length of the fiber L_0. We will discuss this in more detail for each of the experiments of Refs. 1–3.

Reference 1. For the parameters of Ref. 1 we find that ζ_{cr} is more than two orders of magnitude larger than L_0, and consequently, the nonlinearly induced asymmetry should be too weak to be observable. The observed asymmetries and deviations from Gaussian behavior seem to be due rather to "the inherent properties of the mode-locked pulse itself."

Reference 2. For the parameters of Ref. 2 ($L_0 \sim 700$ m, $T \sim 3.5$ ps, $A \sim 500$ SV/cm, and $n_2 \sim 10^{-13}$ esu) we obtain $\zeta_{cr} \sim 8000$ m. This is only one order of magnitude larger than L_0, and a nonlinear asymmetric distortion should be observable, although dispersion does play a non-negligible role in this experiment aimed at studying soliton properties. A quantitative comparison is difficult to make, but asymmetries, especially in the high power pulse spectra of Fig. 2, are indeed observed and commented upon, but no conclusive answer is given. We suggest that the nonlinear effect discussed in the present paper might have played a role in the development of the asymmetries observed in Ref. 2.

Reference 3. For the parameters of Ref. 3 ($L_0 \sim 70$ m, $T \sim 2$ ps, $A \sim 100$ SV/cm, and $n_2 \sim 10^{-13}$ esu) we obtain $\zeta_{cr} \sim 1300$ m. This is again approximately one order of magnitude larger than L_0 and the nonlinear asymmetry should be observable in a carefully performed experiment as that of Ref. 3. As a matter of fact, one of the points emphasized in Ref. 3 is a small but unexplained discrepancy between the experimental results and the numerical solutions of the conventional nonlinear Schrödinger equation. Experimentally, the output pulses exhibit an asymmetry in the form of a slower rising edge than the falling edge.

The possibility of an explanation in terms of asymmetric input pulses seems to be ruled out. ("We have tried many calculations with asymmetric input pulses having different rise and fall times, and with and without an initial frequency chirp, but we cannot calculate this feature of our data.")

A closer look at the experimental results of Ref. 3 (in particular Fig. 4) reveals that the observed asymmetry exhibits the three distinct features characteristic of the nonlinear asymmetric self-steepening, viz., (i) a steepening of the trailing edge, (ii) a flattening of the leading edge, and (iii) a retardation of the peak of the pulse.

Thus although again dispersion plays an important role in the experiment, the qualitative agreement between theoretical predictions and experimental results indicates that the observed pulse asymmetry may be due to the additional nonlinear term in the nonlinear Schrödinger equation arising when the slowly varying envelope approximation has to be amended to account for long-path picosecond-pulse propagation.

Finally, we want to point out the possibility that the presence of dispersion may also enhance the asymmetry in the sense that, e.g., a dispersive spreading of the pulse may tend to further decrease the velocity of the peak. This should be analogous to previous results concerning nonlinear self-steepening processes where the presence of dispersion has been found to significantly shorten the self-steepening distance, see, e.g., Ref. 13.

VII. CONCLUSION

We have demonstrated that nonlinear self-modulation due to finite envelope time variation should play an important role for short-pulse propagation in long optical waveguides. Although the complete solution of the general pulse equation has only been found for the dispersionless case, exact single soliton solutions have also been derived for the general equation and together these results give an indication of the properties of this new nonlinear self-modulation effect.

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[1]R. H. Stolen and C. Lin, Phys. Rev. A 17, 1448 (1978).
[2]L. F. Mollenauer, R. H. Stolen, and J. P. Gordon, Phys. Rev. Lett. 45, 1095 (1980).
[3]H. Nakatsuka, D. Grischkowsky, and A. C. Balant, Phys. Rev. Lett. 47, 910 (1981).
[4]N. Tzoar and M. Jain, Phys. Rev. A 23, 1266 (1981).
[5]D. Anderson and M. Lisak, Opt. Lett. 7, 394 (1982).
[6]F. De Martini, C. H. Townes, T. K. Gustafsson, and P. L. Kelley, Phys. Rev. 164, 312 (1967).
[7]D. M. Bloom, L. F. Mollenauer, C. Lin, D. W. Taylor, and A. M. DelGaudio, Opt. Lett. 4, 297 (1979).
[8]A. Hasegawa and F. Tappert, Appl. Phys. Lett. 23, 142 (1973).
[9]M. Jain and N. Tzoar, Opt. Lett. 3, 202 (1978).
[10]E. Mjölhus, Institute of Mathematical and Physical Sciences, University of Tromsø, P.O. Box 953, N-9001 Tromsø, Norway (private communication).
[11]E. Mjölhus, J. Plasma Phys. 16, 321 (1976).
[12]K. Mio, T. Ogino, K. Minami, and S. Takeda, J. Phys. Soc. Jpn. 41, 265 (1976).
[13]D. Grischkowsky, E. Courtens, and J. A. Armstrong Phys. Rev. Lett. 31, 422 (1973).